Problem-solving teaching strategies from the perspective of dividing the core literacy of mathematical operations

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Abstract. Mathematical operation literacy is one of the six core literacy proposed in the new Chinese senior high school curriculum reform round. Based on the problem-solving teaching link of the high school mathematics conic section, this paper is based on the connotation and method of mathematical operation literacy cultivation. Combined with the problems that high school students have in the problem-solving of conic sections, we innovate the teaching strategies for solving conic sections, improve students' literacy of mathematical operations in classroom teaching, and help teachers teach conic sections better. To enable students to grasp the problem-solving method of the conic section better.

Keywords: mathematical operation literacy; conic section; teaching strategy.

1. Introduction

As one of the six core competencies in the curriculum reform of ordinary senior high schools in China, mathematical operation plays an important role in students' high school mathematics learning and even lifelong learning and development. However, in teaching research, the author found that there is still a big gap between the mathematical operation ability of high school students and the requirements of the new curriculum standard. In response to these problems, the author took the teaching process of conic section problem solving as an example, combined with the cultivation of mathematical operation literacy, innovated the conic section problem-solving teaching strategy, and gave corresponding improvement suggestions, aiming to improve the mathematical operation literacy of high school students and teachers. The methods and paths of cultivating this literacy provide certain theoretical references.

2. Definition of related concepts and research on the current situation

2.1 What are mathematical operations

In 2017, the Ministry of Education recently issued the "Mathematics Curriculum Standards for Full-time Compulsory Education (2017 Edition) " [1] , which proposed six aspects covered by the core literacy of mathematics, namely " logical reasoning, mathematical abstraction, mathematical modeling, intuition. Imagination, Mathematical Operations, Data Analysis". As one of the six core competencies, "a mathematical operation " is not only directly related to the cultivation of other competencies but also has a certain impact on students' theoretical study and practical content in other majors. Then, what is the specific meaning of mathematical operation. The author summarizes it into two aspects: "mathematical operation literacy" and "mathematical operation ability," combined with the latest curriculum standards at this stage.

"Mathematical operation literacy" refers to the literacy of solving mathematical problems according to the algorithm based on clarifying the operation object. That is the personality and psychological quality reflected by the students during the mathematical operation. [2] From the perspective of students' mathematical operation literacy, its main characteristics are innovation, rationality, flexibility, accuracy, and convenience. From the perspective of content, mathematical operation literacy includes symbolic operation literacy and number operation literacy; from the perspective of operation process, mathematical operation literacy consists of analyzing operation

conditions, clarifying operation direction, determining operation steps, selecting operations formulas, etc.

"Mathematical operation ability" refers to the practical process of correctly understanding and analyzing conditions, carrying out simple estimation and approximate calculation, forming operation ideas, selecting reasonable operation rules and methods, and obtaining accurate operation results. Different learning stages have different requirements for mathematical operation ability from elementary school, junior high school to high school, different learning stages have different requirements for mathematical operation ability, but mathematical operation runs through the entire mathematics learning process.

Different from primary and secondary schools, computing ability requirements differ. After entering high school, the breadth and difficulty of knowledge are greatly increased, and the mathematics problems are becoming more and more comprehensive. Solving mathematical operations sometimes require a high level of understanding of the nature of the relevant concepts. When faced with a comprehensive practice question, it is often necessary to choose a strategy after fully understanding the meaning of the question. It is essential to choose a more concise operation method according to different operation properties.

2.2 The importance of conic section in high school mathematics teaching and problem identification

In the "General High School Mathematics Curriculum Standard (2017 Edition)" [3], there are the following requirements for the conic section: be able to establish the equations of plane lines and circles according to different situations, establish the standard equations of the ellipse, parabola, and hyperbola, be able to use algebraic methods to study the basic relationship between the above curves, and be able to solve some simple, practical problems using the idea of plane analytic geometry. Focus on improving the literacy of intuitive imagination, mathematical operations, mathematical modeling, logical reasoning, and mathematical abstraction.

