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The Bounds of Energies of Rough Complemented Graph

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Abstract:

The main objective of this paper is to study the various energies and their bounds of the Rough complemented graph corresponding to the given Rough semiring. In this paper, for a given approximation space I=(U,R) where U is the nonempty finite set of objects and R is an equivalence relation on U, the Rough semiring (T,Δ,∇) is taken for study. The Rough complemented graph of T denoted by GRC(T) is a graph whose vertices are V(GRC(T))={RS(Y)|Y∈ 〚℘(E)〛 ^1 } be the set of equivalence classes induced by I and two distinct vertices RS(X) and RS(Y) are adjacent iff RS(X)∇RS(Y)=RS(∅). Note that there will be 2^n-2 vertices in GRC(T). Also Randic, seidel, minimum dominating, maximal independent and dominating energies of GRC(T) are obtained, the lower and upper bounds of these energies are also established. These energies are obtained through Python programming, and a bar diagram is used to conduct a comparative study for various values of n. All the illustrated concepts are explained with suitable examples.

Keywords: Independent dominating set, Minimum dominating energy, Randic energy, Siedel energy, Python code.

1. Introduction

The concept of energy of a graph was introduced by I. Gutman [1] in the year 1978. In [4] Rajesh kanna et al. compute Milovanovic bounds of the cocktail party graph and crown graph. Different results on independent dominance in graphs are being examined by the authors [2].

A molecular structure descriptor called Randic index was created by Milan Randi in 1975[6]. Later S.B. Bozkurt et al [7] defined Randic matrix and Randic energy. Further discussion on Randic energy can be found in [3], [5]. In [8] the authors find the minimum dominating energy for various graphs like complete graph, star graph etc.

This paper is organized as follows. Section 2 is about preliminaries. In section 3, we introduced the Rough complemented graph $GRC(T)$ and explore the properties of the graph. Also defined various energies like minimal dominating, seidel, randic etc and look at further bounds. In section 4, we provided the python coding for the corresponding graph energies and conclude in section 5.

2. Preliminaries

In this section, the basic definitions required to study the article are listed.

Definition 2.1. Let $G=(V(G),E(G))$ be a simple graph. A set $D \subseteq V(G)$ is s said to be a dominating set if every vertex in $V(G)-D$ is adjacent to atleast one vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality among all dominating sets of G.

Definition 2.2. A set is independent if no two vertices in it are adjacent. An independent dominating set of G is a set that is both dominating and independent in G .The independent domination number of G denoted by α, is the minimum size of an independent dominating set.

Lemma 2.1. [**5**] The Randic spectral radius $\rho_1(G) = 1$.

Lemma 2.2. [**3**] If G posseses isolated vertices, then $detR = detA = 0$. If G does not possess isolated vertices, then $detR = \frac{1}{d_1 d_2 \ldots\ldots\ldots\ldots d_n} detA$.

Let $I = (U, R)$ be an approximation space where U is the nonempty finite set of objects and R is an equivalence relation on U and for any $x \in U$, $[x]_R = \{y \in U | (x, y) \in R\}$ is said to be an equivalence class. For $X \subseteq U, RS(X) = \left(R_-(X), R^-(X)\right)$ be the rough set where $R_-(X) = \{x \in U | [x]_R \subseteq X\}$ is said to be a lower approximation space and the upper approximation space is defined as $R^-(X) = \{x \in U | [x]_R \cap X \neq \emptyset\}$. Also we defined the set of rough sets as $T = \{RS(X) | X \subseteq U\}$. It has been established that if $I = (U, R)$, (T, Δ, ∇) is a Rough semiring [10]. The partition created by R

on U should consist of $\{X_1, X_2, \ldots \ldots X_m, X_{m+1} \ldots \ldots . X_n\}$ where $|X_i| > 1, 1 \leq i \leq m, |X_j| = 1, m + 1 \leq j \leq n$.

