Academic Journal of Science and Technology
ISSN: 2771-3032 | Vol. 6, No. 2, 2023

Study of The Application of Inequalities in Mathematics

Mingzhe Sun[1, a], Heng Cao[1]

[1]Weifang NO.1 Middle School, Weifang 261205, China
[a]845606557@qq.com

Abstract: Inequalities have a very important place in the study of mathematics, and their use can be very convenient in solving many practical problems in our daily lives. Inequalities have become an important part of the basic theory of mathematics and have attracted many professionals to study them. The use of inequalities provides an intuitive mathematical model for understanding inequalities in the real world, making it easier for us to learn mathematics and solve many practical problems in our lives. In this paper, we explore the use of mathematical inequalities in everyday learning.

Keywords: Inequalities, Learning strategies, Applied research.

1. Introduction

Inequalities can be used to solve problems in mathematics such as finding the most value, comparing sizes and finding a range of values to prove a conclusion. Common ideas include: using basic inequalities, using function monotonicity, using deflation, etc. In addition to explaining the theory of inequalities to students and raising their awareness of the application of inequalities to solve problems, you should also do a good job of demonstrating the application of inequalities so that they can accumulate relevant application experience. The students should also be able to gain experience in the application of inequalities.

The use of mathematical ideas to solve problems with inequalities begins with the idea of categorical discussion, which allows complex mathematical knowledge to be divided into smaller problems, so that by solving partial problems, students can gain a deeper understanding of the corresponding knowledge. The second is the idea of permutation. Second is the idea of permutation, which is an important way of thinking in mathematics teaching and learning. It is accustomed to using letters instead of formulas and employing simplification to drive problems to greater simplicity. Third is the idea of combining numbers and rows. The idea of combining numbers and rows refers to the analysis and study of mathematical topics using a combination of numbers and rows, transforming problems that exist in graphs into quantitative relationships and making abstract mathematical problems concrete. The fourth is the idea of function, which is based on the question set, the construction of the appropriate function, through the exploration of the properties of the function, the use of the combination of number and shape thinking to transform research. Finally, there is the idea of reduction, which is the simplification of real concrete problems by means of numbers. In the process of transformation, unsolved problems are transformed into simple and easy to solve problems, and the answers to complex problems are then derived.

2. Application of the Properties of Mathematical Inequalities

Inequalities have their own unique properties which can be used to solve inequality problems and prove inequality relations. Our textbook gives us some basic properties of inequalities from which we can derive some potential properties of inequalities. By understanding the basic properties in the textbook and mastering the conditions for their use and the process of proof, we can grasp the close connection between each property and thus use inequalities flexibly to solve problems.

(a) The conditions under which the properties of inequalities hold. When we use the properties of inequalities to answer some inequality questions we must be skilled in the conditions under which inequalities are established or there will be certain errors in the process of application. The arrows used to represent the properties of inequalities should be clearly seen to note whether they are unidirectional or bidirectional, which simply means that each property should be determined to be reversible.

(b) Use the properties of inequalities to prove inequalities. Using the basic properties of inequalities and some of the properties derived through derivation we can prove some inequality problems. The principle we must follow in solving inequality problems is to understand these properties on the basis of skillful and flexible use so that we can accurately answer the questions.

(c) Use the properties of inequalities to find the range. In the actual learning process we often encounter some problems that require a specific range of inequalities then we can use the range of several inequalities combined to solve them. In answering such questions we should note that "the two sides of an inequality can be added together (subtracted)" but this transformation is not an equivalence deformation plus the repeated use of this transformation in the process of answering a question may invariably increase the true range of values and thus make the calculation wrong. By first establishing the equivalence of the whole of the range to be sought and the whole of the known range, and finally by using the "one-time inequality operation to find the range to be sought" method, we will avoid unnecessary errors.

3. Learning Strategies for Mathematical Inequalities

By analysing and summarising the relevant mathematical theories of education and the basic content of inequalities that we usually learn we can draw some corresponding conclusions. Our learning process is in fact a process of communication, understanding and innovation. When we are learning, we cannot just memorise what the teacher says in our heads, but we must constantly analyse and think about the problems and fully open our brains to combine the theoretical knowledge and solutions we have learned with the practical problems, so that we can learn better.

1. Understanding and mastering mathematical thinking

In the process of learning, we can design our own real-life situations to link our knowledge of inequalities. In fact, mathematical knowledge is systematic and coherent. The inequalities we are learning now are actually a supplement and extension of our knowledge in junior secondary school, and a refinement and enhancement of our previous knowledge. Therefore, we should learn to summarise our knowledge of inequalities at a deeper level in order to improve our cognitive ability.

2. focus on the exploration of inequality solutions, improve thinking skills and enhance the connection between knowledge.

Through our study, we can know the properties of inequalities and inequalities are the basis of the knowledge of inequalities and the solution of inequalities requires us to have strong arithmetic skills, so that we can better use and transfer the mathematical knowledge we have learnt and innovate. In general, we should pay attention to the learning of inequalities with parameters. When learning the whole system of inequalities, we must not learn them in isolation, but in the context of mathematics.

3. through the observation of the reasoning and argumentative process, to develop their own abstract thinking skills.

When we study inequalities, we should observe the reasoning and argumentation process. Through the study of basic inequalities, we can appreciate the combination of number and shape and other methods of thought, thus improving our logical and abstract thinking skills and developing a rigorous and standardised learning ability and the ability to analyse and solve problems.

4. Strengthen the connection of knowledge, the actual life of the problem of mathematical abstraction .

Many problems are based on inequalities and are investigated through a combination of different knowledge. This requires us to strengthen the links between different knowledge and abstract real-life problems into certain basic inequality models to improve comprehensive analysis and problem-solving skills.

