Acta Polytechnica **65**(3):334–340, 2025
https://doi.org/10.14311/AP.2025.65.0334

WAVE PROPAGATION IN A PIPE CONVEYING TWO PHASE FLOW

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Abstract. The study of wave propagation in pipes conveying a two-phase flow is of significant importance in various engineering applications, including the oil and gas, chemical, and nuclear industries. One of the most significant characteristics of a two-phase flow in pipes is the flow velocity whose estimation is achieved by various means. However, such approaches can be very sensitive to the operational and boundary conditions. Therefore, there is a need to develop a reliable approach based on the local flow dynamics and properties of the pipe. In this work, a wave-based approach for estimating the phase and group velocities was developed. The governing equation of the work was derived based on Hamilton, and the Chisolm correlation model was used for the two-phase flow. The dispersion relation of the governing equation is first derived in terms of the traveling wave feature. Then, the exact expressions of the phase velocity and group velocity for the bending waves are obtained. Furthermore, the free vibration frequency characteristics of the two phases was studied. The results indicate that in a single-phase flow, wave dispersion occurs at a wavenumber of 2.1, whereas in a two-phase flow with a vapour quality of 0.2, dispersion occurs at a higher wavenumber of 5.8, demonstrating that increasing vapour quality delays the onset of dispersion. This work contributes to a better understanding of wave propagation in a two-phase flow.

Keywords: Wave propagation, phase velocity, group velocity, two phase, natural frequency.

1. Introduction

Numerous industries, including the production of oil and gas, nuclear and thermal power plants, and the food processing industry, frequently make use of pipes in conveying their product from one point to another [1–4]. Oftentimes, these pipes convey more than one fluid; hence multiphase flow occurs in these pipes [2, 5–7]. Many researchers have investigated the dynamics of these flows and proposed methods for attenuating challenges arising from such multiphase transport [5, 8–12]. In the analysis of these pipes, various methods and solutions have been used to study the dynamics of vibrations in these systems. Nonlinear vibration of fluid-conveying pipes can be analysed using Galerkin discretization [13, 14], numerical tool ANSYS [15], finite element method [8, 9], wave method approach [1, 16], differential transformation method [17], transfer matrix method (TMM) [18, 19], and differential quadrature method (DQM) [20]. Wave propagation approach was used to analyse finite cylindrical pipes that convey dense fluids [21]. He proved that the fundamental frequency transition between circumferential modes happens at various height-radius ratios for various boundary conditions. In addition, wave propagation and band gap properties of long fluid-conveying pipes was investigated [22]. In the first part of the work, the traveling wave characteristics of axially moving structures was used to derive the dispersion relation of the corresponding time-variant uniform pipe system. The second section illustrated the significant effects of flow velocity and deploying speed, propagating and evanescent waves, as well as the phase velocity and group velocity of propagating waves. The dynamics of the fluid-conveying pipe with Functionally graded (FG) materials were explored [16]. In this work, the propagation characteristics of longitudinal and flexural waves in the fluid-conveying pipes were studied. It was found that the temperature and functional gradient exponent decrease the phase and group velocities. On the contrary, the liquid flow velocity increases the phase and group velocities.

It appears that there are relatively fewer works in the literature on wave propagation for a two phase flow in pipes. In order to properly address this lacuna in literature, the present study aims to show that the dynamics of a pipe conveying a two phase flow can be analysed by the elastic wave propagation. The characteristics of flexural waves in the conveying fluid pipes are studied.