Therefore, the conic section is an indispensable link in high school mathematics teaching, and mathematical operation ability is essential for solving problems. "General High School Mathematics Curriculum Standards (2017 Edition)" [4] pointed out that the senior high school mathematics curriculum is the main curriculum of public senior high schools, which is basic, selective, and developmental, creating conditions for students' sustainable development and lifelong learning. Under the "New College Entrance Examination" background, the conic section is arranged in the subject of "Geometry and Algebra" in the optional compulsory course. According to the requirements of the course standard, if students want to continue their studies in colleges and universities in the future, they must study the optional compulsory courses, that is, everyone needs to learn conic sections and no longer require liberal arts and sciences. The conic section is an in-depth study of plane analytic geometry based on the students' knowledge of straight lines and circles. The conic section studies curves and equations, definitions of ellipses, hyperbolas, parabolas, standard equations, and simple geometric properties and examines them together with what you have learned about plane geometry. Therefore, the learning of conic section content has relatively high requirements on students' comprehensive level and can also improve students' mathematical thinking ability and logical ability. The study of the conic section is also very comprehensive, the knowledge you have learned before is often used, and the requirements for students' knowledge and mastery are relatively high, so the conic section has always been a problematic content for students to master. The conic section can be a good test of students' mathematical ability, so it has always been the key test object in the college entrance examination. To sum up, the problem-solving teaching of the conic section is the key and challenging point of teaching, and it has important research value.

By consulting relevant literature and accumulated experience in teaching research, the author summarizes some common problems in high school classrooms exposed by the teaching of conic section problem-solving. The reasons are discussed as follows. Firstly, the ability of mathematical operation is insufficient. Most students' mathematical operation ability in junior high school is not

solid, and when they encounter more complex operations in high school, they are often unable to perform. Secondly, the geometric transformation ability is weak, and most students have a poor grasp of the knowledge of plane geometry in junior high school. Thirdly, it lacks mathematical thinking and does not know how to apply the corresponding mathematical thinking methods when solving problems. Fourthly, they have a feeling of fear of difficulties and tend to choose simple problem-solving methods when encountering complex problems. Fifthly, the study method is not appropriate, do the questions blindly and do not reflect and summarize the question type method. Therefore, optimizing problem-solving teaching and improving students' mathematical operation levels has become an important teaching task.

3. The connection between mathematical operations and the teaching of conic section problem solving

Mathematical operation is one of the six core competencies of high school mathematics. If thinking is the heart of mathematics, then the operation is the blood of mathematics.[8] Core literacy is the correct values, essential qualities, and key abilities that students gradually form in learning. It is the organic unity of knowledge, ability, thinking, methods, emotions, and values. The cultivation and development of core literacy are not achieved overnight. Students' core literacy must pay attention to the teaching process so that the teaching process becomes an effective carrier for cultivating and developing students' core literacy.[7]The definition of the core competency of mathematical operations is given in the curriculum standards: based on clear operation objects, the competencies of solving mathematical problems according to the algorithm. Through the study of high school mathematics courses, students' mathematical operation ability has been further improved, and they can effectively solve practical problems with the help of operation methods. It is the process of transforming literacy into ability. Finally, through operation, the development of mathematical thinking is promoted, and the quality of thinking about problems is formed to create a meticulous, rigorous, and realistic scientific spirit. Mathematical operation is a process in which conjectures and ideas are verified and is a practical result of internalizing theories in the face of complex problems. It is pointed out in the " 2019 National Unified Examination Syllabus for College Admissions (Science Mathematics) " [5] that the calculation includes the calculation of numbers, valuation, and approximation, as well as the combination deformation and decomposition deformation of formulas, but also the calculation of The calculation and solution of various geometric quantities of geometric figures, etc. The conic section has a large number of calculations, many calculation skills, and complex calculation ideas, which is a suitable carrier for cultivating mathematical operation literacy.

