

# Study on the Propagation and Conversion of Evanescent Lamb Waves in Discontinuous Plate Structures

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**Abstract:** In conventional Lamb wave based non-destructive testing (NDT), subwavelength imaging cannot be performed for very small damages due to diffraction limits, which may pose safety hazards to engineering structures and large equipment, and even cause major safety accidents, leading to property damage and casualties. However, in evanescent Lamb waves, there is the necessary subwavelength information that can be converted into propagable Lamb waves, which contain subwavelength damage information that can be received to form super-resolution imaging. Therefore, this article investigates the propagation and transformation characteristics of evanescent Lamb waves in a discontinuous plate structure, providing a research approach for further super-resolution imaging, which has certain scientific research and practical application value.

**Keywords:** Non-destructive testing, Finite element, Plate structure, Lamb wave, Evanescent wave.

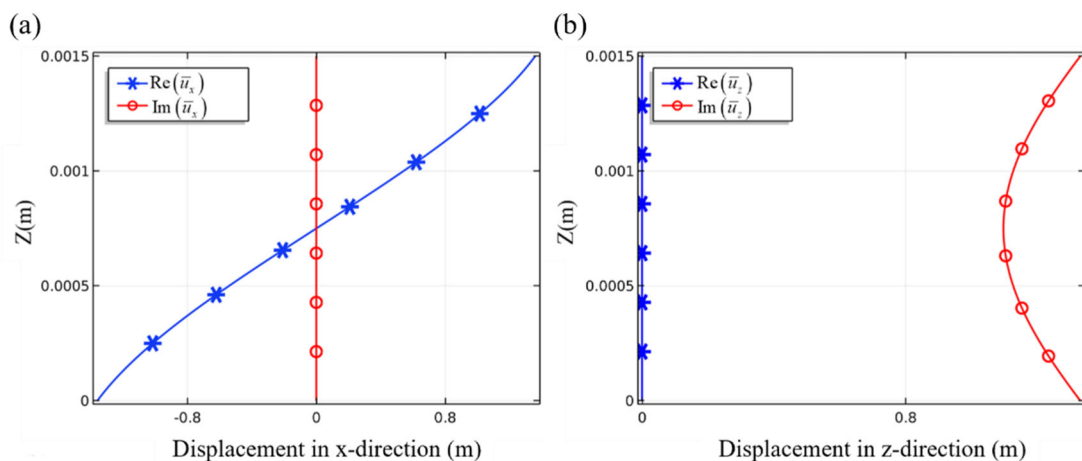
## 1. Introduction

Conventional guided wave imaging technology[1-6] has made significant achievements in suppressing guided wave dispersion, enhancing signal-to-noise ratio of received signals, and improving damage diagnosis sensitivity. However, due to its Born approximation, its essence is to only consider the inverse problem of linear superposition of far-field scattering signals of propagating waves, and cannot reconstruct the sub wavelength features of damage [7]. This article investigates a method for generating single mode Lamb waves by applying a time-harmonic Lamb displacement at the edge of a two-dimensional semi infinite plate. Firstly, the boundary conditions of displacement were extracted, and A1 evanescent Lamb waves were separately excited in the plate structure using this method. Then, the single mode A1 evanescent Lamb is excited in the thick and thin plate structure (discontinuous plate structure), and the propagation characteristics of evanescent Lamb waves in the structure are studied. It is found that there is a conversion phenomenon,

and the evanescent Lamb waves are converted into propagable Lamb waves. Transforming evanescent waves will help to extract sub wavelength damage information, break through diffraction limits[8], reconstruct sub wavelength features of damage, and perform super-resolution imaging, which has certain research significance and value.

## 2. Extraction of Displacement Boundary Conditions

Use finite element frequency domain analysis to verify the generation of Lamb evanescent wave field under edge displacement excitation. In this study, aluminum plates were selected with a thickness of  $h=1.5\text{mm}$  and a density of  $\rho=2710\text{ kg/m}^3$ , Young's modulus  $E=69\text{ GPa}$ , Poisson's ratio  $\nu=0.33$ , grid planning using mapping mode, maximum grid size  $0.1\text{mm}$ , less than  $1/10$  of A0 propagation mode wavelength. We studied the A1 evanescent Lamb wave mode. Figure 1 show the displacement distribution of A1 evanescent Lamb wave mode at the structural boundary at  $f=3.7843\times 10^5\text{ Hz}$ .