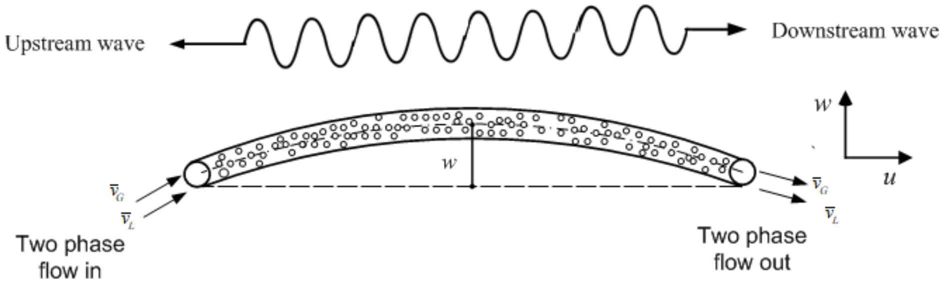


FIGURE 1. Mechanical model of a pipe conveying two phase flow.

2. DERIVATION OF THE EQUATION OF MOTION

The system under consideration consists of a pipe conveying a two phase flow (Figure 1). The governing equation in transverse direction is given as [6, 23]:

$$\begin{aligned}
&(m_p + m_L + m_G)\ddot{\bar{w}} + 2(m_L\bar{v}_L + m_G\bar{v}_G)\dot{\bar{w}}' \\
&+ (m_L\bar{v}_L^2 + m_G\bar{v}_G^2)\bar{w}'' + EI\bar{w}^{\mathrm{IV}} \\
&- EI(2\bar{w}'^2\bar{w}^{\mathrm{IV}} + 8\bar{w}'\bar{w}''\bar{w}''' + 2\bar{w}''\bar{w}''\bar{w}'') \\
&- EA\Bigg(\frac{3}{2}\bar{w}'^2(\bar{w}'' + \bar{Y}_0'') + 3\bar{w}'\bar{w}''\bar{Y}_0' + \bar{Y}_0'^2\bar{w}'' \\
&\quad +2\bar{Y}_0'\bar{Y}_0''\bar{w}' - \alpha\bar{\theta}'(\bar{w}' + \bar{Y}_0') \\
&\quad + \left(\frac{\bar{T}}{EA} - \frac{\bar{P}}{E}\right)(\bar{w}'' + \bar{Y}_0'') \\
&\quad -\alpha\bar{\theta}(\bar{w}'' + \bar{Y}_0'')\Bigg) = 0
\end{aligned} \tag{1}$$

2.1. DIMENSIONLESS QUANTITIES

Introducing dimensionless quantities:

$$\begin{aligned}
&\bar{w} = wr, \quad \bar{u} = uL, \quad \bar{x} = xL, \\
&\bar{Y}_0 = Y_0 r, \quad \bar{t} = t\sqrt{\frac{m_p + m_L + m_G}{EI}}L^2, \\
&\bar{T} = T\frac{EI}{L^2}, \quad \bar{P} = P\frac{EI}{AL^2}, \quad \bar{\theta} = \theta\frac{I}{\alpha AL^2}, \\
&\bar{v}_L = v_L\sqrt{\frac{EI}{m_L}}\frac{1}{L}, \quad \bar{v}_G = v_G\sqrt{\frac{EI}{m_G}}\frac{1}{L},
\end{aligned} \tag{2}$$

and these terms are defined:

$$\begin{aligned}
&\beta_L = \frac{m_L}{m_p + m_L + m_G}, \quad \beta_G = \frac{m_G}{m_p + m_L + m_G}, \\
&\eta = \frac{r}{L}.
\end{aligned} \tag{3}$$

Using the dimensionless quantities defined in Equation (2), the transverse vibration Equation (1) becomes:

$$\begin{aligned}
&\ddot{w} + 2\left(v_L\sqrt{\beta_L} + v_G\sqrt{\beta_G}\right)\dot{w}' + w^{\mathrm{IV}} \\
&+ \left(v_L^2 + v_G^2 - T + P + \theta\right)w'' \\
&- \eta^2\left(2w'^2w^{\mathrm{IV}} + 8w'w''w''' + 2w''w''w''\right) \\
&- \Bigg\{\frac{3}{2}w'^2\left(w'' + Y_0''\right) + 3w'w''Y_0' \\
&\quad +Y_0'^2w'' + 2Y_0'Y_0''w'\Bigg\} \\
&- (T - P - \theta)\,Y_0'' = 0.
\end{aligned} \tag{4}$$

