Foot Structure Enables Strict Locality in Phonological Processes

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1 Introduction

The metrical foot has a long pedigree as a theoretical device in generative phonology (Liberman & Prince, 1977; Halle & Vergnaud, 1978; Selkirk, 1980; Hammond, 1984; Halle & Vergnaud, 1987; Idsardi, 1992; Hayes, 1995). While the motivations for foot structure are typically studied in terms of stress, this paper provides evidence from the principles of formal language theory (Chomsky, 1956; Hopcroft & Ullman, 1979) for foot-based analyses of non-stress processes. Though use of foot structure in these analyses is not novel (see González (2018) for an overview) this paper contributes a precise characterization of what is at stake in terms of the computation of these processes when foot structure is present versus when it is not. This formal computational analysis indicates that feet have measurable implications for the predicted typology of these patterns. Thus, support is provided for a specific substantive phonological proposal based on the well-defined measures of complexity that formal language theoryoffers.

Specifically, this paper demonstrates that segmental and morphological alternations which appear to rely on an overt parity count of the syllables in the word are rendered fundamentally local in the presence of foot structure. In Capanahua, for example, coda glottal stops are deleted in even-numbered syllables (Loos, 1969). Thinking of such alternations as *functions* that take in an input string and return an output string, the addition of feet ensures that an otherwise long-distance, properly *subsequential* (Mohri, 1997) process is computed as the combination of two local functions instead: creation of feet via an *output stricly local* function (OSL) and calculation of the alternation via an *input strictly local* function (ISL) (Chandlee, 2014; Chandlee & Heinz, 2018). This leads to better typological predictions and highlights that these processes are fundamentally local only if the correct representational assumptions are adopted.

The computational analysis further demonstrates that full subsequential power is too weak of a hypothesis for the class of phonological patterns described here. This is because subsequential functions, which are more expressive than OSL or ISL functions, allow for counting modulo some finite number throughout the word. For example, a function that deletes every even-numbered occurrence of a glottal stop is not a natural phonological generalization and can not be derived input- or output-locally, but it *is* describable with a subsequential function. The hypothesis adopted in this paper is thus that attested patterns are limited to an OSL plus ISL map, which excludes such pathological generalizations. This is because OSL and ISL functions can only count in a way that is formally local. For example, a penultimate stress pattern checks only the two input symbols preceding the right word boundary. An iterative stress pattern places further stresses based on the location of a previous stress in the output. While this kind of non-modulo counting that checks some local span is an inextricable aspect of computation in phonology, the modulo counting allowed by subsequential functions is not. Thus, a non-local theory leads to clearly inaccurate typological predictions.

In sum, the study of computational complexity allows us to identify two formally different types of counting and understand why one is a central part of phonological generalizations and the other leads to a less restrictive typology containing unattested pathological patterns. The explicit analysis of the effect of foot structure in this paper thus clarifies both what feet tell us about the nature of phonological generalizations and why this is important.

2 Complexity in phonology

Formal language theory provides a well-defined measure of complexity in the form of complexity classes. The nested hierarchy of complexity classes divide the space of possible functions based on the expressive

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power of those functions. As applied to natural language, the study of FLT complexity delineates function classes that are relevant to natural language processes, helping to establish testable hypotheses about what a possible linguistic generalization is. One important result from this area of research is that phonology does not exceed the power of the *regular* class (Johnson, 1972; Kaplan & Kay, 1994). Intuitively, this means that phonological functions are computed using a finite amount of memory. Further research has shown that the vast majority of phonological generalizations are *sub*regular, belonging to an even more restrictive class (Rogers et al., 2013; Heinz, 2018). The study of the relation of phonology to subregular complexity classes is an ongoing program that delivers precise, mathematically-defined characterizations of the phonological grammar. I provide intuitive descriptions of several relevant complexity classes here.

2.1 *ISL* One relevant class of functions is the input strictly local (ISL) class (Chandlee, 2014; Chandlee & Heinz, 2018). ISL functions are integral to the study of computation in phonology, including common processes such as deletion and epenthesis. A survey of Mielke (2004)'s P-base indicates that around 95% of patterns in the database fall within the class of ISL functions (Chandlee, 2014).