**Figure 1.** The displacement distribution of A1 evanescent Lamb wave mode at  $f=3.7843\times 10^5\text{ Hz}$ (a) Displacement in x-direction(b) Displacement in z-direction

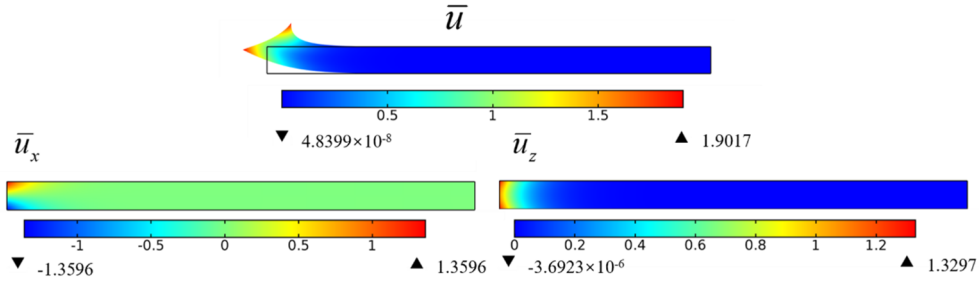
### 3. Excitation of Single Mode Evanescent Lamb Waves

As shown in Figure 1, the displacement is a complex number, so the excitation can be considered as a linear combination of time harmonic excitation with amplitude/and out of phase time harmonic excitation with amplitude. Therefore, the displacement specified at the edge can be written as

$$\bar{u}_x = \text{Re}(\bar{u}_x)e^{-i\omega t} + \text{Im}(\bar{u}_x)e^{j(\pi/2-\omega t)} \quad (1)$$

$$\bar{u}_z = \text{Re}(\bar{u}_z)e^{-i\omega t} + \text{Im}(\bar{u}_z)e^{j(\pi/2-\omega t)} \quad (2)$$

According to Eq. (1) and Eq. (2), Lamb waves can be excited by the specified displacement at the plate boundary. The time harmonic evanescent displacement is applied at the boundary of a two-dimensional semi infinite plate to generate a single mode evanescent Lamb wave.



**Figure 2.** The evanescent Lamb wave displacement field generated by edge displacement excitation at  $f=3.7843 \times 10^5$  Hz(A1 evanescent mode)

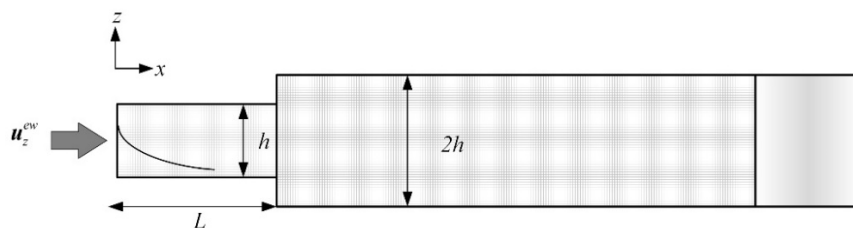
As shown in Figure 2, selecting the A1 evanescent Lamb wave at a single frequency, the displacement field of the A1 evanescent Lamb wave mode generated under the corresponding displacement excitation at the edge can be seen as the total displacement  $\bar{u}$ . From the perspective of deformation, it can be intuitively observed that there is only a small displacement deformation within the near field range, and it quickly decays and disappears. By observing the  $\bar{u}_x$  displacement component, it is shown that it is anti symmetric with respect to the horizontal axis of the plate, while the plate thickness direction  $\bar{u}_z$  is symmetric with respect to the horizontal axis, which is a typical anti symmetric mode. The exponential decay away from the excitation source proves evidence of evanescent Lamb wave fields. Therefore, it can be confirmed through finite element analysis that the specified time harmonic edge displacement excitation at the left boundary can be used to generate a evanescent Lamb wave field, which decays exponentially away from the boundary. This result is consistent with the theoretical formula discussed in the section, that is, there is no propagating Lamb wave present in the mode extension of the excitation field excited by time harmonic displacement.

### 4. The Transformation of Evanescent Lamb Waves in Discontinuous Structures

For the convenience of analysis, the discontinuity of the

structure is simplified to be similar to the connection of two semi infinite plates with height differences, forming a layer difference, which is also known as discontinuity. Here, we first select the case where the centers of the two plates are symmetrical. Using finite element frequency domain analysis, the above method of single mode excitation of evanescent Lamb waves can be used to generate single mode evanescent Lamb waves by applying excitation conditions at the edges. A1 evanescent Lamb wave mode has the smallest virtual wave number and the slowest attenuation among all evanescent Lamb wave modes. In this section, the A1 evanescent Lamb mode is selected to study the conversion of evanescent Lamb waves to propagable Lamb waves.

