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DESIGN OF THE VERTICAL TRAJECTORY FOR ARRIVAL AND DEPARTURE ROUTES AT PRAGUE AIRPORT

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Abstract. Numerous operational methods and related research focus on the efficient use of controlled airspace and thus on the management of its capacity. During the optimisation processes, various legislative limitations or technological constraints might appear. As the aviation industry continues to evolve, it occasionally happens that one of these pillars lags behind the other. This research focuses on a relatively underexplored aspect of using vertical profiles of air traffic service routes within the TMA Praha to optimise traffic flow in terms of flight efficiency and environmental impact.

Virtual points, defining the aircraft's vertical position limits along the given route, were computed to establish a vertical profile on already published standard departure or arrival routes. These points define the aircraft's vertical position limits along the given route. Along with the proposal of a vertical profile, newly designed arrival and departure routes were implemented into the air traffic control simulation tool Escape Light and validated simultaneously within X-Plane simulator. Simulation studies conducted in these environments demonstrated significant fuel savings, averaging 13.2 %, with even greater savings observed in the Boeing 737-800, exceeding 25 %. These results confirm that applying vertical profiles positively impacts fuel consumption and contributes to more environmentally friendly operations.

Keywords: Continuous climb operations, continuous descent operations, standard instrument arrival, standard instrument departure, vertical profile.

1. Introduction

State-published routes have been fundamental since the early days of air navigation. They allow for the determination of aircraft positions, streamlining traffic flow, and ensuring safe separation from airspaces and other aircraft. With the advent of new technologies, the methods of utilizing these routes have evolved. However, from the early days to the present, routes have predominantly been viewed as horizontal, except for determining minimum safe altitudes. In the vicinity of airports, aircraft often experience frequent changes in their vertical position. If not executed smoothly and simultaneously, the frequency of these changes directly impacts flight efficiency, the environment, and the operational workload of both flight crews and air traffic controllers. The Continuous Descent Operations (CDO) and Continuous Climb Operations (CCO) procedures address this issue to some extent [1]. By applying these procedures, fuel consumption can be reduced by up to a few percentage points [2]. A specific and increasingly desirable form of CDO is the Continuous Descent Approach (CDA), sometimes referred to as the Continuous Descent Final Approach (CDFA), which involves a continuous descent from cruise altitude to the touchdown point. However, to effectively utilize CDO/CCO methods, flight execution must also be adapted. For example, when descending from cruise altitude to the destination airport, initiating the descent at a reduced speed is more advantageous than transitioning to level flight to reduce speed during a later approach segment. This operational limitation is less significant than in flights with horizontal phases [3]. However, there might be further restrictions from ATC, which may mandate a reduction in forward speed, causing the aircraft to level off, thereby disrupting the entire concept of this method. In areas with higher traffic or complex airspace configurations, their use can be challenging and may not serve the intended purpose of smooth vertical movement [4]. During pre-flight planning, flight crews calculate their optimal descent, which primarily considers published ATS procedures and regulations, as well as the needs of the aircraft operator. Subsequently, non-published factors such as air traffic control workflow, operational conditions, and traffic composition also play a role, aside from force majeure influences. In order to secure equally fair handling of flights by air traffic service providers, individual needs cannot be the sole consideration. The higher the demand for services and the number of flights, the greater the set of restrictions that must utilized, and vice versa. The scope and degree of measures taken are determined by individual ATC units. Each air traffic controller (ATCO) also has an individual style of service delivery. Therefore, it cannot be expected that a universal air traffic control method will apply everywhere [5].

This research primarily focuses on the design of arrival routes with integrated information about the expected vertical position of the aircraft on standard routes to and from LKPR airport. It also evaluates the impact of these methods on flight efficiency. Vertical restrictions are currently applied at many international airports with very high traffic in Europe and globally. An example is London Heathrow Airport, where, given the airspace structure and the volume of traffic utilizing it, there is limited manoeuvring space outside the standard routes [6].

The research aimed to design and validate the vertical profiles of departure and arrival routes at LKPR airport while respecting the existing airspace structure and the sectorisation at the LKPR APP control unit.

An APP unit is a part of the air traffic services facility that includes one or more workstations, each fulfilling specific tasks within a defined scope [7]. Prague APP's area of responsibility (AoR) is classified as CTA 1 Prague, a Class E airspace, and Prague TMA, a Class C airspace [8].

2. Materials and methods

The practical part of the research consisted of four key steps:

- Analysis of the current state of the art.
- Identification and calculation of key points along the route, where newly calculated restrictions will be implemented.
- Determination of values for vertical restrictions.
- Implementation of the vertical restrictions into the simulation environment and validation.

2.1. Analysis of the current state of the art

The Analysis involves a detailed traffic composition that operates within the airspace. This includes an assessment of aircraft types based on the collection of real data, utilisation of individual flight paths, frequency and representation of various aircraft types, as well as other relevant factors that influence operations in this space.

There are two established methods of designing arrival and departure routes. 2D and 3D strategic de-confliction. In aviation, area navigation is a four-dimensional discipline that combines lateral and vertical navigation with speed and time. TMA Praha has established standard departure and terminal arrival routes (SIDs and STARs) that utilise solely lateral navigation. This reflects the 2D strategic de-confliction of these routes. The only level constraint can be found on turning departures with the first turn defined by an altitude of 1 700 ft AMSL. By adding level constraints to these routes, that would correspond to the optimal levels of aircraft on them, full 3D (vertical and lateral) strategic de-confliction would be created [9].

To establish the vertical profile on an already published departure or arrival route, it is essential to define points that will determine the limits of the aircraft's vertical position on the given route. These limitations must respect CDOs and be part of the buffer zone, which physically separates arrival and departure trajectories to continuously ensure vertical separation between aircraft. These trajectories essentially form virtual tunnels from which an aircraft cannot deviate unless explicitly directed otherwise. This allows flight crews to know precisely when and where they can ascend or descend. The currently separate clearances to change level incrementally could be executed based on a single ATC clearance aiding in.

There are two types of arrival routes, and Prague is adapted to both. First one, called open procedure, provides track guidance to a downwind position from which the aircraft is guided by ATC using radar vectoring to intercept the final approach track. Second route type, closed procedure, continues with track guidance after passing the downwind position up to the final approach point or fix of an instrument approach to the active runway. Closed procedure is nowadays used in LKPR by adding an RNAV path to the initial approach connecting the downwind positions with the final approach [10].

The standard situation for most traffic to and from LKPR is as follows. IFR departure clearances include a standard departure track and an initial climb altitude of 5 000 ft. The departure track is normally assigned according to the first waypoint in the flight plan. This point should be identical to the last point on the departure track. For climbs above 5 000 ft AMSL the clearance is issued by the controller of APP Praha (the aircraft is located in TMA Praha – Class C area). He also ensures the aircraft is separated from surrounding traffic and from other controllers' areas of responsibility. The last level assigned by this controller is normally FL140 for propeller aircraft and FL160 for jet aircraft, or their cruise level if this is lower. The aircraft is then instructed to contact ACC Praha for further climb and other clearances [11].