Definition 2.3 [**11**] **Rough Zero Divisor Graph of Rough Semiring**

The zero divisor graph of the Rough Semiring (T, Δ, ∇) is $T(G) = (V, E)$ where V is the set of vertices in T(G) consists of nonempty zero divisors i.e, $V = \{RS(X) \in T | RS(X) \neq RS(\emptyset)$ is a zero divisor of T$\}$ and E is the set of edges connecting the elements of V such that there is an edge connecting $RS(X)$ and $RS(Y)$ in V iff $RS(X) \nabla RS(Y) = RS(\emptyset)$. This graph T(G) is called a Rough zero divisor graph of the Rough Semiring (T, Δ, ∇).

3. Bounds of various energies of Rough complemented graph

Throughout this section, we consider an approximation space $I = (U, R)$ along with the Rough semiring (T, Δ, ∇). Let $E = \{X_1, X_2, \ldots \ldots \ldots \ldots . X_n\}$ be the equivalence classes induced by R in which $\{X_1, X_2, \ldots \ldots \ldots \ldots . X_m\}$ are the equivalence classes with cardinality greater than 1.

In this section, the Rough complemented graph $GRC(T)$ of the Rough Semiring is introduced. Properties of $GRC(T)$ is studied and the various energies along with their bounds are dealt in detail.

3.1 Minimum dominating energy

Definition 3.1. Rough complemented graph

Let (T, Δ, ∇) be a Rough semiring. The Rough complemented graph of T denoted by $GRC(T)$ is a graph whose vertices are $V\left(GRC(T)\right) = \{RS(Y) | Y \in (\wp(E))^1\}$ where $(\wp(E))^1 = \wp(E) - \{RS(U), RS(\emptyset)\}$ and two distinct vertices $RS(X), RS(Z)$ are adjacent iff $RS(X) \nabla RS(Z) = RS(\emptyset)$.

Remarks. It is to be noted that

- The number of vertices in Rough complemented graph $GRC(T)$ is $2^n - 2$ where n denotes the number of equivalence classes in E.
- The number of edges in Rough complemented graph $GRC(T)$ is $\frac{1}{2}(3^n - 2^{n+1} + 1)$.

- For any $RS(X) \in V\big(GRC(T)\big)$, the degree of $RS(X)$ is $2^{n-r}-1$ where $1 \leq r < m$.
- For any $RS(X), RS(Y) \in V\big(GRC(T)\big)$,

$$d_2(RS(X)) = \big\{RS(Y) \in V\big(GRC(T)\big) | d(RS(X), RS(Y) = 2)\big\}$$

$$|d_2(RS(X))| = 2^r - 2 + (2^r - 1)(2^{n-r} - 2).$$

- For any $RS(X), RS(Y) \in V\big(GRC(T)\big)$,

$$d_3(RS(X)) = \big\{RS(Y) \in V\big(GRC(T)\big) | d(RS(X), RS(Y) = 3)\big\}$$

$|d_3(RS(X))| = 2^r - 2$. Here d represents the distance.

- The diameter of $GRC(T)$is 3.

Definition 3.2. Minimum dominating set

Consider the Rough complemented graph $GRC(T) = \Big(V\big(C(T)\big), E(C(T))\Big)$. A subset $D\big(C(T)\big)$ is called the dominating set of $GRC(T)$ if every vertex in $V\big(C(T)\big) - D\big(C(T)\big)$ is adjacent to some vertex in $D\big(C(T)\big)$. The minimum cardinality of a dominating set $D\big(C(T)\big)$ is called the domination number of the graph $GRC(T)$, denoted by $\gamma\big(C(T)\big)$.