Intuitively, an ISL function is a function that is calculated based entirely on information in the input. To demonstrate, consider a process of word-final consonant deletion. This could be modeled using an ISL function that checks the two symbols at the end of the input string for a 'C#' sequence and returns an output string where the final consonant is deleted. The following example shows the input string and corresponding output string, as well as a description of the information needed to calculate the pattern. The relevant material is underlined in the input for clarity. Other derivations throughout the paper follow this format as well:

(1) input: CV. CV. CV. CV$\underline{\text{C \#}}$
output: CV. CV. CV. CV
information required: 'C#' input structure

Taking the input string, the function checks for a 'C#' configuration at the end of the word. When such a configuration is present, the final consonant is deleted, as in (1). A consonant followed by another consonant, a vowel, or a syllable boundary will not be deleted. The function is ISL because it only over needs to see the final two input symbols, and thus can be calculated based exclusively on local input information. In the proceeding analyses, it is shown that the presence of foot structure allows phonological generalizations that are otherwise non-local to be encoded locally with an ISL function.

2.2 *OSL* Also relevant are the output strictly local (OSL) functions (Chandlee, 2014; Chandlee & Heinz, 2018). OSL is the output-oriented counterpart of ISL, where functions are calculated based on information in the output string, rather than the input. Processes where application of a rule is iterative, such as spreading of a feature, are in general OSL (Chandlee, 2014; Dolatian et al., 2021). Intuitively this is because the next application of the rule depends on where it last applied, and so this information can only be located in the output. As an example, consider a process that applies stress iteratively to a string of syllables:

(2) input: $\# \sigma\ \sigma\ \sigma\ \sigma\ \sigma$
output: $\acute{\sigma}\ \underline{\sigma\ \acute{\sigma}}\ \underline{\sigma\ \acute{\sigma}}$
information required: 'σ' output structure

The trigger for any iterative application of stress is when an unstressed syllable has just been seen in the *output*. This process is local, depending only on a single element to determine if the process should apply again. Note that since no changes are made to the input string, there is nothing there to condition the iterative pattern – it depends solely on the output and is OSL.

2.3 *Subsequential* A third relevant class of functions is the subsequential class (Mohri, 1997). A subsequential pattern is any pattern that can be derived by a one-way deterministic function. Thus, subsequential functions are more expressive than ISL or OSL functions, as the requirement to adhere to input (or output) locality no longer applies. Subsequential functions represent a important divide in the space of possible functions that includes many phonological patterns and excludes many non-phonological ones (Heinz & Lai, 2013; Chandlee, 2014; Jardine, 2016; Payne, 2017; Luo, 2017; McCollum et al., 2020; Hao & Anderson, 2019; Koser & Jardine, 2020).

The subsequential class is a stronger hypothesis on the power of phonological computation than the rewrite rules of *SPE* (Chomsky & Halle, 1968), which are formally regular modulo certain restrictions (Johnson, 1972; Kaplan & Kay, 1994). It is also more restrictive than classical Optimality Theoretic grammars (Prince & Smolensky, 1993), which can be non-regular (Frank & Satta, 1998; Buccola, 2013). Conversely, subsequential functions are too *weak* of a hypothesis for phonology on their own, permitting a range of patterns that are pathological from a phonological standpoint. This is in part because subsequential functions can carry out an explicit long-distance count modulo some finite number for the entire length of the string, such as an even/odd parity count. As an example, consider a hypothetical pattern that deletes every even-numbered glottal stop that is found in the word:

(3) input: $\underline{\text{\# ʔV. CV}\textbf{ʔ.}\text{ CV. CV. ʔV. CV. CV}\textbf{ʔ}\text{. CV}}$
output: # ʔV. CV . CV. CV. ʔV. CV. CV . CV
information required: even/odd parity value of current ʔ

The function tracks the parity of the glottal stops it encounters, deleting even-numbered instances of the segment. Tracking of parity must span the entire word – note that the entire input form is underlined. In (3), deletion occurs in the second syllable and seventh syllable, demonstrating that it cannot be determined by the parity of the syllable. The process is not ISL, because there is no local input structure that can determine the parity of a given glottal stop. The process is also not OSL for analogous reasons – no local output structure can determine how a particular glottal stop should surface, as it may be separated from a previous glottal stop by an, in principle, unbounded number of intervening segments that do not participate in the process. However, the process *is* subsequential – subsequential functions have access to the kind of long distance memory necessary to determine that the current input symbol is an even-numbered glottal stop, no matter where it occurs in the word. It is, in part, this ability to count modulo some number that makes the subsequential class a weak hypothesis for the expressive power of phonology.