Arriving IFR traffic is transferred to the APP Praha frequency typically in descent to FL170 (jet aircraft) or FL150 (propeller aircraft) with an already assigned arrival route. Taking into account the airspace layout, sectorisation and operational situation, the pilot is cleared to descend progressively down to 4 000 ft AMSL – the intermediate approach altitude to one of the LKPR runways [11].

Pilots never know in advance what level they can expect at certain points (except at the final approach point). This leads them to plan climbs and descents according to the optimum profile to reduce fuel consumption and maximise efficiency. However, if the aircraft is descending along the optimum descent path, its level at the route intersection may be dangerously close to the level of the aircraft climbing along the optimum path. This shall be prevented by ATC by ensuring vertical separation between aircraft. For safety reasons, both aircraft will stabilize at instructed levels and await clearance for further climb/descent. It is this so-called level-off that has a big impact on flight efficiency, fuel consumption, emissions and completely neglects the principles of CDO/CCO [2, 12].

Therefore, the goal of this paper is to reach a compromise and separate the aircraft vertically at the intersections in advance. This is done by establishing fixes with vertical restrictions already on the arrival and departure routes and their charts. This method will get as close as possible to 3D strategic route de-confliction.

2.2. IDENTIFICATION AND CALCULATION OF KEY POINTS ALONG THE ROUTE

The second step involves the identification and calculation of key points along the route, where newly calculated restrictions will be implemented. The shortest distance between two points on the surface of a sphere is a great circle, with the centre of the circle located at the centre of the sphere, known as the orthodrome. However, the Earth is not a perfect sphere but rather resembles an oblate spheroid, which is important in cartography [8]. Aircraft equipped with Performance Based Navigation (PBN) specifications, such as RNAV 1 applied on STAR arrival routes into Prague, follow orthodromic segments between the points along the route [13, 14]. To establish a buffer zone that ensures the minimum separation is not violated during continuous altitude changes on the flight route, it is necessary to calculate the coordinates of the points that define this space. This calculation involves several steps and transformations between coordinate systems. Initially, it is essential to verify that all segments between which the intersection point is being sought have a length greater than zero. If the length is zero, a solution is not possible [13, 15]. Additionally, it is necessary to work with geographic coordinates in radians. An important condition is the conversion of coordinates from the sexagesimal system to the decimal system if they are not already in decimal form [16].

To determine the intersection of two planes containing orthodromes, vectors must be created from the points defining the orthodromic segments. These vectors are then used to calculate the intersection of the planes, which determines the intersection of the orthodromes [16]. The calculation of this intersection is crucial for identifying points where aircraft should cross.

The calculation process is as follows:

(1.) The coordinates in decimal form are converted to radians according to the Eq. 1, where ϕ and λ represent the latitude and longitude respectively.

$$\begin{pmatrix} \phi \\ \lambda \end{pmatrix} = \begin{pmatrix} lat \cdot \frac{\pi}{180} \\ lon \cdot \frac{\pi}{180} \end{pmatrix} \tag{1}$$

(2.) The next conversion is to the Cartesian coordinate system, where two sections of orthodromic lines are expressed by four points: *P1*, *P2*, *P3* and *P4*. These points are expressed in longitude and latitude coordinates in radians. The conversion to the Cartesian coordinate system is as follows in the Eq. 2.

$$P = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} N(\phi)\cos\phi\cos\lambda \\ N(\phi)\cos\phi\sin\lambda \\ \frac{b^2}{a^2}N(\phi)\sin\phi \end{pmatrix} \tag{2}$$

(3.) Here the radius of curvature in the plane perpendicular to the meridian (transverse radius of curvature) is calculated in the Eq. 3.

$$N(\phi) = \frac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin\phi}} = \frac{a}{\sqrt{1 - e^2\sin^2\phi}} \tag{3}$$

The variables a, b are called the major and minor semi-axes of the meridian ellipse. The constant e is then the (first) eccentricity. The values of a, b, e depend on the geodetic reference system. For international aviation, this is the WGS-84 system according to the ICAO Annex 15 [17] (4).

$$\begin{aligned} a &= 6\ 378\ 137\ m \\ b &= 6\ 356\ 752{,}31425\ m \\ e &= 8{,}1819190842622 \cdot 10^{-2} \end{aligned} \tag{4}$$

(4.) To determine the intersection, it is necessary to find the intersection of the two planes of which these orthodromes are a part. This intersection will be in the form of a line which, intersecting the surface of the ellipsoid, defines 2 points on the ellipsoid. The general equation of the plane is as depicted in the Eq. 5.

$$ax + by + cz = 0 \tag{5}$$

(5.) From the points P_1 and P_2 (defining the segment of the first orthodrome) a vector $\vec{V_1}$ is created and from the points P_3 and P_4 a vector $\vec{V_2}$ is created (6).

$$\vec{V_1} = P_1 \times P_2 = \begin{pmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{pmatrix} = \begin{pmatrix} y_1z_2 - y_2z_1 \\ x_2z_1 - x_1z_2 \\ x_1y_2 - x_2y_1 \end{pmatrix} \tag{6}$$

(6.) From the vectors $\vec{V_1}$ and $\vec{V_2}$ come unit vectors $\vec{U_1}$ and $\vec{U_2}$ (the individual components of the vector are divided by its length) (7).

$$\begin{aligned} \left|\vec{V_1}\right| &= \sqrt{{v_{1x}}^2 + {v_{1y}}^2 + {v_{1z}}^2} \\ \vec{U_1} = \begin{pmatrix} u_{1x} \\ u_{1y} \\ u_{1z} \end{pmatrix} &= \begin{pmatrix} \frac{v_{1x}}{\left|\vec{V_1}\right|} \\ \frac{v_{1y}}{\left|\vec{V_1}\right|} \\ \frac{v_{1z}}{\left|\vec{V_1}\right|} \end{pmatrix} \end{aligned} \tag{7}$$

(7.) To calculate the equations to which the orthodromes belong and which determine the intersection we use the following formula (8)

$$\begin{aligned} \rho_1 &: u_{1x}x + u_{1y}y + u_{1z}z = 0 \\ \rho_2 &: u_{2x}x + u_{2y}y + u_{2z}z = 0 \end{aligned} \tag{8}$$

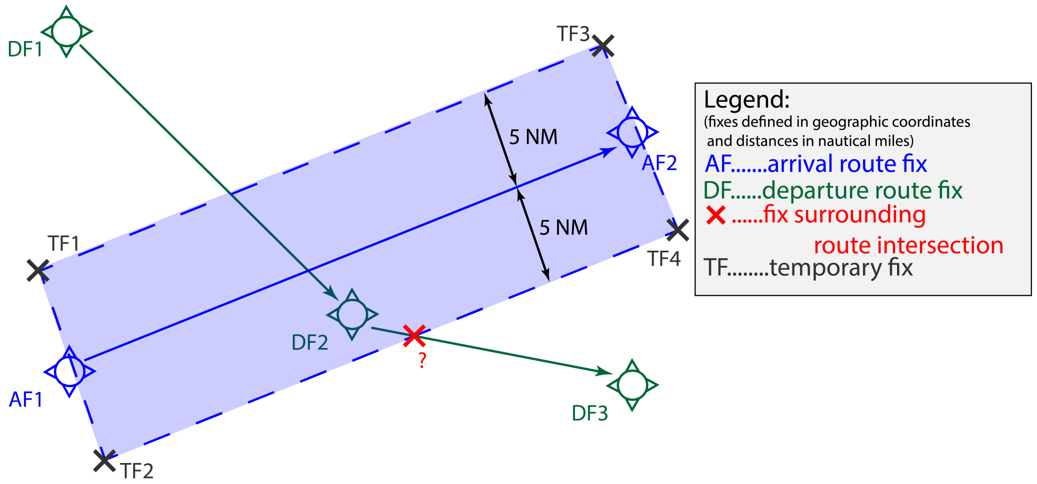


Figure 1. Buffer zone around a straight route. The red intersection indicates the position of the first possible point where an altitude change can be made in the expected direction of flight.