As the chief expert on the development of core competencies for Chinese students' development, Professor Lin Chongde has carried out a series of pioneering and ground-breaking work around the construction of textbooks for universities and middle and primary schools. He is the first in China to divide the level of mathematical operation ability: ① understanding and understanding level; ② mastery and application-level; ③ synthesis and evaluation level [6]. The three corresponding levels are 1, 2, and 3, respectively. Students at level 1 can initially understand the meaning of mathematical operations, identify relevant mathematical problems and methods in time, and understand the rules of operations, laws, formulas of operations, etc. Students at level 2 learn a certain amount of operation skills and methods afterward. They have a certain computing ability and can use mathematical operations to deal with general mathematical problems. Level 3 students can apply different mathematical operation methods and formulas, flexibly handle various mathematical problems, and use efficient and fast computing methods to solve problems. Therefore, the level 3 computing ability is the highest level.

The main challenge of problem-solving teaching lies in the teaching of comprehensive problem-solving methods, but high school students' learning and mastery of conic curve comprehensive problems are not ideal, and the scoring rate in the college entrance examination is low. Therefore, in order to enable students to grasp better the comprehensive problem-solving method of conic section,

the teaching method of conic section problem solving is a hot topic in academic circles to explore the strategy innovation of conic section problem-solving teaching, and to form teaching cases as teaching reference, in order to provide a particular wisdom scheme for cultivating students' mathematical operation literacy, improving students' conic section problem-solving ability and teachers' teaching ideas and methods.

4. Teaching strategies for integrating mathematical operations into the teaching of conic section problem solving

Good design of exercises in the teaching of conic section problem solving can cultivate students' proficiency, accuracy, and flexibility in operation and help them further improve their mathematical operation ability. In the teaching process of conic section problem solving, teachers promote the development of students' mathematical thinking by guiding students to reasonably analyze the operation conditions, accurately grasp the direction of operation, effectively select the operation formula, and correctly output the operation result to develop the habit of thinking programmatically. Therefore, the following is case-based, combined with the three-level mathematical operation ability classification and the training and guidance strategies of mathematical ability at all levels, and then gives a conic curve problem-solving teaching plan that promotes the improvement of mathematical operation literacy.

4.1 Master the basic question types and reach the level 1 maths operation level

The problem-solving of conic sections involves a variety of core mathematical competencies, including intuitive imagination, mathematical modeling, logical reasoning, and mathematical operations. The ability of mathematical operation determines the overall grasp of the questioner's ability to grasp the question. The examination of the basic question type of conic section in the college entrance examination is mainly in the following aspects: First, examine the definition of the conic section, such as the relationship between the conic section and the ellipse. The combination of focus triangles can solve related problems such as ellipses and triangles. The second is to examine the standard equations of conic sections, combined with the relationship between the basic quantities of conic sections, and the undetermined coefficient method is used to solve them; the third is to examine the geometric properties of conic sections. Examining the eccentricity problem; the fourth is to examine the positional relationship between the straight line and the conic section, which is comprehensive, often combined with the vector, involving simultaneous equations, the discriminant of the root, the relationship between the root and the coefficient, the chord length problem, inequality, etc.

The following uses an example problem as a teaching case to illustrate:

Example 1: (2021 National College Entrance Examination Questions) Known to be $F_1, F_2$the two focal points of the Mellipse, if the point is $C: \frac{x^2}{9} + \frac{y^2}{4} = 1$on the Ctop, $|MF_1| \cdot |MF_2|$the maximum value is ( )

$A.\ 13$ $B.\ 12$ $C.\ 9$ $D.\ 6$

The teacher's method guide:

①Confirm the question type: This question involves the distance between the point on the ellipse and the two focal points of the ellipse. This kind of question often starts with the definition of the ellipse, which is called a "definition question" and is one of the common basic question types. Starting with the definition, since the problem requires finding the maximum value, the solver should pay attention to the flexible use of basic inequalities or remember the theorem: two positive numbers, when the sum is equal, the product is the largest, the product is constant, and the sum is the smallest when they are equal, or you can Quick solution.

②Using the corresponding problem-solving method to solve the problem: After confirming the question type, the problem solver uses the definition method, which can be obtained from the

definition of the ellipse $|MF_1| + |MF_2| = 2a = 6$. After $|MF_1|, |MF_2|$the relationship between the sum and the, so the solver reuses the variant in the fundamental inequality $|MF_1| \cdot |MF_2| \leq \left(\frac{|MF_1+MF_2|}{2}\right)^2 = 9$(if and only if the equal sign holds). $|MF_1| = |MF_2| = 3$In teaching, students with a poor foundation, may not be able to derive the variants of basic inequalities directly. At this time, teachers need to deduce it again to deepen the impression of the problem solver. Therefore, the author found that mastering the various variants of basic inequalities is essential for solving conic sections. The fixed value problem can achieve the effect of solving the problem faster and make the calculation easier.