Note that while OSL processes such as iteration of stress as in (2) could be described as a parity counting pattern i.e. stress every odd syllable, the computational properties of the function tell us that this is an incorrect descriptive characterization of the process. Calculation of iterative stress is carried out by a local count up to the previous stress in the output string, rather than a long-distance count of the parity of each syllable over the duration of the word. The difference between these two kinds of counting matters for theories of phonology – local counting as in ISL or OSL is an integral part of phonological computation, whereas I argue that the long-distance parity counting possible with a subsequential function is not. Allowing this kind of explicit counting into phonology leads to a less restrictive typology featuring pathological patterns as in (3), while the limitation to OSL and ISL proposed here excludes such patterns.

It should also be noted that 'subsequential' is not synonymous with 'long-distance'. Not all long-distance processes are subsequential (Jardine, 2016; McCollum et al., 2020). There are also long-distance processes that are formally *less* complex than subsequential, such as those belonging to the *strictly piecewise* class (Rogers et al., 2010; Burness & McMullin, 2020). When the patterns below are analyzed without foot structure, it is a combination of the long-distance *and* parity-counting that require a properly subsequential function to compute.

2.4 *Feet and computation* The presence of feet affects the formal complexity of phonological processes in a non-trivial way. When foot structure is present in the input string, it provides a local reference point from which processes may be calculated. Consider the following input-output pairs from (2), one footed and one unfooted:

(4) a. input: $\#\ \sigma\ \sigma\ \sigma\ \sigma\ \sigma\ \sigma$
output: $\acute{\sigma}\ \underline{\sigma\ \acute{\sigma}}\ \underline{\sigma\ \acute{\sigma}}\ \sigma$
information required: 'σ' output structure

b. input: $\underline{(\ \sigma}\ \sigma\ )\ \underline{(\ \sigma}\ \sigma\ )\ \underline{(\ \sigma}\ \sigma\ )$
output: $\acute{\sigma}\ \sigma\quad \acute{\sigma}\ \sigma\quad \acute{\sigma}\ \sigma$
information required: '(σ' input structure

In (4a), no foot structure is present in the input. This means that iterative placement of stress has no input structure to relies on presence of stress in the output, as demonstrated in §2.2. In (4b), however, foot structure is now present in the input, and so no reference to the output is required. Instead, stress can be placed correctly whenever a '(*σ*' input-sequence is observed. Thus, feet give the input string structure that an ISL function can use to calculate phonological processes locally based on the position of foot boundaries.

Importantly, creation of feet is itself an iterative OSL process, proceeding in the same way as placement of stress in (4a). Consider the following input-output pair:

(5) a. input: $\# \sigma\ \sigma\ \sigma\ \sigma\ \sigma\ \sigma$
output: $\underline{(\ \sigma\sigma\ )}\,(\ \sigma\ \sigma\ \underline{\ )}\,(\ \sigma\ \sigma\ )$
information required: '($\sigma\sigma$' and ')' output structure

Further foot boundaries are inserted depending on the location of the previous foot boundary in the output. When a left foot boundary is two syllables away i.e. the output structure '($\sigma\sigma$' is present, a right foot boundary is placed. When a right foot boundary is seen in the output, a left foot boundary is placed immediately afterwards. So, whether a derivation employs iteratively created feet, or a process such as stress placement occurs iteratively by itself, the total map in each case is subject to output locality.

While iterative stress is output local with or without feet, there are cases where the presence or lack of foot structure has measurable implications for locality and the complexity of the function that computes the process. These types of patterns only apply in syllables of a certain parity, such as deletion of coda-ʔ in even numbered syllables in Capanahua (Loos, 1969; Safir, 1979; González, 2009). Unlike iterative stress, which is placed on every syllable in the alternating count of the pattern, processes like in Capanahua apply in any arbitrary syllable of the correct parity (6th, 8th, 10th, etc.) where the structural description of the rule is met. For example, the declarative marker /taʔ/ surfaces as [ta] in the sixth syllable in the following word:

(6) / ʔú.tʃi.ti.ma.raʔ.**taʔ.**ki / ↦ [ʔú.tʃi.ti.ma.raʔ.**ta.**ki] 'It's probably not a dog'

This is the first occurrence of coda-ʔ deletion in the word. As a consequence of this, such processes are *not* iterative – there is no local structure in the output (or input) that can determine the alternation alone. Thus, computation of a process like coda-ʔ deletion in Capanahua requires a properly subsequential function to compute.