(8.) The planes ρ_1 a ρ_2 intersect in one line which has a direction vector $\vec{D}$ (9).

$$\vec{D} = \vec{U_1} \times \vec{U_2} = \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \begin{pmatrix} u_{1y}u_{2z} - u_{2y}u_{1z} \\ u_{2x}u_{1z} - u_{1x}u_{2z} \\ u_{1x}u_{2y} - u_{2x}u_{1y} \end{pmatrix} \tag{9}$$

(9.) This line already directly corresponds to the searched orthodromic intersections. And from the direction vector, we can directly calculate their geographic coordinates in degrees according to the Eq. 10, where the angle measured from the origin of the coordinate system to $\vec{D}$ is the angle θ – geocentric latitude.

$$\begin{aligned} \tan\theta &= \frac{d_y}{d_x} = \left(1 - e^2\right)\tan\lambda \\ \lambda &= \tan^{-1}\left(\frac{d_y}{d_x\left(1 - e^2\right)}\right) \\ \phi &= \tan^{-1}\left(\frac{d_z}{\sqrt{(1 - e^2)({d_x}^2 + {d_y}^2)}}\right) \end{aligned} \tag{10}$$

(10.) The resulting point is the point X with coordinates ϕ and λ after correct conversion back from radians to degrees (11).

$$X = [\phi; \lambda] \tag{11}$$

The whole calculation was then written in the Matlab programming language into the function intersection, where the input parameters are the coordinates of the points defining the beginning and the end of the orthodrome in the decimal system. The output parameters are also the coordinates in the decimal system of the intersection of these orthodromes.

For safety reasons, an ATCO cannot issue a clearance that would reduce the minimum vertical or horizontal separation. Aircraft must remain vertically separated by 1 000 feet or horizontally by 5 NM at all times [18]. Therefore, there have been created protection areas to their corresponding routes with a minimum width of 5 NM. Once the aircraft are at a point 5 or more miles from the intersection, they may change their level, ensuring their trajectories remain non-conflicting. The protection area around the routes determines the new intersection point along the route direction, which is the first possible point where a change in vertical restriction can be defined. However, determining such a point on the routes also involves another limitation: identifying individual points and turns along the routes. To ensure the minimum separation of 5 NM around fly-by and fly-over fixes, Vincenty's equations were used [19, 20]. An additional buffer must be created in turn areas to allow for further protection area required by turns at fly-by waypoints [21]. These turns are defined by the ETP (Earliest Turning Point), which marks the beginning of the turn's buffer zone. This ensures that it meets all criteria for maintaining the minimum separation, essential for the safe and efficient operation of aircraft along the route [18, 21]. A graphical representation of the buffer zone for a straight segment of the route is shown in Figure 1 and for a route with a turn in Figure 2.

2.3. DETERMINATION OF VALUES FOR VERTICAL RESTRICTIONS

In the third step, it was necessary to determine the values for vertical restrictions. These must be defined with sufficient precision, and the actual traffic at LKPR must be taken into consideration. The goal is not just to ensure a safe change in altitude without any probability of infringing on the minimum prescribed separation. The proposed concept must also reflect the performance characteristics of the aircraft operating in this space and consider the airspace organisation, including possible sectorisation.

To determine the most commonly used altitudes at the newly established intersection points that define the buffer zone, data from the OpenSky network were

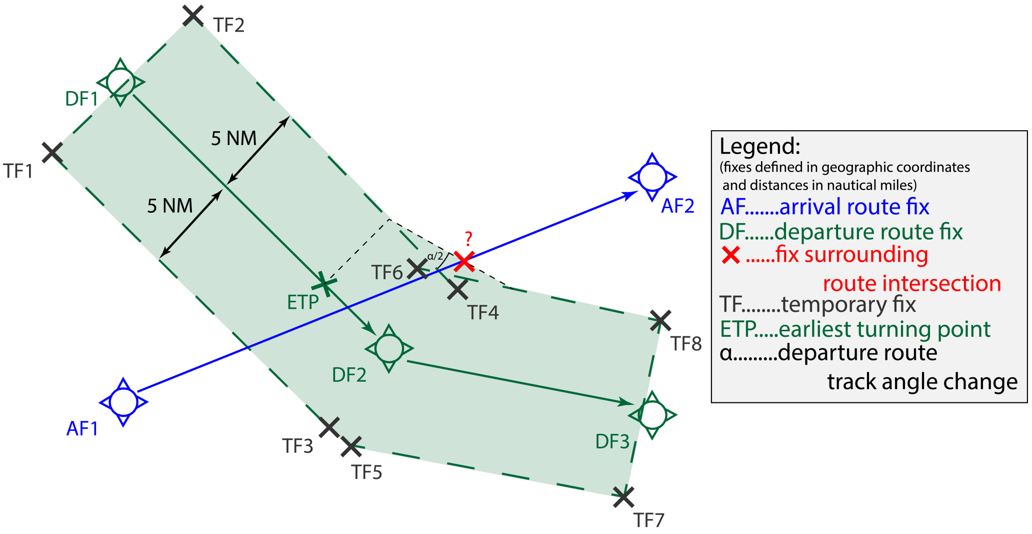


Figure 2. Buffer zone around a route with a turn. The red intersection indicates the position of the first possible point where an altitude change can be made in the expected direction of flight.