Definition 3.3. For every $RS(X), RS(Y) \in V\big(C(T)\big)$, the minimum dominating matrix is

$$A_D(C(T)) = \begin{cases} 1 \; if RS(X) \nabla RS(Y) = RS(\emptyset) \\ 1 \; if \; RS(X) = RS(Y) and RS(X) \in D(C(T)) \\ 0 \; otherwise \end{cases}$$

Definition 3.4. Minimum dominating energy

The minimum dominating energy of $GRC(T)$is defined as $\mathcal{E}(D(C(T))) = \sum_{i=1}^{2^n-2} |\mu_i|$ where $\mu_1, \mu_2 \dots\dots\dots.. \mu_{2^n-2}$ are the spectrum of $A_D(C(T))$. The spectral radii of $A_D\big(C(T)\big)$ are in non-increasing order i.e., $\mu_1 \geq \mu_2 \geq \dots\dots\dots\dots \geq \mu_{2^n-2.}$

Theorem 3.1. In $GRC(T)$, the minimum dominating set is $D\big(C(T)\big) = \{RS(X_i) | i = 1,2, \dots\dots\dots.. n\}$

Proof. Let $RS(Y) \in V\big(C(T)\big) - D\big(C(T)\big)$.

Then the edge set $\xi = \{(RS(Y), RS(Z)) | Z \in E - Y\}$

Next to prove that $D\big(C(T)\big)$ is the minimum dominating set.

Note that if we remove any $RS(X_i), i = 1,2, \dots\dots\dots.. n$ from $D\big(C(T)\big)$

then $RS(X_1 X_2 \dots\dots\dots. X_{i-1} X_{i+1} \dots\dots\dots.. X_n)$ will not be adjacent to any of the elements in $D\big(C(T)\big) - RS(X_i)$ which will affect the dominating property. Hence removal of any element from $D\big(C(T)\big)$ will affect the dominating property.

Therefore $D\big(C(T)\big) = \{RS(X_i) | i = 1,2, \dots\dots\dots.. n\}$ is the minimum dominating set.

Theorem 3.2. Let $GRC(T)$ be the Rough complemented graph and $A_D(C(T))$ be the minimum dominating matrix of $GRC(T)$ and $\gamma\big(C(T)\big)$ denotes the minimum domination number then

- $\sum_{i=1}^{2^n-2} \mu_i = |\gamma(C(T))|$
- $\sum_{i=1}^{2^n-2} {\mu_i}^2 = |3^n - 2^{n+1} + 1| + |\gamma(C(T))|$

Proof. It is known that the sum of eigen values of $A_D(C(T))$ is the trace of $A_D(C(T))$.

$$\sum_{i=1}^{2^n-2} \mu_i = \sum_{i=1}^{2^n-2} d_{ii} = |\gamma(C(T))|$$

It is find that $\sum_{i=1}^{2^n-2} \mu_i^2$ of $A_D(C(T))$ is the trace of $[A_D(C(T))]^2$

$$\sum_{i=1}^{2^n-2} {\mu_i}^2 = \sum_{i=1}^{2^n-2} d_{ij} \sum_{j=1}^{2^n-2} d_{ji}$$

$$= \sum_{i=1}^{2^n-2} {d_{ii}}^2 + \sum_{i\neq j} d_{ij} d_{ji}$$

$$= \sum_{i=1}^{2^n-2} {d_{ii}}^2 + 2\sum_{i<j} {d_{ij}}^2$$

$$= |\gamma(C(T))| + |(3^n - 2^{n+1} + 1)|$$

Theorem 3.3. In $GRC(T)$, $D(C(T))$ be the minimum dominant set and Δ_D denote the determinant of $A_D(C(T))$ then

$$\sqrt{\left(3^n - 2^{n+1} + 1 + \gamma(C(T))\right) + (2^n-2)(2^n-3){\Delta_D}^{\frac{2}{(2^n-2)(2^n-3)}}} \leq \mathcal{E}_D(GRC(T)) \leq \sqrt{(2^n-2)\left(3^n - 2^{n+1} + 1 + \gamma(C(T))\right)}$$