However, if we assume that processes like coda-ʔ apply to a string where foot structure is present, then the properly subsequential pattern is rendered ISL instead. This is because, like in (4b), feet provide an input local reference point that conditions the application of the process without any need for explicit parity counting. Thus, breaking patterns like Capanahua down into an OSL step of foot creation and an ISL step of glottal stop deletion reveals the fundamental locality that the patterns are subject to – a result that is completely obscured when looking at the map as a single holistic step from input to output.

This approach is related to previous work from subregular phonology demonstrating that more complex patterns can be fruitfully broken down into simpler primitives that underlie a wide range of phonological processes (Rogers et al., 2013; Heinz, 2014). It encapsulates a classic idea in phonology expressed succinctly by McCarthy (1988)'s claim that: "the goal of phonology is the construction of a theory in which cross-linguistically common and well-established processes emerge from very simple combinations of the descriptive parameters of the model" (p.84). Studies of formal complexity in linguistics help us evaluate how a preference for combinations of simpler generalizations – rather than a more complex single-step approach – directly affects the predicted typology of phonological patterns. For example, if we do not adopt a stepwise analysis for coda-ʔ deletion in Capanahua, the pattern is properly subsequential. This is an implicit claim that, without any further restrictions, we might expect to find any possible subsequential function in the typology of segmental phenomena. As demonstrated via the pathological pattern in (3), this typological claim is a bad one. Crucially, a pattern that deletes segments of a certain parity cannot be derived by the same breakdown into local functions as proposed here for Capanahua. Consider the following attempt to derive (3) via the application of foot structure:

(7) input: # (ʔV. CV**ʔ.**) (CV. CV.) (ʔV. CV.) (CV**ʔ**. CV)
output: # ʔV. CV . CV. CV. ʔV. CV. CV . CV
information required: even/odd parity value of current ʔ

Deletion occurs in the second and seventh syllables. The process is no longer tied to the parity of the syllable, and so foot structure is no longer helpful. The local input structure it provides does not offer the same computational aid as it does in attested patterns like Capanahua. Deleting every odd/even occurrence of a segment does not seem phonological in an intuitive sense, and the measures of formal complexity provided by formal language theory tell us why – it is beyond the expressive power of the hypothesized breakdown of such processes into simpler pieces that are fundamentally local. While it is also ultimately necessary to state a restriction on combinations of local processes such as OSL and ISL, I argue that, in terms of typological predictions and learnability, they provide a better starting hypothesis for these processes that can be tested and refined as more data is observed and research in this vein progresses.

3 Analyses

In this section, I provide analyses of patterns from four languages that demonstrate how foot structure serves to render a long-distance, parity counting pattern formally local. These patterns that support the presence of foot structure are notable in that they are segmental and morphological alternations, rather than stress, where the use of feet is typically motivated. For an excellent overview of such patterns, see González (2018).

3.1 *Huariapano* In the now-extinct Panoan language Huariapano (Parker, 1994, 1998), a process of coda /h/ epenthesis occurs in an odd-numbered syllable when the following syllable has a voiceless obstruent onset i.e. when the structure ‘V.T’ occurs. Consider the following pairs of underlying and surface forms:

(8)	UR	SR	gloss
	a. / ja.na.pa.kwin /	[jà.na.pah.kwín]	‘I will help’
	b./ βo.no.si.kæj /	[βò.no.síh.kæj]	‘they will take, carry’
	c./ jo.mɯɹ.ra.ka.ti.kæj /	[jò.mɯɹ.ràh.ka.tíh.kæj]	‘they hunted’
	d./ paj.ri.rah.kaj /	[pàj.ri.ráh.kaj]	‘still’

From the data, we observe that epenthesis only occurs when the following syllable has a voiceless obstruent onset. In (8a) for example, the third syllable surfaces with a coda [h] because of the following [k], but no epenthesis occurs on the first syllable, which is followed by a nasal onset. Epenthesis is also restricted to syllables of odd-parity – in (8c), the fourth syllable is followed by the voiceless obstruent [t], but no epenthesis occurs. Note that the process is also independent of stress – it can occur in syllables with main (8c,d), secondary (8c), or no stress (8a).