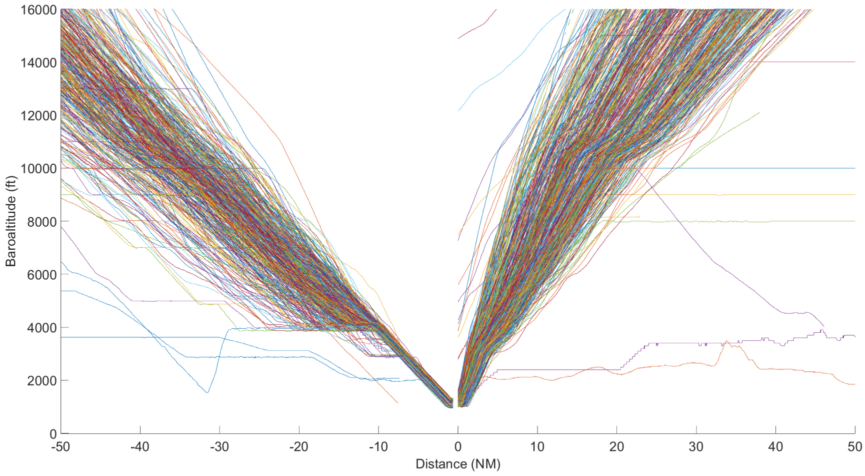


Figure 3. Graphical representation of aircraft vertical position relative to LKPR [22].

utilised [23]. OpenSky collects data from secondary surveillance radar transponders and Automatic Dependent Surveillance – Broadcast (ADS-B). Data from aircraft arriving and departing from LKPR were selected from days with the highest traffic, as during peak times, aircraft are forced to use the entire length of the arrival route and gradually descend or climb according to available altitudes. The data was filtered to include only aircraft flying under IFR (Instrument Flight Rules) departing from or arriving at LKPR. The graphical representation in Figure 3 provides an overview of all the mentioned arrivals and departures. The y-axis represents altitude in feet above the standard pressure setting of 1 013.25 hPa. The x-axis represents the distance of a given arriving aircraft from the touchdown point. For departures, the x-axis shows the distance from takeoff. This illustrates the relationship between the aircraft's altitude and their distance from takeoff or landing. Flights that did not have Prague Ruzyně as their departure or destination airport were intentionally filtered out from this analysis. The resulting vertical restrictions will be determined by rounding to the nearest available level to ensure that arriving and departing aircraft maintain a minimum separation of 1 000 feet. The restrictions will be plotted on charts at fixes at least 5 NM from the intersecting routes. If any fixes are too close to already existing ones, they will be merged, and vertical restrictions will be added to them.

The graph in Figure 3 shows that some flights reach the altitude of the intermediate approach before the final approach fix. An aircraft performing an ILS approach must intersect the glide slope from below rather than catching up with it during descent. For departures, minor fluctuations can be observed, primarily due to acceleration and the conversion of vertical speed into forward speed. When regression is applied to these data, an average level is obtained,

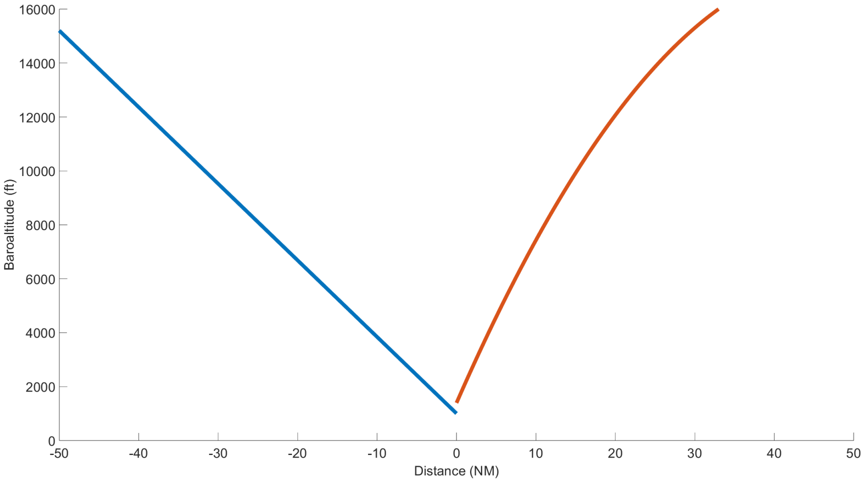


Figure 4. Display of Vertical Profile Regression for Arrivals (Blue) and Departures (Red) at LKPR [22].

representing the altitudes at which the aircraft are located. The graphical representation of this curve in Figure 4 illustrates the dependence of aircraft altitude on their distance from takeoff/landing, where the descent for arrivals is linear, and the climb for departures follows a second-degree polynomial, representing the ideal altitude for all flights at LKPR. This data determines the optimal altitude at previously identified intersection points. The traffic composition at LKPR is very consistent in terms of aircraft types. Therefore, the measured values are very similar and realistically achievable for the vast majority of types without any issues. Based on this data, vertical restrictions can be applied to the calculated intersection points of the buffer zone. It is also necessary to ensure that another ATC sector is not infringed upon. The aircraft must always remain within the sector of a single controller, except when handed over to the next one. The crew can then adjust the descent profile considering these limiting points and prepare the aircraft for the next phase of flight according to operational needs. These flight phases are also ideal for ATC to manage forward speed. Before the aircraft reaches the point for the next level change, any intervention in forward speed will have a minimal impact on the descent profile. The earlier the forward speed is managed, the less it will affect the reacquisition of the ideal descent profile.

2.4. Implementation of the vertical restrictions into the simulation environment and validation

The fourth and final step is to implement the points and calculated vertical restrictions into the ESCAPE Light simulation environment and validate them. After creating a standalone model of vertical paths and verifying feasibility, actual traffic, including its vertical positions on the paths, was compared with the model in the BADA (Base of Aircraft Data) system [24]. Buffer zone conditions were then established to ensure that aircraft at no point on the paths fall below the minimum prescribed separation, which is either a horizontal distance of 5 NM or a vertical distance of 1 000 feet [18].

Thanks to the lateral strategic deconfliction combined with the vertical profile, complete traffic separation is achieved, where each aircraft is pre-informed of the range of flight levels within which it may operate, and when it may leave the limiting altitude. Ideally, the crew adjusts the vertical speed so that the aircraft remains in constant vertical motion and reaches the limiting point at an altitude that allows it to continue vertical movement without needing to stop the climb at the published limiting altitude. The air traffic controller can then issue a clearance to climb up to the boundary of their area of responsibility, with the aircraft required to comply with the vertical restrictions duly marked on the chart. An exception is the ATCO's clearance, which directly cancels the vertical restriction at specific points, allowing unrestricted climb at those points. The restriction is also lifted in the case of route shortening beyond the points with altitude limitation or providing navigation guidance through vectoring [18].

Similarly, an arriving aircraft managed by the Arrival control unit may be cleared for descent to the intermediate approach altitude with a single instruction at the beginning of communication. This instruction obliges the crew to adhere to the limiting altitudes at each point unless otherwise specified. In such a case, the aircraft cannot move outside the specified range, and the crew is informed when the limitation expires and can continue changing the vertical position in the expected direction of flight [25]. The entire concept's feasibility from the ATC perspective was validated in a synthetic ATS environment using the Eurocontrol ESCAPE Light simulator. This simulator calculates the 4D trajectory of aircraft in real-time, modelling according to the BADA dataset, and stores the results at intervals of 5 seconds, corresponding to the refresh rate of the surveillance system. Calculations are based on the distance travelled and the individual weight of each aircraft, considering the average amount of fuel on board. The aim is to assess the real-world data against the values obtained from the BADA model. If these values do not differ significantly, the proposed values can be considered valid. A significant deviation is considered when values fall outside the overall range of the box plot.