Proof. By Cauchy Schwarz inequality

$$\left(\sum_{i=1}^{2^n-2} |\mu_i|\right)^2 \leq \left(\sum_{i=1}^{2^n-2} 1\right)\left(\sum_{i=1}^{2^n-2} {\mu_i}^2\right)$$

$$\left[\mathcal{E}_D(GRC(T))\right]^2 \leq (2^n-2)\left(3^n - 2^{n+1} + 1 + \gamma(C(T))\right)$$

And from arithmetic –geometric mean inequality

$$\sum_{i\neq j} |\mu_i||\mu_j| \geq (2^n-2)(2^n-3){\Delta_D}^{\frac{2}{(2^n-2)(2^n-3)}}$$

$$\left[\mathcal{E}_D(GRC(T))\right]^2 = \left(\sum_{i=1}^{2^n-2} |\mu_i|\right)^2$$

$$= \sum_{i=1}^{2^n-2} |\mu_i|^2 + \sum_{i\neq j} |\mu_i||\mu_j|$$

$$\left[\mathcal{E}_D(GRC(T))\right]^2 \geq \left(3^n - 2^{n+1} + 1 + \gamma(C(T))\right) + (2^n-2)(2^n-3){\Delta_D}^{\frac{2}{(2^n-2)(2^n-3)}}$$

$$\mathcal{E}_D(GRC(T)) \geq \sqrt{\left(3^n - 2^{n+1} + 1 + \gamma(C(T))\right) + (2^n-2)(2^n-3){\Delta_D}^{\frac{2}{(2^n-2)(2^n-3)}}}$$

3.2. Maximal independent and dominating energy

Definition 3.5. The maximal independent and dominating set of the Rough complemented graph $GRC(T)$ is denoted by $ID(C(T))$. Note that the elements of $ID(C(T))$ will be a maximum independent set as well as a dominating set.

Definition 3.6. Let $ID(C(T))$ be a maximal independent and dominating set of a graph $GRC(T)$, then the corresponding adjacency matrix is denoted by $A_{ID}(C(T))$ is defined as

$$A_{ID}(C(T)) = \begin{cases} 1 \; if RS(X) \nabla RS(Y) = RS(\emptyset) \\ 1 \; if \; RS(X) = RS(Y) and RS(X) \in ID(C(T)) \\ 0 \; otherwise \end{cases}$$

Definition 3.7. The maximal independent and dominating energy of $GRC(T)$ is defined by $\mathcal{E}_{ID}(C(T)) = \sum_{i=1}^{2^n-2} |\omega_i|$ where ω_i are the eigen values of $A_{ID}(C(T))$ and are in non - increasing order.

Theorem 3.4. For the Rough complemented graph $GRC(T)$, $|ID(C(T))|$ is $2^{n-1} - 1$.

Proof. Consider $ID(C(T)) = \{RS(X_i \cup Y) | Y \in \wp(E - X_i)\}$

To prove $ID(C(T))$ is an independent set.

Let $RS(X), RS(Z) \in ID(C(T))$, where $RS(X) = RS(X_i \cup Y_1)$ and $RS(Z) = RS(X_i \cup Y_2)$ where $Y_1, Y_2 \in \wp(E - X_i)$

This implies $RS(X_i) \in RS(X) \nabla RS(Z) \neq RS(\emptyset)$

There is no edge between $RS(X) and \; RS(Z)$

$\therefore$ $ID(C(T))$ is an independent set.

It is clear to verify that addition of any vertex to $ID(C(T))$ will affect the independence property.

Hence $ID(C(T))$ is the maximal independent set.

Next to prove that $ID(C(T))$ is a dominating set.

Let $RS(Z) \in V(GRC(T)) - ID(C(T))$

Since the elements of $V(GRC(T)) - ID(C(T))$ are from $\wp(E - X_i)$ and so $RS(Z)$ is adjacent to $RS(X_i)$ and hence it is a dominating set.