For a straight pipe, $Y_0 = 0$, and η has been established to have negligible effect on the dynamic response of the pipe conveying the fluid, hence, Equation (4) becomes:

$$\begin{aligned}
&\ddot{w} + 2\left(v_L\sqrt{\beta_L} + v_G\sqrt{\beta_G}\right)\dot{w}' + w^{\mathrm{IV}} \\
&+ \left(v_L^2 + v_G^2 - T + P + \theta\right)w'' - \frac{3}{2}w'^2w'' = 0.
\end{aligned} \tag{5}$$

In a two-phase flow, the relationship between the velocity of the gas with liquid is given as [8, 23]:

$$v_G = K_s v_L\sqrt{\frac{\rho_G\alpha_G}{\rho_L(1-\alpha_G)}}, \tag{6}$$

where α_G is the gas void fraction, ρ_G and ρ_L is the density of the gas and the liquid, respectively, and K_s is the slip ratio. The relationship between dimensional term β_G and β_L is [5]:

$$\beta_G = \frac{\rho_G\alpha_G\beta_L}{\rho_L(1-\alpha_G)}. \tag{7}$$

Inserting Equations (6) and (7) into Equation (5) yields the governing equations of motion as:

$$\begin{aligned}
&\ddot{w} + 2v_L\sqrt{\beta_L}\left(1 + K_s\frac{\alpha_G\rho_G}{\rho_L(1-\alpha_G)}\right)\dot{w}' + w^{\mathrm{IV}} \\
&+ \left(v_L^2 + v_L^2\frac{\rho_G}{\rho_L}\frac{K_s^2\alpha_G}{left(1-\alpha_G)} - T + P + \theta\right)w'' \\
&- \frac{3}{2}w'^2w'' = 0.
\end{aligned} \tag{8}$$

The gas void fraction plays a significant role in determining several multiphase flow parameters [24]. The void fraction and slip correlations according to Chisolm [25] will be adopted in this work:

$$\begin{aligned}
&\alpha_G = \left[\sqrt{1 + x_g\left(\frac{\rho_L}{\rho_G} - 1\right)}\left(\frac{1-x_g}{x_g}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}, \\
&K_s = \sqrt{1 + x_g\left(\frac{\rho_L}{\rho_G} - 1\right)},
\end{aligned} \tag{9}$$

where x_g is the void quality, ρ_L is the density of the liquid, and ρ_G, then the density of the gas.

3. Wave propagation approach

We first consider the wave propagation problem, assuming that the equation has a traveling waves solution. The harmonic solution in the form of a traveling wave is given as [22]:

$$w = W_n e^{i(\kappa x - \omega t)}, \tag{10}$$

where W_n and ω are the amplitude and angular frequency of the wave, respectively. κ is the wavenumber and i the imaginary unit. In addition, according to the stress wave theory, the wave phase velocity is equal to the circular frequency divided by the wave number, i.e.:

$$C_p = \frac{\omega}{\kappa}. \tag{11}$$

By rearranging Equation (11), we obtained:

$$\omega = \kappa V_p, \tag{12}$$

the group velocity is equal to the derivative of the circular frequency with respect to the wavenumber, hence:

$$V_g = V_p + \kappa \frac{dV_p}{d\kappa}. \tag{13}$$

Substitution of Equation (10) into Equation (8) leads to the dispersion equation as:

$$\begin{aligned}&\kappa^4 - \omega^2 + 2\kappa\omega v_L \sqrt{\beta_L}\left(1 + K_s v_L \frac{\alpha_G \rho_G}{\rho_L (1-\alpha_G)}\right)\\ &- \kappa^2\left(v_L^2 + v_L^2 \frac{\rho_G}{\rho_L}\frac{K_s^2 \alpha_G}{(1-\alpha_G)} - T + P + \theta\right) = 0.\end{aligned} \tag{14}$$

There are four different wavenumber roots for Equation (14), involving two real and a conjugate pair of imaginary ones. The two real describe the downstream and upstream propagating waves, respectively, and the imaginary describe evanescent waves.