The subjects involved in the experiment were individuals with real-world air traffic control experience at the APP Prague unit or were in the training phase for this unit. Each participant had at least 200 hours of experience at the APP Prague unit, either with real-world operations or in a simulated environment. This level of experience was deemed sufficient for this experiment. The experiment consisted of two simulations: one using the current air traffic control methods and the other using the proposed vertical profiles. The operational conditions, scenario, and traffic composition were identical in both simulations. Each subject chose the order in which the simulations were conducted individually. The simulated scenario was designed based on the traffic mix operating at LKPR. According to 2019 data, the most frequently operated aircraft types were the Boeing 737 series (38.3 % of total traffic) and the Airbus A320 series (30.4 % of traffic). Turboprop aircraft, such as ATR 72 and Dash-8, accounted for 10.3 % of operations. Other aircraft types, including long-haul flights and smaller aircraft for individual transport, comprised only a few percent of total traffic. In terms of traffic distribution by wake turbulence categories, the Medium category was the most represented in 2019, with a 92.8 % share of total traffic. The Light and Heavy categories were represented equally, at 3.6 % each [26, 27]. This statistic indicates that aircraft with similar performance and weight characteristics dominate LKPR. Their aerodynamic and physical properties during flight are thus very similar.

To verify feasibility from the perspective of pilots and aircraft operations, the Prepar3D and X-Plane 12 flight simulators were used. Simulations were conducted with Boeing 737-800 and Airbus A320 models, which, including their various variants, represent the majority of movements at LKPR. Runway 24 was selected for the design, as it is the most frequently used runway at LKPR, with a maximum capacity of 46 movements per hour [7, 26].

3. Results

The final proposal of procedures with the vertical profile is illustrated in Figure 5. The procedures include:

- Intersections determining a five-mile safety zone of the routes (INT1 through INT4).
- New waypoints (LVL1 through LVL8).
- Vertical restrictions (labelled as FL80).

New points were created by establishing a circle with a diameter of 5 NM centred on each INT and standard routes. These were named LVL1 through LVL8 and determine vertical limitations on the routes. They also respect the boundaries of responsibility areas, which are always 2.5 NM from the point whose centre is the standard route. Except for LVL5, all these points are derived based on the same principle.

LVL5 was defined differently, as it is part of the standard departure route for propeller aircraft, where the first turn is determined by a point corresponding to an altitude of 1 700 feet AMSL. However, reaching such a turning point is irrelevant in terms of time and space, as more powerful aircraft will reach it sooner and with less distance travelled from the airport than lower-performing aircraft. Moreover, the first fix of this departure route is located at such a distance from the airport that vertical restrictions could not be applied.

To create a buffer space for aircraft on the LOMKI8S arrival route, which intersects with these departures, and to ensure continuous descent and ascent, LVL5 was established as an intersection located 5 NM from the line connecting the points forming the arrival route. Since the aircraft's position before reaching the first fix of the departure route can vary, LVL5 was determined to be 11.5 NM from VOR/DME OKL, located at LKPR. This distance is sufficient to ensure that an aircraft in any possible position is at least 5 miles from other traffic.

The process of creating all LVL points was designed to avoid creating redundant points that could complicate procedures and make them unclear. An increased risk of errors could also arise in onboard spatial navigation systems, mainly if LVL points are located near fly-by fixes. In cases where such situations occurred, the LVL point was cancelled and merged with the nearest fix from the INT point in the flight direction.

For each INT point, the flight level at which aircraft are located in real operations was determined, which served as the basis for setting vertical restrictions. The procedure is illustrated in Figure 6a. The red line represents the median of the data, while the black horizontal lines above and below the box plots indicate the maximum and minimum altitudes. The rectangular outline shows the range of flight levels in which 75 % of the traffic is located. Based on this information, vertical restrictions would be set at FL90 for departures and FL100 for arrivals after consulting the median with other values, such as the average.

Values from the BADA database were included in the box plots, as shown in Figure 6b. According to BADA simulations, blue dots represent the ideal flight level for each aircraft type. This approach allowed for the precise design of optimal flight levels, determining vertical restrictions at the given point. Based on these values, the optimal level for each LVL point was set, considering the most frequent levels and safety and

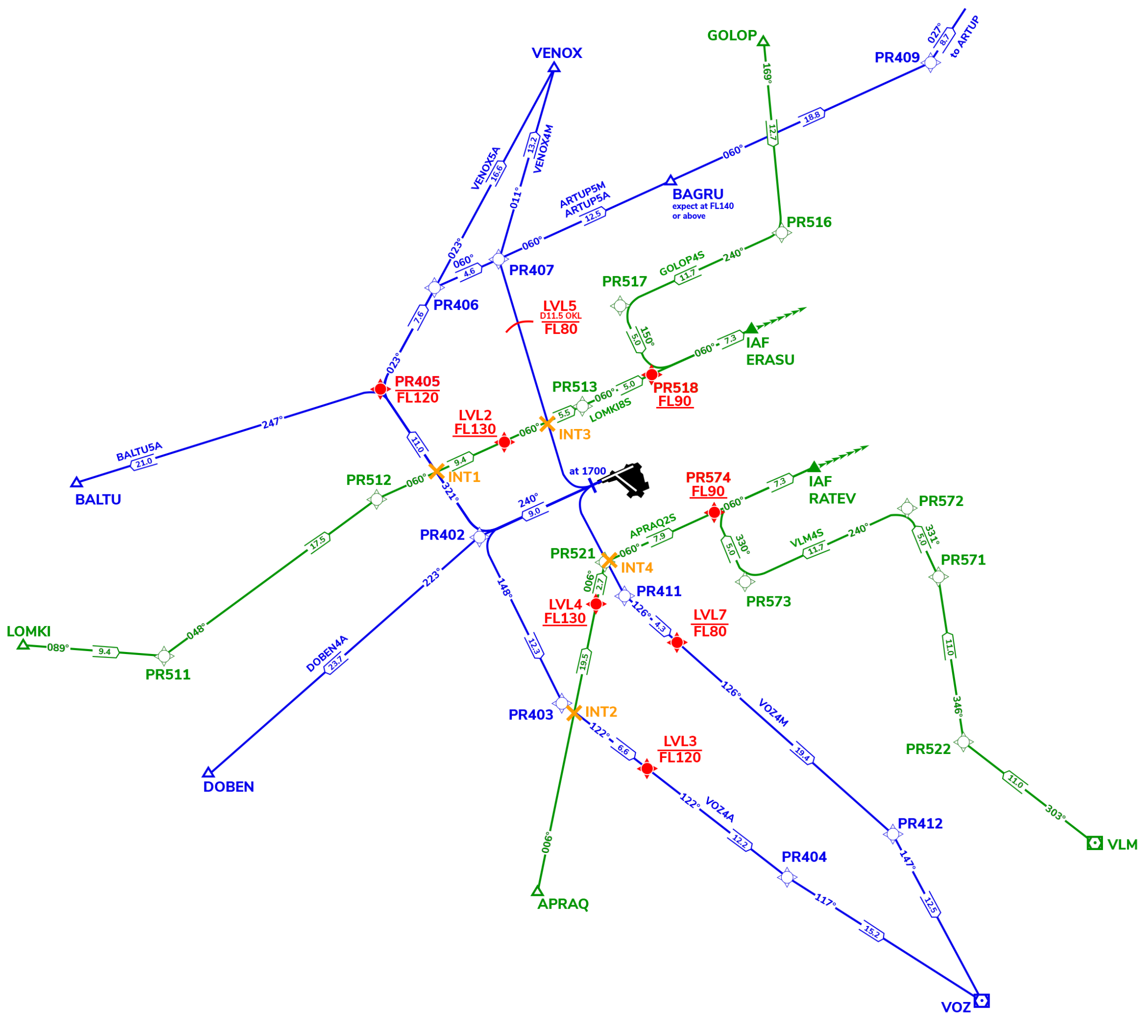
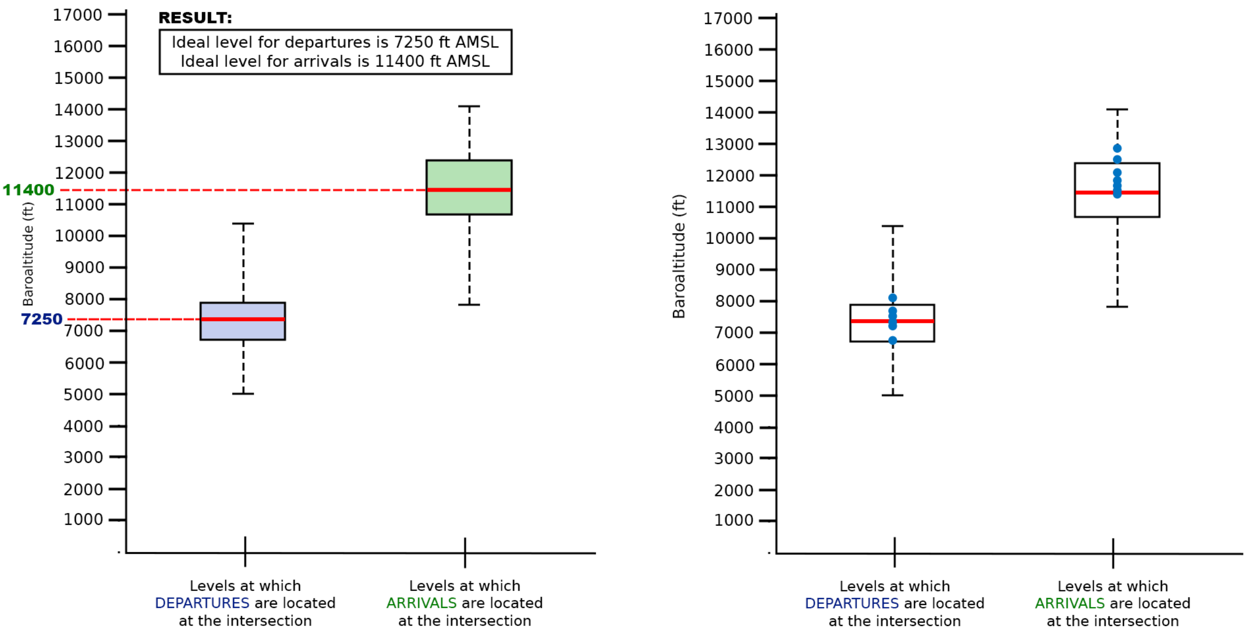