3.1.1 *Without feet* In the absence of foot structure, h-epenthesis in Huariapano must know the parity of each syllable, starting from the left edge and maintain the count for the duration of the word. This is because, with no other structure to refer to, an explicit parity count is the only method to track where h-epenthesis may or may not apply. In the following diagrams, the portion of the word where the relevant information is tracked is underlined:

(9) a. input # ja. na. pa. kwin.
output ja. na. pah. kwin.

b. input # jo. mɯɹ. ra. ka. ti. kæj
output jo. mɯɹ. rah. ka. tih. kæj
information required: σ parity, presence of ‘V.T’ input structure

Starting from the left edge, the function checks for a voiceless obstruent following the coda of every odd syllable. This occurs in the third syllable of (9a), and in the third and fifth syllable of (9b), leading to surface [pah], [rah], and [tih] for underlying /pa/, /ra/, and /ti/. Though the first syllable is always odd, the following onset is a nasal in both forms, and so h-epenthesis does not apply. It also does not apply in even syllables such as the fourth syllable of (9b), even though it is followed by a voiceless obstruent.

An odd-even count of syllable parity must start at the left edge, as indicated by the underlined word boundary symbol in the input in (9). The count must be maintained as the word grows, as syllables in the first, third, fifth – and so on – positions are potential targets for h-epenthesis. Thus, the behavior of further input syllables is affected by information all the way at the opposite edge. This is reflected in the fact that the entire input of both forms in (9) is underlined – parity is tracked through the entire word.

As a result, the process is not ISL – the trigger in the input (the word boundary) is separated from potential targets by an, in principle, unbounded number of intervening syllables. Crucially, it is also not OSL because it is not iterative – h-epenthesis may apply for the first time in any arbitrary odd syllable where the structural description of the rule is met. If it applied consistently to *every* odd syllable, then this would provide local output information for further propagation of the process, but this is not the case. Instead, a footless analysis of Huariapano h-epenthesis requires a properly subsequential function, which can track long-distance information such as parity.

3.1.2 *With feet* For patterns like Huariapano that are subsequential as a mapping between bare strings of segments, the inclusion of feet serves to make the patterns ISL. This is because feet provide a reference point that is local in the input string which a phonological process can use to check if it should apply. Consider the analysis here in (10), which follows the analysis in (9) but crucially assumes that the word has been parsed into feet by an OSL foot placement process, as described in §2.4:

(10) a. input # (ja. na.) (pa. kwin.)
output ja. na. pah. kwin.

b. input # (jo. mɯ.) (ra. ka.) (ti. kæj)
output jo. mɯ. rah. ka. tih. kæj
information required: presence of '(V.T' input structure

Reading the string left to right, a left foot boundary informs the function that the following syllable is odd, *without* relying on an explicit count from the beginning of the word. Every time a left foot boundary is seen, the next two syllables determine the behavior of the function. If a '(V.T' sequence is found, epenthesis will occur. Otherwise, parsing of the string continues without altering the input. Thus, Huariapano h-epenthesis modulo foot structure is input local, as its application is conditioned entirely by a specific '(V.T' sequence in the input string. The presence of feet reduces a non-local, subsequential process to an ISL one.

3.2 *Capanahua* In Capanahua, a process of coda glottal stop deletion occurs in even numbered syllables, counting from the left (Loos, 1969; Safir, 1979; González, 2009). Consider the alternation of the declarative suffix /taʔ/ and morpheme /raʔ/ 'maybe' in the data below, written in bold for clarity:

(11)	UR	SR	gloss
	a. / ho.no.**taʔ**.ki /	[hó.no.**taʔ**.ki]	'It is a wild pig'
	b. / ho.no.ma.**taʔ**.ki /	[hó.no.ma.**ta**.ki]	'It is not a wild pig'
	c. / ʔu.tʃi.ti.**raʔ**.**taʔ**.ki /	[ʔú.tʃi.ti.**ra**.**taʔ**.ki]	'It's probably a dog'
	d. / ʔu.tʃi.ti.ma.**raʔ**.**taʔ**.ki /	[ʔú.tʃi.ti.ma.**raʔ**.**ta**.ki]	'It's probably not a dog'