4.2 Flexible use of operation rules to solve conic section problems, breaking through the level of 2nd-level mathematical operation

In the college entrance examination, the conic section questions are generally more comprehensive, the content of the investigation is relatively rich and comprehensive, and the characteristics of the proposition are relatively distinct. Therefore, the problem solver should master the basic operation rules and formulas and use them flexibly in solving problems. The main features of this type of proposition are: firstly, a "chain problem" is designed based on the intersection of a straight line passing through a particular point and a conic curve. Combined with the definition and geometric properties of a curve, the standard equation of the curve is determined in advance by the method of undetermined coefficients, and further research is carried out. Chord length, figure area, maximum value, range of values, etc. Secondly, design "chain questions" based on the positional relationship of different curves (ellipse, hyperbola, parabola), combine the definition and geometric properties of curves and use the method of undetermined coefficient. Firstly, it determines the standard equation of the curve and further studies the chord length, figure area, maximum value, value range, etc. The third is the positional relationship between the straight line and the conic section, which is comprehensive.

The following uses an example problem as a teaching case to illustrate:

Example 2: (Guangdong Province 2022 Senior High School One Modular Mathematics Test Question 2 2) Given an ellipse $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b > 0)$, its right focus is $F(\sqrt{3}, 0)$, the point is Mon, the circle $x^2 + y^2 = b^2$but not yon the axis and Mthe tangent of the circle through the point intersects the ellipse at Ptwo Qpoints, when When clicked Mon an xaxis, $|PQ| = \sqrt{3}$.

(1) the standard equation of the ellipse C

(2) When the point Mmoves on the circle, try to explore $\triangle$ FPQthe value range of the perimeter.

The teacher's method guide:

①Confirm the question type: Although this question seeks the range of values, it is a further investigation of the problem of finding position parameters by combining conic section equations, and it is highly comprehensive. Learners often learn many formulas about conic sections when learning conic sections further. These formulas will help us save many steps in solving problems and are also the requirements that solvers need to meet in the second-level operation level, such as the chord length formula $\sqrt{[(x_1 + x_2)^2 - 4x_1x_2](1 + k^2)}$.

② Using the corresponding problem-solving method to solve the problem: set $P(x_1, y_1)$, $Q(x_2, y_2)$. It is indicated $\triangle$ FPQthe perimeter, that is, $|PQ|$the $|PF|$length $|QF|$of , , and then classifies and discusses the absence and existence of the slope of the straight line. It is combined the straight line and the ellipse equation to obtain the root and the ellipse equation. The relational expression of the coefficient, the obtained expression $\triangle$ FPQof the perimeter: $4 + \frac{8\sqrt{3}}{\frac{m}{k}+\frac{3k}{m}}$, which combines the basic inequality, because $km > 0$, so $\frac{m}{k} + \frac{3k}{m} \geqslant 2\sqrt{3}$, if and only if $\frac{m}{k} = \frac{3k}{m}$, that is

$\begin{cases} m = \frac{\sqrt{6}}{2} \\ k = \frac{\sqrt{2}}{2} \end{cases}, \begin{cases} m = -\frac{\sqrt{6}}{2} \\ k = -\frac{\sqrt{2}}{2} \end{cases}$ take the equal sign. Therefore $4 < 4 + \frac{8\sqrt{3}}{\frac{m}{k}+\frac{3k}{m}} \leqslant 8$, $\triangle$ FPQthe value range of the perimeter is $(4, 8]$, and the answer is obtained.