Figure 5. Schematic map of LVL point locations including aircraft vertical position restrictions [28].



(A). Box plot based on the air traffic at LKPR from the OpenSky database.

(B). Box plot based on the air traffic at LKPR compared to BADA.

Figure 6. Illustrations of the box plots on the basis of which the level restrictions were determined.

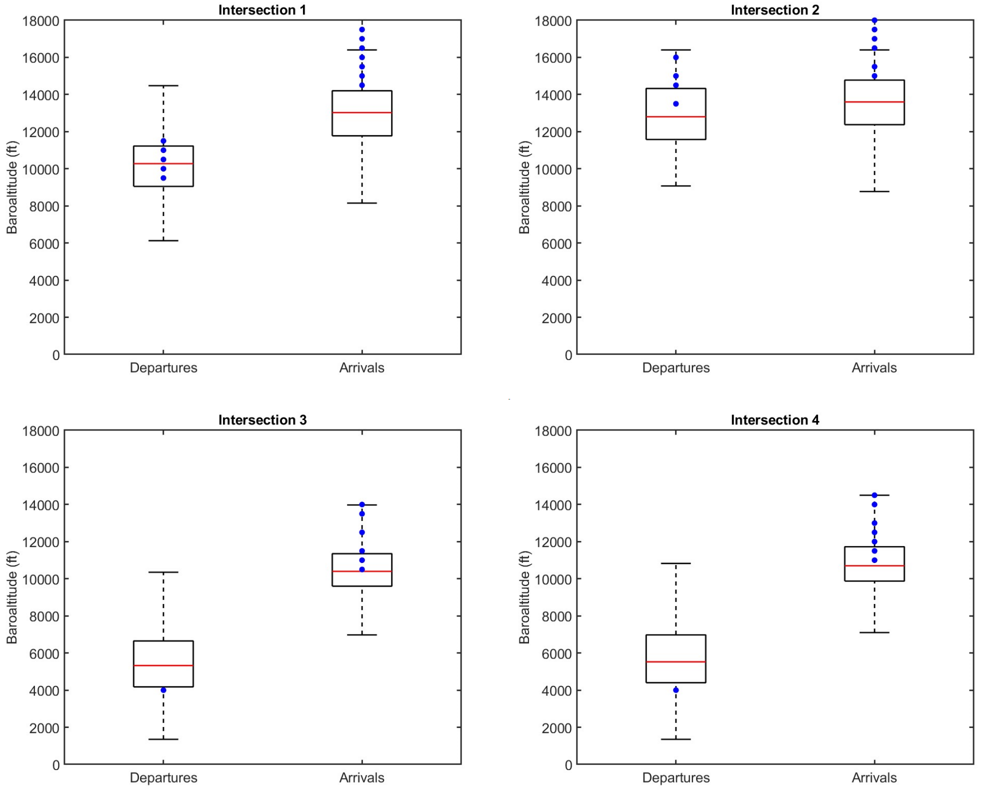


FIGURE 7. Boxplots of the vertical profile of traffic at LKPR compared to BADA.

operational factors. This level was included in the standard arrival and departure route maps as a fixed vertical restriction limit. Aircraft must adhere to these restrictions unless otherwise directed by ATC. This information must also be part of the map.

After assessing the actual traffic and the BADA database, the information was systematically processed and integrated into analogous box plots. A box plot was created for each INT intersection to represent the vertical position of aircraft during departures and arrivals at LKPR. These box plots, illustrated in Figure 7, served as the basis for determining vertical restrictions at points referred to as LVL. According to the median (red line) and the average, most departures are near flight level FL100. This level also corresponds to the optimal values that departing aircraft should reach according to the BADA database (blue dots). Arrivals, on average and according to the median, are approximately at FL130 above the intersection of the routes, while the BADA database recommends an optimal level even higher, around FL160. Therefore, the appropriate vertical restrictions on the routes were set at flight levels FL130 for arrivals and FL120 for departures. The same approach was used to determine restrictions for other routes.