In (11a), the declarative suffix /taʔ/ occurs in an odd syllable, and so it surfaces with its underlying glottal coda. In (11b), however, the addition of the negation affix *-ma-* pushes /taʔ/ into an even-numbered syllable position, and so the coda glottal stop is deleted. The same pattern is observed with both /taʔ/ and /raʔ/ in (11c-d). The addition of the negation affix in (11d) pushes the two into a position with different syllable parity, and so the behavior of the coda glottal stop changes. It should also be noted that this process is divorced from stress assignment – stress in Capanahua is bound to a two-syllable window at the beginning of the word, and no secondary stresses are reported to occur.

3.2.1 *Without feet* As in Huariapano, calculation of coda glottal stop deletion in Capanahua without reference to foot structure requires an explicit parity count of all the syllables in the word. Starting from the left edge, the function checks for the configuration 'Vʔ.' in every even syllable and deletes /ʔ/ when it is found. Consider the following example using (11c) from above, where underlining once again represents portions of the word where relevant information is tracked by the process:

(12) a. input: # ho. no. ma. taʔ. ki
output: ho. no. ma. ta. ki

b. input: # ʔu. tʃi. ti. raʔ. taʔ. ki
output: ʔu. tʃi. ti. ra. taʔ. ki
information required: σ parity, presence of ‘Vʔ.’ input structure

In (12a), the glottal stop of the fourth syllable deletes, surfacing as [ta]. In (12b), deletes in the fourth syllable, giving [ra], but is preserved in the fifth syllable, leading to [taʔ]. Once again parity must be tracked through the entire word, as the process can apply at any point when a glottal coda occurs in an even numbered syllable. Like Huariapano, this means it is not ISL or OSL. Since the word boundary that determines the parity count may occur an unbounded distance away from a given even syllable, it is not input local. It is also not output local, as it is not iterative – the process could apply for the first time in any even syllable. Instead, calculation of coda-ʔ deletion in Capanahua with no foot structure requires the power of a subsequential function.

3.2.2 *With feet* In Capanahua, if feet are constructed first before coda deletion applies, then the presence of foot structure in the input renders the process ISL. This is because it again removes the need for an explicit parity count – instead of checking for coda-ʔ in even syllables, it checks for a coda-ʔ that immediately precedes a right foot boundary. Consider the following reanalysis of (12) with feet:

(13) a. input: # (ho. no.) (ma. taʔ.) ki
output: ho. no. ma. ta. ki

b. input # (ʔu. tʃi.) (ti. raʔ.) (taʔ. ki)
output ʔu. tʃi. ti. ra. taʔ. ki
information required: presence of ‘Vʔ.)’ input structure

Reading through the string, the structure ‘Vʔ.)’ is encountered in the fourth syllable in both forms, resulting in output [ta] and [ra], respectively. Other syllables that precede the right foot boundaries in the word lack a glottal coda, and so the process does not apply. Thus, just as in Huariapano, the insertion of foot structure renders the otherwise subsequential process of coda-ʔ deletion in Capanahua ISL.

3.3 *Shipibo* In Shipibo, the emphatic suffix /-riβ/ surfaces variously as [-ri.ba] or [-ri.bi], depending on how many syllables precede it (Lauriault, 1948; Elias-Ulloa, 2006; González, 2009). If an odd number of syllables precede the suffix, the allomorph [-ri.ba] appears. After an even number of syllables, [-ri.bi] is selected instead:

(14)

UR	SR	gloss
a. / a.**riβ**.kɯ /	[a.**ri.ba**.kɯ]	‘did it again’
b. / a.ma.**riβ**.kɯ /	[a.ma.**ri.bi**.kɯ]	‘made (him) do it again’
c. / pu.ta.ma.**riβ**.kɯ /	[pu.ta.ma.**ri.ba**.kɯ]	‘made (him) throw again’
d. / pu.ta.ya.ma.**riβ**.kɯ /	[pu.ta.ya.ma.**ri.bi**.kɯ]	‘did not throw again’

In (14a, c), the emphatic suffix is separated from the beginning of the word by an odd number of syllables, and so it surfaces as [-ri.ba]. In (14b, d), an even number of syllables intervene instead, and so the allomorph [-ri.bi] appears.