4. Application of Inequalities in Mathematical Problem Solving

4.1. Used to find the maximum value

In particular, when it comes to finding the maximum value of a single parameter, it is common to separate the parameters and then combine them with known conditions to create a basic inequality, which can be solved using basic inequalities. The use of basic inequalities to find the most value should ensure that it meets the "one positive, two definite, three equal", especially not to be rigidly applied, otherwise you may get the wrong.

Example 1 Knowing that x, y are positive real numbers and satisfy the relation $x + y + 3 = xy$, and that $(x + y)^2$-a(x + y) + 6 ≥ 0 is constant for any x, y, the maximum value of the real number a is ____.

The question does not satisfy the condition of using the basic inequality, at this point you need to use the function properties to break through.

Analysis Since the positive real numbers x, y satisfy the relation

x+y+3=xy, $xy \le \left(\frac{x+y}{2}\right)^2$, so $x+y+3 \le \left(\frac{x+y}{2}\right)^2$

That is, -4 (x+y) -12≥0. Solve for x+y≥6 or x+y≤-2 (round off).

And because $(x+y)^2$-a (x+y) +6≥0 is always true, $a \le x+y+\frac{6}{x+y}$ is always true.

The problem is converted to finding the minimum value of $x+y+\frac{6}{x+y}$.

Let t= x+y (t≥6), then. $\int(t) = t + \frac{6}{t}$.

From the property of the check function, it can be seen that it is monotonically increasing on [6, +∞), then $f(t)_{min} = 7$.

Therefore, the maximum value of a is 7.

4.2. Used to compare sizes

There are two main ways to compare sizes using inequalities: Train of thought 1, using the monotonicity of the learned function to make a direct comparison. Train of thought 2, construct a new function, use the derivative to study its monotonicity in the corresponding interval, and then compare the size.

Example 2 Let $a=3^{\pi}$, $b=\pi^3$, $c=3^3$, then the magnitude relation of a, b, c is ( ).

A.b > a > c B.c > a > b C.a > b > c D.b > c > a

Observation shows that a and c, b and c can be directly compared by using the exponential function power function property. To compare the size of a and b, we need to construct the function and use the derivative knowledge analysis.

Analysis The power function y= x^3 increases monotonically on (0, +), and π > 3, b > c.

And because y=x^3 increases monotone over R, π > 3, so a > c.

Since $a=3^{\pi}$, $b=\pi^3$, take the logarithm of both sides of the equation to get lna=πln3, lnb=3lnπ.

Let f (t) $=\frac{\ln x}{x}$, then, $\int'(x) = \frac{1-\ln x}{x^2}$, then f (x) is monotonically decreasing.

I.e. f (3) > f (π), i.e. $\frac{\ln 3}{3} > \frac{\ln \pi}{\pi 3}$, i.e. πln3 > 3lnπ, lna > lnb

That is, a > b, so a > b > c, so choose C.

4.3. Used to find the value range

Solving the range of parameter values is an important problem in high school mathematics. The situation of related exercises is complex and changeable, and the ideas of solving problems are flexible and diverse, which can sometimes get unexpected results by cutting into the perspective of non-equality.

Example 3 Given x > 0, y > 0, x+y+xy=4, then the range of $\frac{xy+1}{x^2y^2+2xy+17}$ value is.

This problem has a certain skill, it needs to use the knowledge of inequality to construct a new inequality relation, and then combine the properties of function to find its maximum value.

Analysis Because x > 0, y > 0, x+y+xy=4, $x+\frac{1}{2}y+xy \ge 2\sqrt{\frac{1}{2}xy}+xy=\sqrt{2}*\sqrt{xy}+xy$.

That is, $(\sqrt{xy})^2 + \sqrt{2}*\sqrt{xy}-4 \le 0$.

Solve for $0 < \sqrt{xy} \le \sqrt{2}$,That is,$1 < xy+1 \le 3$.

Therefore $\frac{xy+1}{x^2y^2+2xy+17}=\frac{1}{xy+1+\frac{16}{xy+1}}$.

Let t=xy+1, then f（t）=t+$\frac{16}{t}$.

From the property of the check function, we know that it is monotonically decreasing on (1,3], then f (t) $\in[\frac{25}{3}, 17)$.

Then the value range of the original formula is ($\frac{1}{17}$, $\frac{3}{25}$].

5. Conclusion

In this paper, the application of inequality and how to master and use inequality in the ordinary learning process are simply discussed. By learning the basic knowledge of inequalities and training basic skills, we can help improve our logical thinking ability and the ability to analyze and solve practical problems, hoping to provide some help for cultivating and improving our comprehensive ability.

References

[1] Nan Shan. Cauchy Inequality and Ordering Inequality [M]. Shanghai: Shanghai Education Press, 2012.

[2] Peng Zhifeng. Analysis of Mathematical Thinking in the Teaching of Mathematical Inequality in Senior High School [J]. Mathematical Physics and Chemistry of Middle School Students (Graduate Edition),2015,06:22.

[3] [3]Zhang Ping, Xie Xiaoping. [3] Effective Strategies for Teaching Inequality in High school Mathematics [J]. Navigation of the Arts and Sciences (Middle),2015,10:25.

[4] Liang Songlin. Some Suggestions on the Teaching of Mathematical Inequalities in high school [J]. New Curriculum Learning (Basic Education),2010,02:84.

[5] Qin Xiuhong. Question Analysis and Teaching Strategy Research of Senior High School Mathematics Inequality Test [J]. Mathematics Learning and Research,2021(13) :2-3.

[6] LIU Qian. Research on the solution of Inequality Problem in High School Mathematics [J]. Reference for Middle School Mathematics Teaching,2021(09) : 49-50.