The aircraft's ability to meet the new route requirements was verified in the ESCAPE Light and X-Plane simulation environments, with the obtained data subsequently processed into graphs shown in Figures 8 and 9. In both cases, a reduction in segments with horizontal flight was observed. During descent, the horizontal segment appears mainly at an altitude of 4 000 ft or just before reaching it. This is the part of the flight where the crew prepares the aircraft for the final approach phase. The aircraft must be slowed down, which is related to the deployment of the mechanism and achieving the descent plane of the ILS from below.

Considering the aerodynamic characteristics of the most frequently used aircraft at LKPR, it should be expected that aircraft will not be able to reduce speed and descend simultaneously. Some crews, therefore, prefer to reach the intermediate approach altitude earlier so that they can lose speed by inertia before reaching the final approach point. The ILS approach was not included in the measured data, as the ESCAPE system lacks sufficient accuracy in calculating the vertical trajectory on the final approach route. However, this fact represents only a limitation of the simulation environment considered in the analysis. Based on data obtained from actual operations, it can be assumed that aircraft would descend similarly after passing the final approach point, even when implementing the proposal discussed in this article. The vertical trajectory shown in Figures 8 and 9 reveals a somewhat irregular descent after passing the final approach point, located approximately 10 miles from Runway 24 in Prague. Since aircraft would continue to approach and follow the same trajectory as in current

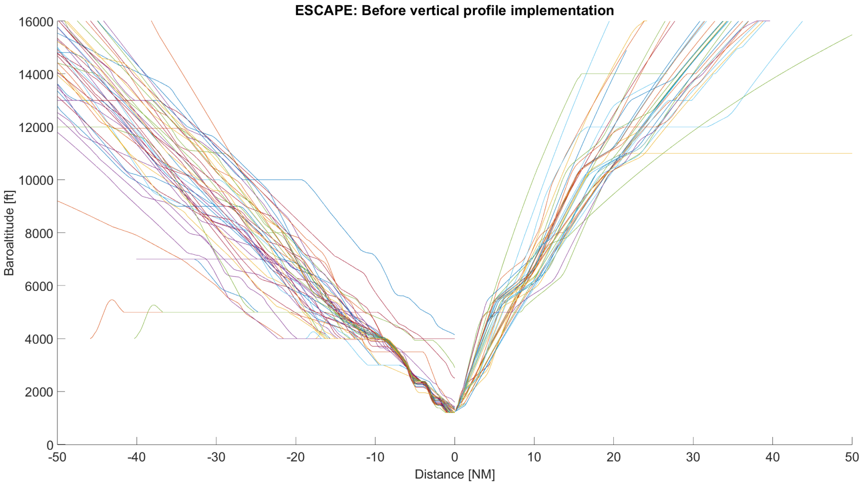


Figure 8. Graphical representation of the vertical position of simulated aircraft relative to LKPR before the implementation of new procedures.

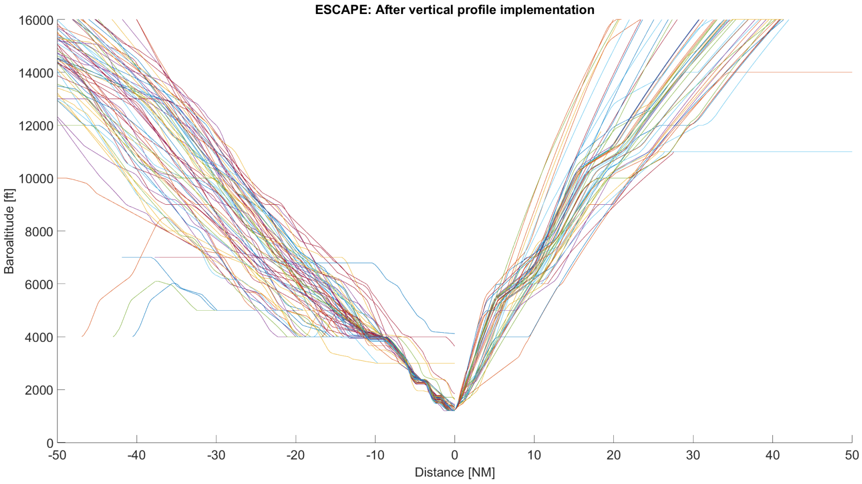


Figure 9. Graphical representation of the vertical position of simulated aircraft relative to LKPR after the implementation of new procedures.

practice, the values after passing the final approach point are not considered relevant for this research.

The same procedure was applied in the X-Plane simulator environment on a Boeing 737-800 aircraft. The vertical profiles shown in Figure 11 also show a significant reduction in horizontal segments before reaching the altitude of 4 000 ft AMSL, where the aircraft configuration and speed reduction subsequently occurred to initiate the next approach phase.

4. Discussion

The analysis of the box plots for departing aircraft suggests that their trajectories in real operations align with the optimal performance profile. This alignment is facilitated by onboard computers that automatically adjust thrust regulation systems based on real-time calculations. These findings align with expectations, as modern aircraft are equipped with advanced flight optimisation systems that automatically manage all phases of flight to maximise safety and efficiency. The resulting trajectories and performance profiles closely match the mathematical BADA models, confirming these systems' high accuracy and reliability in current air traffic operations.

However, the analysis of BADA models for turboprop aircraft, particularly concerning points INT3 and INT4, revealed a significant discrepancy in the climb gradient between simulated models and actual operations. Specifically, BADA models show a lower climb gradient at high aircraft weights than what is observed in real operations. As a result, the BADA model lags behind the real-world data box plot. Since BADA models operate with standardised values, deviations from reality occur, particularly in scenarios requiring a higher climb gradient, such as noise abatement procedures. At LKPR, the operational requirement for a higher climb gradient is set at 10 % up to 3,500 feet for departure routes heading north through INT3. This requirement can lead to even higher climb gradients in actual operations, reflecting the difference between simulated and real-world data. Real operations show that arriving aircraft are at slightly lower altitudes than expected according to optimal trajectories per the BADA models. This deviation can be attributed to pilots initiating descent earlier to reach the lowest permitted altitude before starting the initial phase of instrument approach while also decelerating and configuring the aircraft for landing. In practice, aircraft often deviate from the entire planned trajectory of the initial and intermediate approach, with vectoring onto the runway axis for the final approach already occurring when the aircraft is parallel to the runway. When vectoring begins, onboard systems for maintaining the aircraft's optimal vertical position disconnect, as the aircraft no longer follows the planned route. Similarly, the aircraft onboard systems bypass such limitations during trajectory adjustments by vectoring to a point outside the prescribed limitations.

After implementing the vertical profile, changes were observed in pilot behaviour during departures and arrivals. Simulations indicate that departing traffic ascends at a lower vertical speed. Where feasible and desirable in terms of operational procedures, energy is converted into higher forward speed at the expense of faster vertical speed. This change results in smoother operations, fuel savings, and improved passenger comfort. Conversely, arriving traffic remains longer at higher flight levels, using the published vertical profile to determine optimal flight levels, which should better align with the aircraft's performance profile.