Hence it is clear that $|ID(C(T))| = 2^{n-1} - 1$.

Remarks 3.1. For the Rough complemented graph $GRC(T)$, $|ID(C(T))| \leq 2^n - 3$. Also $|ID(C(T))| \geq \frac{2^n-2}{n}$.

3.3. Siedel Energy

Definition 3.8. The Siedel matrix of $GRC(T)$ is defined by

$$S(C(T)) = \begin{cases} -1 \; if RS(X)\nabla RS(Y) = RS(\emptyset) \\ 1 \; RS(X)\nabla RS(Y) \neq RS(\emptyset) \\ 0 \; if \; RS(X) = RS(Y) \end{cases}$$

The Siedel energy of $GRC(T)$ is defined as $SE\big(C(T)\big) = \sum_{\mathrm{i}=1}^{2^{\mathrm{n}}-2} |\lambda_{\mathrm{i}}|$,

$i = 1,2 \dots\dots\dots 2^{\mathrm{n}} - 2$, where $\lambda_{\mathrm{i}}{}^{\mathrm{s}}$ are the eigen values of $S\big(C(T)\big)$.

Lemma 3.1. Let $\lambda_1, \lambda_2 \dots\dots\dots\dots\dots \lambda_{2^n-2}$ denote the siedel eigenvalues of $S\big(C(T)\big)$ then

- $\sum_{i=1}^{2^n-2} \lambda_i = 0$
- $\sum_{i=1}^{2^n-2} \lambda_i{}^2 = (2^n - 2)(2^n - 3)$

Proof. It is known that $\sum_{i=1}^{2^n-2} \lambda_i = \sum_{i=1}^{2^n-2} s_{ii} = 0$

Sum of squares of eigen values of $S\big(C(T)\big)$ is the trace of $\Big(S\big(C(T)\big)\Big)^2$.

$$\sum_{i=1}^{2^n-2} \lambda_i{}^2 = \sum_{i=1}^{2^n-2} s_{ij} \sum_{j=1}^{2^n-2} s_{ji}$$

$$= \sum_{i=1}^{2^n-2} (s_{ii})^2 + \sum_{i \neq j} s_{ij} s_{ji}$$

$$= \sum_{i=1}^{2^n-2} (s_{ii})^2 + 2\sum_{i<j} s_{ij}{}^2$$

$$= 2\left\{\frac{1}{2}(3^n - 2^{n+1} + 1)(-1)^2 + \left(\frac{(2^n-2)^2 - (2^n-2)}{2} - \frac{1}{2}(3^n - 2^{n+1} + 1)\right)(1)^2\right\}$$

$$= (2^n - 2)^2 - (2^n - 2)$$

$$\sum_{i=1}^{2^n-2} \lambda_i{}^2 = 2^n(2^n - 5) + 6$$

Theorem 3.5. In $GRC(T)$, $\Delta_S = |detS\big(C(T)\big)|$ then

$$\sqrt{2^n(2^n - 5) + 6 + (2^n - 2)(2^n - 3)\Delta_S^{\frac{2}{2^n-2}}} \leq SE\big(C(T)\big) \leq \sqrt{(2^n - 2)\{2^n(2^n - 5) + 6\}}$$

where $|detS\big(C(T)\big)|$ means the absolute value.