3.3.1 *Without feet* As in Capanahua and Huaripano, alternation of /-riβ/ in Shipibo can only be determined by an explicit parity count of the syllables in the word. Starting from the left edge, the process counts syllables until the morpheme is found, at which point the current parity value determines which allomorph to select. Consider the examples of (14c,d) above:

(15) a. input: # pu. ta. ma. -riβ. kɯ
output: pu. ta. ma. -ri.ba. kɯ

b. input: # pu. ta. ya. ma. -riβ. kɯ
output: pu. ta. ya. ma. -ri.bi. kɯ
information required: σ parity, presence of '-riβ' in the input

With no feet, determining the alternation requires a properly subsequential function to track the parity of the syllables leading up to the suffix. The locus of the parity count is separated from the relevant morpheme in the input by an, in principle, unbounded distance. The process is also not output local, as there is nothing overt in the output string that indicates the parity of a given syllable.

3.3.2 *With feet* If foot structure has first been placed into the word, then the pattern of Shipibo is input local. Instead of tracking parity, the alternation of the /-riβ/ morpheme is determined instead by its location relative to the right foot boundary. If '-riβ.)' is seen in the input, this indicates that an odd number of syllables preceded the suffix, and so [-ri.ba] surfaces. Otherwise, [-ri.bi] surfaces instead. Consider the following examples from (15), now with foot structure:

(16) a. input: # (pu. ta.) (ma. -riβ.) kɯ
output: pu. ta. ma. **ri.ba.** kɯ

b. input: # (pu. ta.) (ya. ma.) (-riβ. kɯ)
output: pu. ta. ya. ma. **ri.bi** .kɯ
information required: presence of '-riβ.)' input structure

Once /-riβ/ is encountered, the function checks the next input symbol. in (16a), the morpheme is immediately followed by a right foot boundary, indicating a preceding odd-parity string of syllables. This results in the surface form [-ri.ba]. In (16b), the morpheme is instead followed by another syllable, indicating a preceding even sequence, and so [-ri.bi] is output instead. Foot structure again helps derive a pseudo-parity count in a way that is fundamentally local, rendering the properly subsequential alternation in Shipibo ISL.

3.4 *Urarina* In Urarina, an isolate spoken in Peru, the nominal plural suffix /-ʉ.rʉ/ surfaces either as [-ʉ.rʉ] or [-kʉ.rʉ], depending on the number of preceding morae (Cajas Rojas et al., 1987; Olawsky, 2006; González, 2011). After an even number of morae, [-ʉ.rʉ] is the form that surfaces. After an odd number of morae, [-kʉ.rʉ] surfaces instead:

(17)

UR	SR	gloss
a. / le.ra.no.**-ʉ.rʉ** /	[le.ra.no.**-kʉ́.rʉ**]	'fishes'
b. / ha.re.hee.ri.**-ʉ.rʉ** /	[ha.re.hée.ri.**-kʉ.rʉ**]	'thin person'
c. / ka.la.mi.na.**-ʉ.rʉ** /	[ka.la.mi.ná.**-ʉ.rʉ**]	'zinc roofs'
d. / ka.tʃa.**-ʉ.rʉ** /	[ka.tʃa.**-ʉ́.rʉ**]	'men'

In (17a,b), an odd number of morae precede the nominal plural suffix – three in (a) and five (b) – and so it surfaces as [-kʉ.rʉ]. In (17c,d), an even number of morae precede the morpheme instead – four in (c) and two in (d) – and so the allomorph [-ʉ.rʉ] appears instead. The allomorphy occurs independent of main stress, and no secondary stress is reported in the language.

3.4.1 *Without feet* As with the other patterns described above, Urarina requires a long-distance parity count of the syllables in the word to determine the alternation observed in the data if no foot structure is present. Counting from the left, the correct allomorph is selected based on the parity of the syllable that occurs before the morpheme. Consider the following examples from (17a,c):

(18) a. input: # le. ra. no. -ʉ.rʉ
output: le. ra. no. -kʉ.rʉ

b. input: # ka. la. mi. na. -ʉ.rʉ
output: ka. la. mi. na. -ʉ.rʉ
information required: σ parity, presence of '-ʉ.rʉ' in the input

In (18a), the preceding count of syllables when /-ʉ.rʉ/ is encountered is odd, and so [kʉ.rʉ] surfaces. In (18b), the preceding string of syllables is of even parity instead, and so [ʉ.rʉ] is selected. The derivation mirrors that of Shipibo, where the parity count is maintained until the morpheme is encountered. It requires a properly subsequential function to calculate, and is neither ISL nor OSL for the same reasons as in Shipibo.