The realism of simulated data may have been affected by the inherent limitations of the simulation environments. The ESCAPE simulation system does not have access to all the parameters that pilots in real cockpits typically use. The calculations performed within the simulations depend on input data corresponding to BADA models and any interventions by pseudo-pilots, who simulate the behaviour of aircraft and crews and are responsible for controlling all aircraft and communicating with air traffic controllers. Given the large number of tasks and the higher number of aircraft pseudo-pilots must control simultaneously in the simulator, their ability to realistically account for all aspects of the situation in the aircraft is limited. Simulators used for measuring on Boeing 737 and Airbus A320 aircraft also have limitations. The simulations did not include any non-standard operational states, such as system degradation, non-standard flight conditions, weather impact, or special crew requests. Such situations could significantly affect the research results, with each change forcing air traffic controllers to revert to traditional control methods.

Comparing the situation before and after the introduction of vertical restrictions, as illustrated in Figures 10 and 11, there was a noticeable decrease in segments where aircraft flew in level flight (with "kinks" in the trajectory) during arrivals. Conversely, the segments where aircraft remained at the intermediate approach altitude of 4 000 feet AMSL were extended. This change was likely driven by two main factors. First, the relatively high landing weight of the aircraft, around 65 tons, required more time to reduce forward speed before beginning the final approach phase. This could only be achieved in level flight. Second, the test pilots' lack of experience with the new procedure led to a preference for descending before braking, with speed reduction occurring only in the last phase of arrival after reaching the intermediate approach altitude. This descent method contributed to fuel savings, as minimal engine thrust was applied to reduce speed even during level flight.

After implementing the new procedures, the average fuel consumption across all simulations decreased by 13.2 %. The most significant fuel consumption reduction was observed in the Boeing 737-800 simulations in the X-Plane environment, where consumption decreased by nearly 25 % compared to values before implementing the vertical profile. On the other hand, due to the time-consuming nature of the measurements, a relatively limited data set was obtained. Acquiring a more extensive dataset, whether in a simulation environment or real operations, could provide additional valuable insights into the vertical movement of aircraft and their fuel consumption. The simulations almost matched the expected behaviour of air traffic on arrival and departure routes before and after implementing the vertical profile. Nevertheless, slightly higher fuel savings were expected in the ESCAPE environment, which did not reach the anticipated levels. While simulations and models revealed some deviations and limitations, they also showed that the new methodology offers promising possibilities for further improvement and adaptation in real operations. Further research should focus on various scenarios, operational conditions, or the impact of weather on feasibility to obtain even more accurate and comprehensive data.

5. Conclusion

The proposed system of vertical profiles for arrival and departure routes at TMA Prague significantly improves air traffic efficiency and safety. This system includes the establishment of points that limit the vertical positions of aircraft, leading to the adaptation of their trajectories, which allows for a smoother flow of arriving and departing aircraft. By setting vertical limitations, the risk of infringing on the minimum required separation between aircraft at crossing routes is minimised. Optimisation processes, such as those discussed in this article, contribute to the safe flow of air traffic and the sustainability of aviation. The concept under study is particularly beneficial during

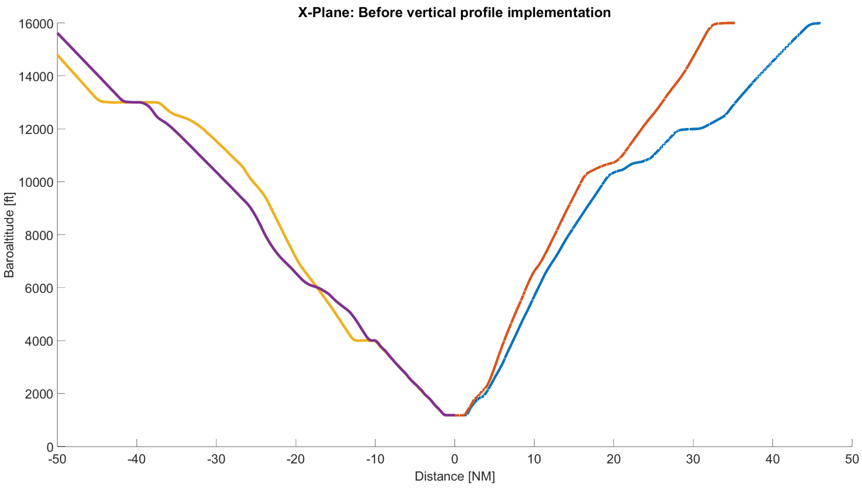


Figure 10. Graphical representation of the vertical position of simulated aircraft relative to LKPR before the implementation of new procedures.

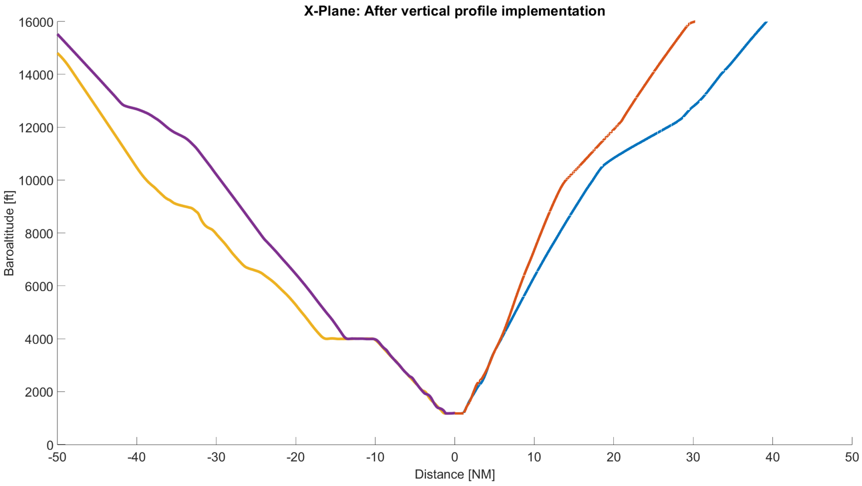


Figure 11. Graphical representation of the vertical position of simulated aircraft relative to LKPR after the implementation of new procedures.

peak operational times, where its advantages are most pronounced.

Simulation studies conducted in the ESCAPE and X-Plane environments demonstrated substantial fuel savings, averaging 13.2 %, with more significant savings of over 25 % observed in the Boeing 737-800. These results confirm that applying vertical profiles positively impacts fuel consumption and contributes to more environmentally friendly operations. Although the specific fuel savings when using the new methods may vary, the results indicate that this system is advantageous in terms of both the workload on flight crews and fuel consumption. While the simulations revealed some deviations and limitations, primarily due to inaccuracies in simulated conditions, the results show a clear positive trend. Future research should focus on obtaining a broader spectrum of data and including various operational scenarios that could negatively impact the operational situation. If further research yields positive results, the introduction of vertical profiles in TMA Prague could be realistically considered. Prague Airport would then move among the most modern airports of the 21st century, offering more efficient and environmentally friendly air traffic control services.

List of symbols

ϕ	Latitude [°]
λ	Longitude [°]
$N(\phi)$	Transverse radius of curvature [m]
a	Semi-major axis [m]
b	Semi-minor axis [m]
e	Eccentricity
θ	Geocentric latitude [°]

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