By rearranging Equation (14), we can easily get:

$$\begin{aligned}&\kappa^2 - C_p^2 + 2C_p v_L \sqrt{\beta_L}\left(1 + K_s v_L \frac{\alpha_G \rho_G}{\rho_L (1-\alpha_G)}\right)\\ &- \left(v_L^2 + v_L^2 \frac{\rho_G}{\rho_L}\frac{K_s^2 \alpha_G}{(1-\alpha_G)} - T + P + \theta\right) = 0\end{aligned} \tag{15}$$

Then, the expressions for phase velocity and group velocity are defined as:

$$\begin{aligned}C_p = \frac{1}{2}&\sqrt{4\left(v_L\sqrt{\beta_L} + v_L\sqrt{\beta_L}\frac{\alpha_G\rho_G}{\rho_L(1-\alpha_G)}\right)^2 + 4\Gamma}\\ &+ 2v_L\sqrt{\beta_L}\left(1 + \frac{\alpha_G\rho_G}{\rho_L(1-\alpha_G)}\right),\end{aligned} \tag{16}$$

$$V_g = C_p + \frac{2\kappa^2}{\sqrt{4\left(v_L\sqrt{\beta_L} + v_L\sqrt{\beta_L}\frac{\alpha_G\rho_G}{\rho_L(1-\alpha_G)}\right)^2 + 4\Gamma}}, \tag{17}$$

where

$$\Gamma = \kappa^2 - P + T - \theta - v_L^2 - v_L^2 \frac{\rho_G}{\rho_L}\frac{K_s^2\alpha_G}{(1-\alpha_G)}.$$

For the free vibration problem, based on the Navier solution method, we assume Equation (8) has the following form solutions:

$$w(x,t) = \sum_{n=1}^{\infty} W_n e^{i\omega_n t}\sin\left(\frac{n\pi}{L}x\right). \tag{18}$$

Substituting Equation (18) into Equation (8), the expression for the frequency relation is obtained as:

$$\begin{aligned}\Omega_n = in\pi&\sqrt{\begin{aligned}&L^2K_s^2v_L^2\alpha_G\rho_G\\ &+\Big(-n^2\pi^2 + L^2(P-T+\theta)\\ &\quad - L^2v_L^2(\alpha_G - 1)\\ &\quad + (n^2\pi^2 + L^2(T-P))\alpha_G\Big)\rho_L\end{aligned}}\\ &\times\left(\sqrt{L^2(1-\alpha_G)\rho_L}\right)^{-1}.\end{aligned} \tag{19}$$

4. Results and discussion

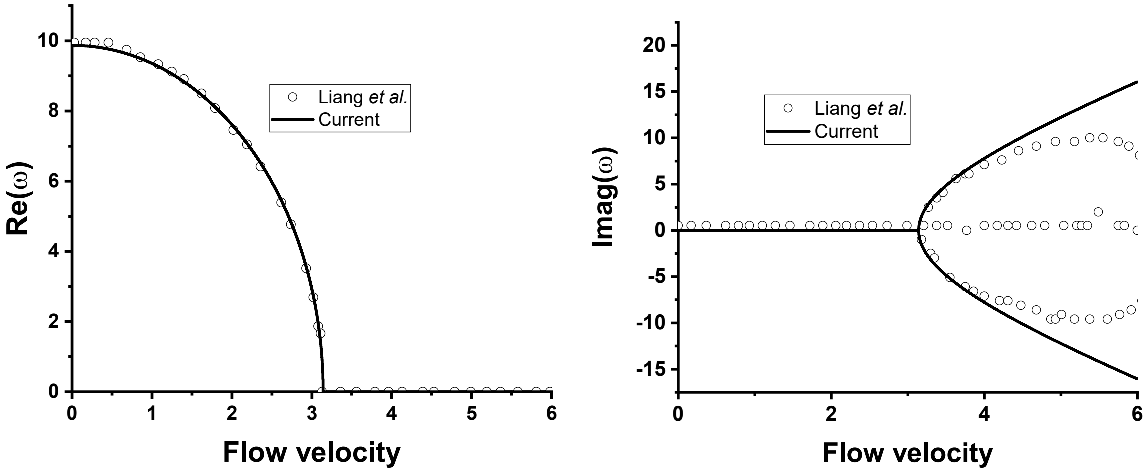
In the following numerical examples, the two-phase flow constituents; water and air, have these densities $\rho_L = 1\,000\,\mathrm{kg\,m^{-3}}$ and $\rho_G = 1.2\,\mathrm{kg\,m^{-3}}$, respectively. These parameters are used except stated otherwise, $P = 0$, $T = 0$, $\theta = 0$, $\beta = 0.5$.