3.4.2 *With feet* With foot structure, however, the pattern is describable with an ISL function. If the sequence '-ʉ.)rʉ' is seen in the input, this indicates that an odd number of syllables preceded the suffix, leading to surface [kʉ.rʉ]. If no foot boundary immediately follows the first syllable of the suffix, then the preceding string of syllables was of even parity, leading to [ʉ.rʉ] instead. The following examples provide a reanalysis of (18) with feet:

(19) a. input: # (le. ra.) (no. -ʉ.) rʉ
output: le. ra. no. -kʉ. rʉ

b. input: # (ka. la.) (mi. na.) (-ʉ. rʉ)
output: ka. la. mi. na. -ʉ. rʉ
information required: presence of '-ʉ.)' in the input

When the beginning of the suffix is encountered, the function checks the next symbol. If it is a right foot boundary, as in (19a), this indicates a preceding odd-parity string of syllables, and so the chosen allomorph is [-kʉ.rʉ]. If no foot boundary is found, an even string of syllables preceded the suffix, and so [-ʉ.rʉ] surfaces instead. As in the other analyses discussed above, the presence of foot structure makes a properly subsequential process ISL.

4 Discussion

In the preceding section, four analyses were presented that show how foot structure serves to make a long-distance parity-counting process formally local. Specifically, the analyses demonstrate the exact computational effect of feet: making a process that requires a properly subsequential function with parity counting into one that requires only an ISL function. The key difference is in the type of counting that occurs – OSL and ISL counting is inseparable from computation in phonology, while the modulo counting available to properly subsequential functions results in non-phonological typological predictions.

Though some properly subsequential patterns can be described by the composition of an OSL and ISL function, limitation to OSL and ISL *does* exclude patterns such as the one described above that deletes every even-numbered glottal stop. So, while a restriction on compositions of OSL and ISL processes that provides the most accurate predicted typology remains to be stated, the adherence to local functions provides a better initial hypothesis than full subsequential power.

It should be noted that proposing a foot-based analyses of these non-stress processes, especially where this creates a tier distinct from what is necessary for the stress pattern of the language, is a matter of contention (Parker, 1994; González, 2007; Bennett, 2013). I have demonstrated that there is good evidence from formal complexity that a foot-based analysis is indeed appropriate as it allows for a more restrictive theory involving the interaction of simple constraints. Additionally, I suggest here that if the foot is a device that languages do employ, there is no *a priori* reason to think that other types of processes do not have access to feet as well, especially if the specific language employs feet for stress.

Some previous work treats parity counting as a natural property of phonology for which feet are useful to represent. McCarthy (1979) in an analysis of Arabic stress, for example, describes parity counting in stress assignment as "reasonably familiar" (p.448) and "stipulated by" the presence of feet (p.451). Hayes (1995), referring to the same pattern, states that the count of syllables is "carried out by the bimoraic foot structure" (p.70). However, the explicit computational analysis in this paper indicates that parity counting is not a property of phonological patterns *at all*. Placement of feet allows for reference to local input structure to compute phonological processes, completely obviating any need for explicit parity counting. The argument from formal complexity also tells us why this is a desirable result – adopting properly subsequential power as a hypothesis for phonology makes worse typological predictions than a hypothesis that instead adheres to input and/or output locality. Thus, the preceding analysis demonstrates not only precisely what is at stake in terms of computation when feet are present, but why this matters for substantive proposals in phonological theory relating to foot structure.

5 Conclusion

This paper demonstrated that placement of foot structure in certain phonological processes renders otherwise properly subsequential patterns ISL. Using examples from four languages, it was shown that feet remove the need for any parity counting and demonstrate that, broken down into simpler parts, the processes are fundamentally local. Thus, this analysis based on formal language theory complexity provides support for a substantive element of phonological theory i.e. metrical structure.

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