4.3 Multiple solutions to one question, learn to reflect and break through the level of mathematical operations at level 3

Conic questions are highly comprehensive and involve many knowledge points. Often, the solution to a question is not unique. Therefore, teachers should not be limited to standard answers when teaching but should expand students' thinking and encourage students to try different solutions. Methods to solve problems, broaden the direction of thinking, deal with various mathematical problems flexibly, use efficient and fast calculation methods to obtain calculation results quickly, and improve problem-solving efficiency.

The following uses an example problem as a teaching case to illustrate:

Example 3 : (2020·Hainan College Entrance Examination Question) If the slope is $\sqrt{3}$the line passing through the focus of the parabola $C: y^2 = 4x$, and Cintersecting at A, Btwo points, then $|AB| =$_____.

The teacher's method guide:

①Confirm the question type: This question examines the intersection of the straight line and the parabola in the positional relationship between the straight line and the parabola and examines the definition and application of the parabola and the geometric properties of the parabola, which is a basic question type.

②Using the corresponding problem-solving method to solve the problem: However, this problem is not unique in terms of solution. It can be solved by either the algebraic method or the image method. Therefore, when choosing a problem-solving method, the problem solver can choose according to his expertise. Choice of problem-solving methods.

(Algebraic method) Because the equation of $y^2 = 4x$the parabola is , the coordinate $F(1,0)$F of the focal point of the parabola is, and because the straight line passes ABthrough the focal point Fand the slope is $\sqrt{3}$, the equation of $y = \sqrt{3}(x-1)$the straight line is AB. Substitute the equation of the straight line into the equation of the parabola to eliminate yand simplify $3x^2 - 10x + 3 = 0$, and the solution is obtained $x_1 = \frac{1}{3}, x_2 = 3$. So.$|AB| = \sqrt{1+k^2}|x_1 - x_2| = \sqrt{1+3} \cdot |3 - \frac{1}{3}| = \frac{16}{3}$

(geometric method) Since the equation of $y^2 = 4x$the parabola is , the coordinates of $F(1,0)$the focal point of the parabola are F, and because the line ABpasses through the focal point Fand has a slope of $\sqrt{3}$, the equation of $y = \sqrt{3}(x-1)$the line is: AB. Substituting the line equation into the parabola equation to eliminate y and simplify to get $3x^2 - 10x + 3 = 0$, $\Delta = 100 - 36 = 64 > 0$.

Let $A(x_1, y_1), B(x_2, y_2)$, then $x_1 + x_2 = \frac{10}{3}$, go through the vertical A, Blines that are respectively $x = -1$the directrix, let the vertical feet be respectively C, D. As shown .$|AB| = |AF| + |BF| = |AC| + |BD| = x_1 + 1 + x_2 + 1 = x_1 + x_2 + 2 = \frac{16}{3}$.

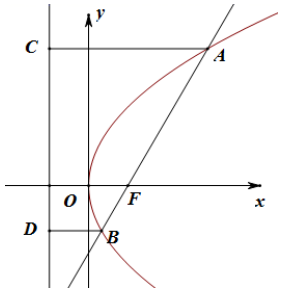

Fig 1. Parabolic equation

5. Summary and Outlook

By analyzing the important value and role of mathematical operations in the teaching of conic section solving under the background of the new curriculum reform, this paper summarizes the common problems of high school students about conic section solving. In response to these problems, the author puts forward the improvement and innovation of the problem-solving teaching strategy of the conic section based on the mathematical operation. The optimization of the problem-solving teaching strategy and the case study method expounds on how teachers can better carry out the teaching of conic section problem-solving. In order to guide and train students, it is to use the problem-solving method of conic section comprehensive problems flexibly. Therefore, the author hopes that the teacher should put forward the requirements of cultivating students' mathematical operation thinking and other core literacy from the top-level design. From the school's point of view, the author hopes to put forward the requirements for teachers to cultivate students' problem-solving skills and understand the general idea of the problem. It aims to strengthen their awareness of the cultivation of students' mathematical thinking so that students not only can solve problems but also have the ability to think independently.

With the continuous enrichment of middle school mathematics teaching content and the improvement of quality requirements, in the future, we will continue to innovate problem-solving teaching strategies to guide high school students to continuously strengthen their thinking and literacy, and improve their learning in mathematics and various subjects. Comprehensive quality and level.

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