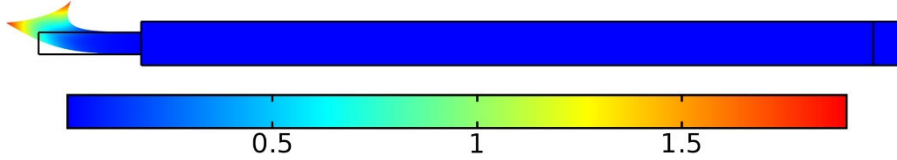
In this study, materials consistent with the above were selected. Select aluminum plate, thickness  $h=1.5\text{mm}$ , density  $\rho=2710\text{ kg/m}^3$ , Young's modulus  $E=69\text{GPa}$ , Poisson's ratio  $\nu=0.33$ , grid planning using mapping mode, with a maximum grid size of  $0.1\text{mm}$ . In order to simulate the interference caused by reflection on the boundary of a semi infinite plate, a perfect matching layer is set at the tail of the thick plate to eliminate reflection. In order to investigate the influence of the length  $L$  of the thin plate on the propagation and transformation of evanescent Lamb waves,  $L$  is set to a variable length. Select the displacement excitation to excite the A1 evanescent Lamb wave, and for convenience, select the A1 evanescent Lamb wave mode excited above ( $f=3.7843 \times 10^5$  Hz).



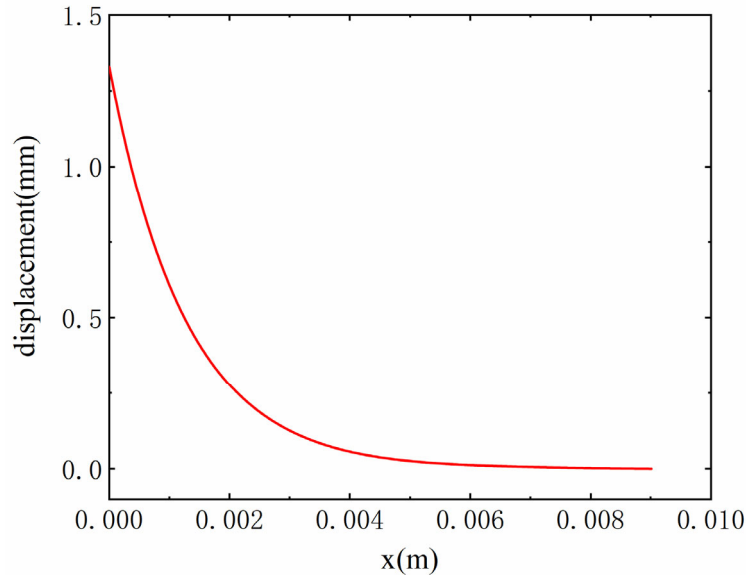
**Figure 3.** Model for studying the propagation and transformation characteristics of evanescent waves in discontinuous boundary structures(horizontally symmetrical)

Firstly, when the length  $L$  of the thin plate is relatively large compared to its thickness, as shown in Figure 4, where  $L=9\text{mm}$ , it can be observed through the displacement distribution diagram that the A1 mode evanescent Lamb wave is excited at the edge of the thin plate in the same situation. However, as in the previous section, evanescent Lamb waves rapidly decay in the near field, and there are no modal waves in the latter half of the thin plate length. It can also be seen that there are no evanescent Lamb waves or other waves in

thicker plates in the far field. Extracting the off plane displacement of the upper surface in the thin plate is shown in Figure 5. It is also found that when reaching the discontinuous boundary, the evanescent Lamb wave has already depleted and cannot reach the interface. So this is a better understanding of the situation where there are no waves in the far field. So it can be concluded that when  $L$  is relatively large, the evanescent Lamb wave decays, cannot reach the interface, and no conversion phenomenon occurs.