4.1. Validation of the result

The accuracy of the wave propagation approach for the pipe conveying a two phase flow is validated with the work of Liang et al. [26]. For the single-phase flow, the results obtained from the present model are in good agreement with those from the reference (Figure 2). These results show that the formulation can give a reliable description for wave propagation in a pipe conveying a two phase flow.

4.2. Effect of two-phase flow on phase and group velocities

The effects of the two-phase flow on wave propagation are shown in Figures 3 and 4 at a liquid velocity of 3. It can be observed that the wave phase velocity remains constant across all phases for a range of wavenumbers until dispersion occurs. For the constant phase velocity, the wave is non-dispersive and the wave velocity depends only on the medium's bulk properties and not on frequency. For instance, in a single-phase flow, the wave velocity is constant from zero to a wavenumber of 2, with dispersion occurring at a wavenumber of 2.1. However, for a two-phase flow with a vapour quality $x_g = 0.2$, wave dispersion occurs at a wavenumber of 5.8. At the dispersion points, the wave velocity depends on the frequency of the propagation. Therefore, as vapour quality increases, the wavenumber at which the propagating wave becomes dispersed also increases. The corresponding group velocity for the same wave is shown in Figure 4. In Figure 4, there is a critical point where the group velocity jumps to infinity for a particular wavenumber. This point likely corresponds to stationary waves or indicates that the wave has a very localised energy distribution [22].



(A). Real. (B). Imaginary.

Figure 2. The first-order frequency diagrams for the fluid conveying pipes.

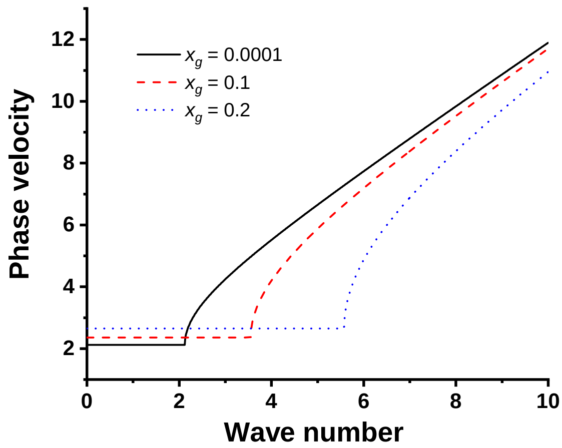


Figure 3. Phase velocity versus wavenumber with different vapour quality $P = 0$, $T = 0$, $v = 3$, $\beta = 0.5$.

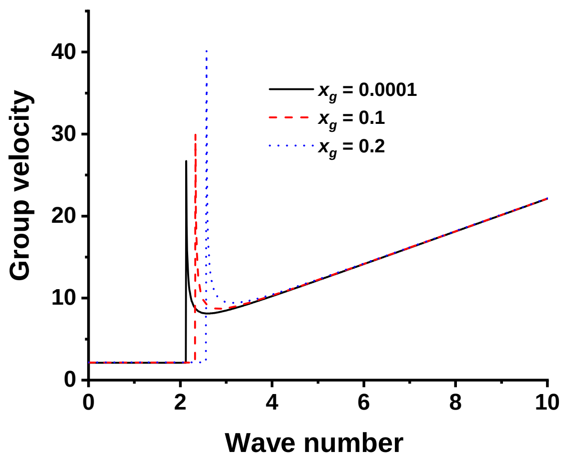


Figure 4. Group velocity versus wavenumber with different vapour quality $v = 3$.

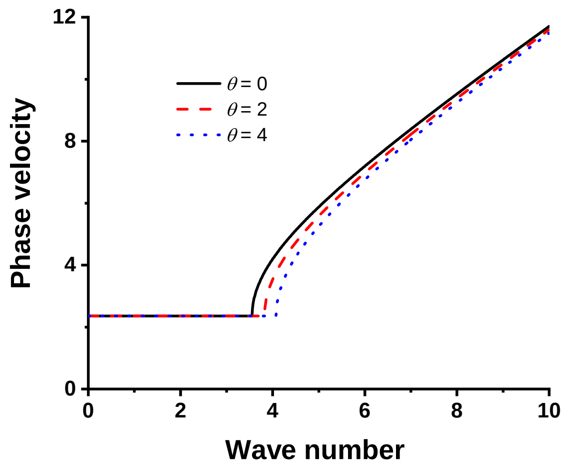


Figure 5. Group velocity versus wavenumber for different temperature values $x_g = 0.1$, $v = 3$.

4.3. Effect of temperature on phase and group velocities

The evolution of the phase and group velocities with increasing wave number is shown in Figures 5 and 6 for various temperature changes. It is observed that an increase in temperature affects the transition point between the non-dispersive and dispersive wave behaviour. For the two-phase flow, an increase in temperature tends to shift the dispersive position to a higher wavenumber.

4.4. Effect of wavenumber on phase velocity

Figures 7 and 8 illustrate how the phase velocity of a wave in a two-phase flow system varies with the velocity of the liquid and how this relationship is influenced by the wavenumber. When there is no real wave propagation ($\kappa = 0$), there is a linear relationship between the fluid velocity and both the wave phase velocity and wave group velocity. For non-zero wavenumbers, the phase velocity initially increases with the liquid velocity, reaches a peak, and then

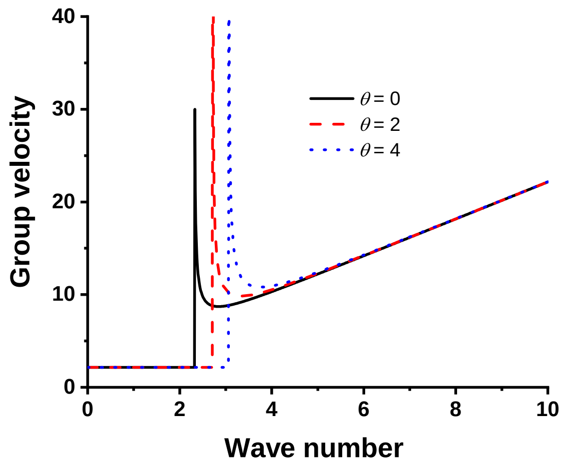


Figure 6. Group velocity versus wavenumber for different vapour quality values, $x_g = 0.1$, $v = 3$.

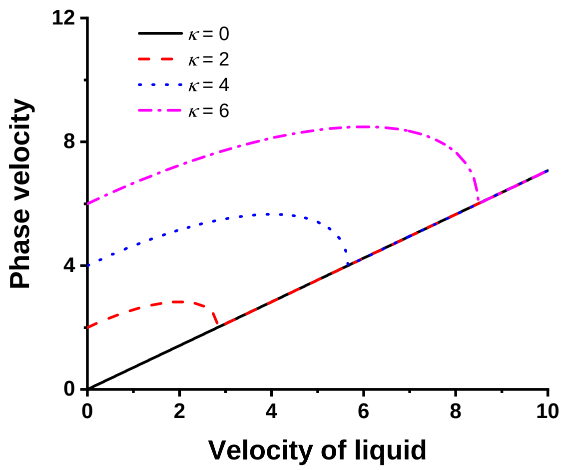


Figure 7. Phase velocity versus wavenumber for different wavenumber values $P = 0$, $T = 0$, $x_g = 0.0001$, $\beta = 0.5$.