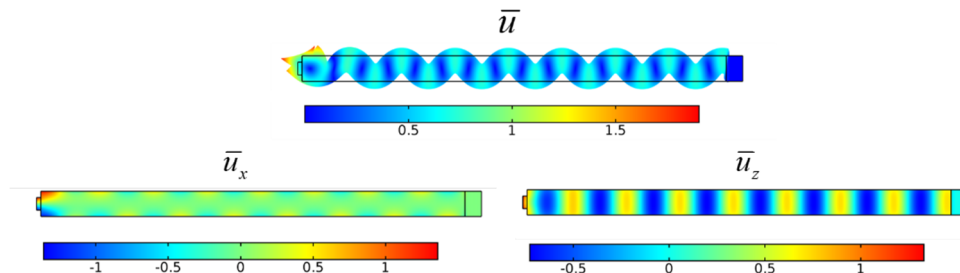
**Figure 4.** Displacement field of A1 evanescent Lamb wave propagation process in discontinuous structures with a thin plate length of 9 mm (horizontally symmetrical)



**Figure 5.** Off plane displacement sampling of thin plates with a length of 9 mm (horizontally symmetrical)

Therefore, in order to facilitate further observation and analysis,  $L$  is reduced to make the fluctuations in thick plates more pronounced. When  $L=0.5\text{mm}$  is selected here, as shown in Figure 6, it is also evident that there are near-field evanescent Lamb waves in the thin plate, reaching discontinuous interfaces. At the same time, it can be observed that there are propagating Lamb waves in thick plates, which are more pronounced than those in thin plates when they are longer. This indicates that the evanescent waves that rapidly decay in the near field of discontinuous boundary structures are transformed into propagating Lamb waves, and a considerable portion of the energy is limited to the near field.

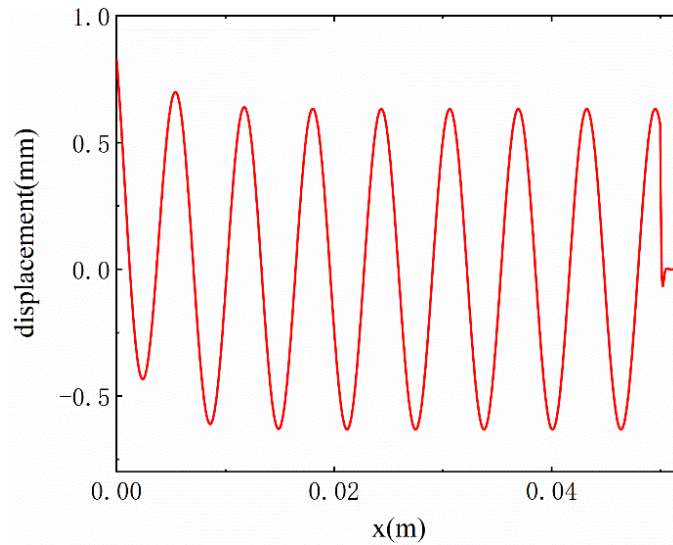
It can be intuitively observed from the total displacement deformation that the wave propagates throughout the structure until it is absorbed by the perfectly matched layer, and its total displacement field is antisymmetric to the horizontal central axis of the plate. By observing its  $\bar{u}_x$  displacement component, it is shown to be anti symmetric with respect to the horizontal central axis of the plate, while the plate thickness direction  $\bar{u}_z$  is symmetric with respect to the horizontal central axis, which is a typical anti symmetric mode.



**Figure 6.** The displacement field of A1 evanescent Lamb wave conversion process in a discontinuous structure with a thin plate length of 0.5 mm (horizontally symmetrical)

The off plane displacement sampling of the thick plate is shown in Figure 7, and its wavelength is calculated. It is found that the A1 evanescent wave in the thin plate at this frequency

is converted into the corresponding A0 mode propagable Lamb wave in the corresponding thick plate.



**Figure 7.** Off plane displacement sampling of thick plates with a thin plate length of 0.5 mm (horizontally symmetrical)

Using the same method, A1 evanescent Lamb waves in different frequency modes achieve consistent results. In summary, when the evanescent Lamb wave reaches the discontinuous boundary before it decays completely, it will be transformed into a propagating Lamb wave

## 5. Conclusion

In this article, by extracting the displacement boundary conditions of single mode Lamb waves and applying them to the structural boundary in a specific combination, single mode Lamb waves can be excited. Selecting A1 evanescent Lamb wave excitation and studying its propagation and conversion characteristics in discontinuous plate structures, we found that when evanescence reaches the discontinuous boundary before attenuation, it can be transformed into a propagable Lamb wave. This will help transform and extract subwavelength information from evanescent Lamb waves, facilitate super-resolution imaging, and reduce errors in damage imaging detection in conventional NDT, as evanescent Lamb waves scatter at small damages and are converted into propagating Lamb waves.

## Acknowledgment

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