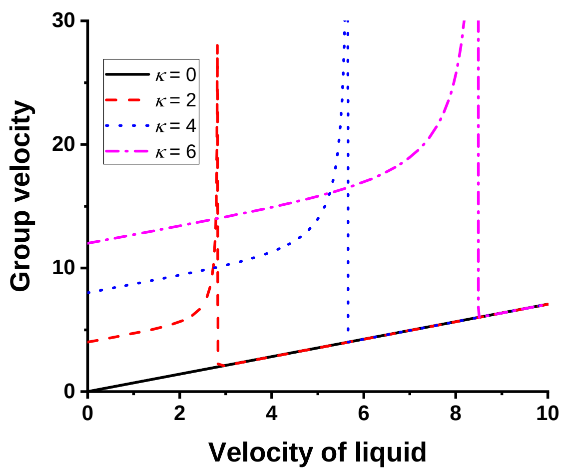


Figure 8. Group velocity versus wavenumber for different wavenumber values $P = 0$, $T = 0$, $x_g = 0.0001$, $\beta = 0.5$.

decreases, indicating a dispersive behaviour. For illustration, when the wavenumber is 4, the wave phase velocity increases from zero to 5 as the velocity of the liquid increases. Then, between liquid velocities of 5 and 5.5, the phase velocity decreases with increasing liquid velocity. At a liquid velocity of 5.5, the relationship between the liquid velocity and the phase velocity becomes linear again.

4.5. Natural frequency of two-phase flow

The relationships between flow velocity and natural frequency are shown in Figure 9, in which the number of mode is fixed to be $n = 1$. For a single phase flow ($x_g = 0.0001$), the critical velocity is 3.14. As the vapour quality increases in the flow, i.e. the flow turns to two phase flow, the critical velocity reduces. When $x_g = 0.3$, the critical velocity is 1.1. The imaginary frequency plots confirm the critical velocity point as shown in the real part.

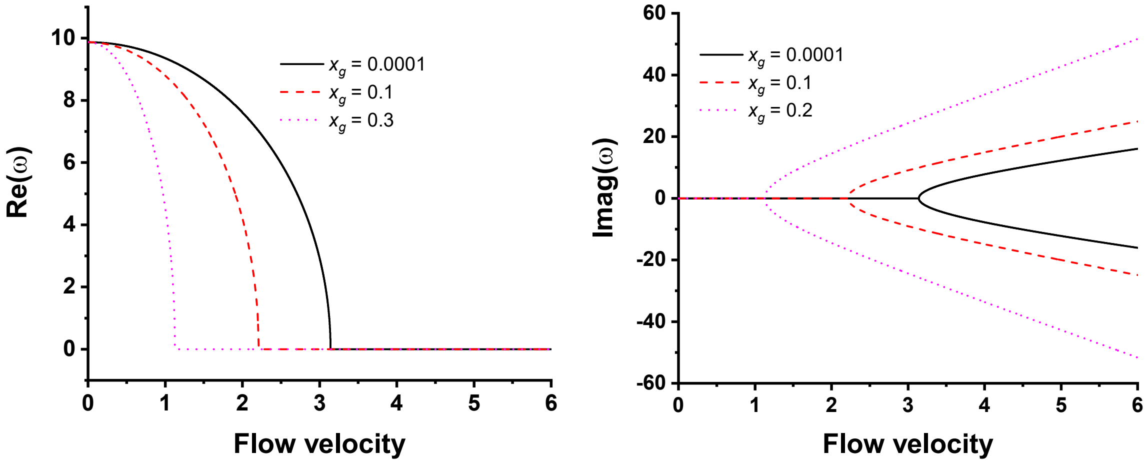
The influence of temperature on the first three modes of the natural frequency is shown in Figure 10. Temperature reduces the critical velocity of the flow, as can be observed for both the real and imaginary frequencies in Figure 10.

5. Conclusion

The characteristics of wave propagation in a pipe conveying a two phase flow is studied in this paper. Chisolm correlation model was used to relate the two-phase flow in the governing equation. A harmonic solution in the form of a traveling wave was used to investigate the phase and group velocities of the two-phase flow dynamics. In the numerical result, the phase velocity remains constant over a range of wavenumbers for the single-phase flow, while for the two-phase flow, increased vapour quality delays the onset of dispersion to higher wavenumbers, and the group velocity exhibits critical behaviour, indicative of localised energy distribution. Temperature further modulates this behaviour by shifting the transition between non-dispersive and dispersive regimes to higher wavenumbers, and variations in liquid velocity and wavenumber reveal complex dispersive effects on the phase velocity. Additionally, the natural frequency analysis shows that as the flow transitions from single-phase to two-phase, the critical velocity decreases, with temperature exerting a reducing effect on the critical velocity across different modes, underscoring the intricate interplay between flow parameters and wave dynamics in two-phase systems.

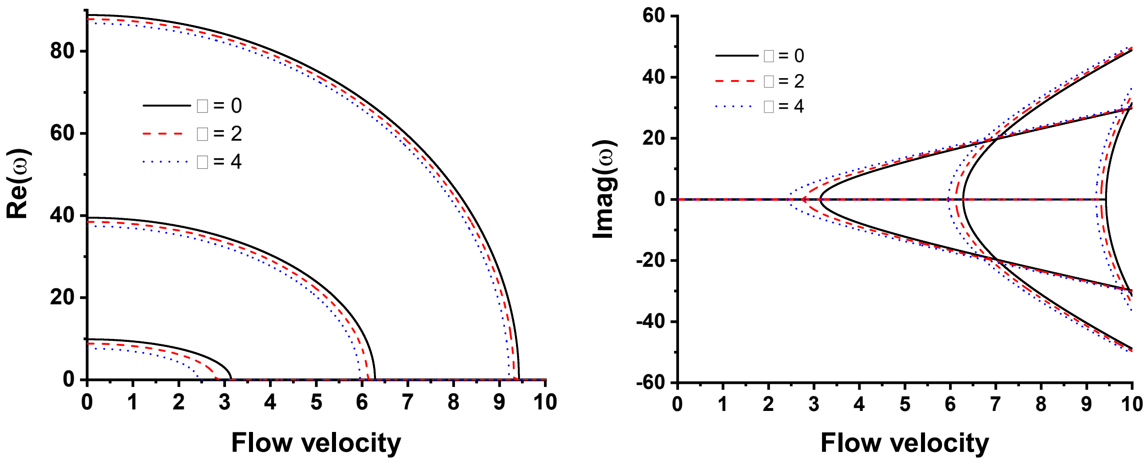
List of symbols

- L Length of the pipe
- E Elastic modulus
- I Moment of inertia
- A Cross-sectional area
- $\bar{P}$ Pressure
- $\bar{T}$ Tension



(A). Real. (B). Imaginary.

FIGURE 9. The first-order frequency diagrams for the fluid conveying two phase flow.



(A). Real. (B). Imaginary.

FIGURE 10. The first three order frequency diagrams for the fluid conveying two phase flow with varying temperature parameters $x_g = 0.0001$.

m_p Mass of the pipe
m_G Mass of the gas
m_f Mass of the fluid
$\bar{w}$ Transverse displacement
$\bar{Y}_0$ Initial curvature
θ Temperature
$\bar{v}_G$ Velocity of the gas
$\bar{v}_L$ Velocity of the liquid
x_g Void quality
α Thermal coefficient
K_s Slip ratio

ACKNOWLEDGEMENTS

We are grateful for the invaluable support provided by the University of Lagos throughout the research process.

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