Proof. Taking $a_i = 1, b_i = |\lambda_i|$ Cauchy Schwarz inequality becomes

$$\left(\sum_{i=1}^{2^n-2} |\lambda_i|\right)^2 \leq \left(\sum_{i=1}^{2^n-2} 1\right)\left(\sum_{i=1}^{2^n-2} \lambda_i{}^2\right)$$

$$\Big(SE\big(C(T)\big)\Big)^2 \leq (2^n - 2)(2^n(2^n - 5) + 6) \quad (by\ lemma\ 3.1)$$

$SE(C(T)) \leq \sqrt{(2^n-2)(2^n(2^n-5)+6)}$

By arithmetic geometric mean inequality

$\sum_{i\neq j} |\lambda_i||\lambda_j| \geq (2^n-2)(2^n-3)\Delta_S^{\frac{2}{2^n-2}}$

$\left(SE(C(T))\right)^2 = \left(\sum_{i=1}^{2^n-2} |\lambda_i|\right)^2$

$= \sum_{i=1}^{2^n-2} |\lambda_i|^2 + \sum_{i\neq j} |\lambda_i||\lambda_j|$

$\left(SE(C(T))\right)^2 \geq 2^n(2^n-5)+6+(2^n-2)(2^n-3)\Delta_S^{\frac{2}{2^n-2}}$ $SE(C(T)) \geq \sqrt{(2^n-2)(2^n-3)\left(1+\Delta_S^{\frac{2}{2^n-2}}\right)}$

3.4. Randic Energy

Definition 3.9.

The Randic matrix $R(C(T)) = R_{xy}$ of $GRC(T)$ is a square matrix of order 2^n-2 whose (x,y) entry is

$$R_{xy} = \begin{cases} \frac{1}{\sqrt{d_x d_y}} & if RS(X) \nabla RS(Y) = RS(\emptyset) \\ 0 & otherwise \end{cases}$$

Where d_x denote the degree of the vertex $RS(X)$.

The eigenvalues of $R(C(T))$ are called Randic eigenvalues and are denoted by $\rho_1, \rho_2 \ldots\ldots\ldots..\rho_{2^n-2}$. If all the ρ_i's , $1 \leq i \leq 2^n-2$ are distinct then the Randic spectrum of $GRC(T)$ can be denoted as

$$Spec\left(R(C(T))\right) = \begin{pmatrix} \rho_1\ \rho_2 \ldots\ldots\ldots..\rho_{2^n-2} \\ m_1\ m_2 \ldots\ldots\ldots\ m_{2^n-2} \end{pmatrix}$$

Where m_j indicates the algebraic multiplicity of the eigenvalue ρ_j , $1 \leq j \leq 2^n-2$ of $GRC(T)$.

The Randic energy of $GRC(T)$is defined as $RE(C(T)) = \sum_{i=1}^{2^n-2} |\rho_i|$, $i = 1,2,\ldots\ldots\ldots.2^n-2$.

Theorem 3.6. For $GRC(T)$,

$RE(C(T)) \leq 1 + \sqrt{\frac{(2^n-3)(2^n-2-\delta(C(T))}{\delta(C(T))}}$ where $\delta(C(T))$ is the minimum degree.

Proof. It is known that

$\sum_{i=1}^{2^n-2} {\rho_i}^2 = 2\sum_{x,y\in E(C(T))} \frac{1}{d_x d_y}$

$= \sum_{x=1}^{2^n-2} \frac{1}{d_x} \sum_{x,y\in E(C(T))} \frac{1}{d_y}$

$\leq \sum_{x=1}^{2^n-2} \frac{1}{\delta(C(T))} \sum_{x,y\in E(C(T))} \frac{1}{d_y}$ $\quad Since\ d_x \geq \delta(C(T))$

$$= 2^n - 2$$

$RE(C(T)) = \sum_{i=1}^{2^n-2} |\rho_i| = 1 + \sum_{i=2}^{2^n-2} |\rho_i|$ (*by lemma* 2.1)

$\leq 1 + \sqrt{(2^n-3)(\sum_{i=1}^{2^n-2} \rho_i{}^2 - 1)}$ (by Cauchy Schwarz inequality)

$$\leq 2^n - 2$$

Theorem 3.7. In $GRC(T)$, with maximum degree $\Delta(\,C(T))$

$$RE(C(T)) \geq 1 + \sqrt{\frac{2^n-2}{\Delta(\,C(T))} - 1 + (2^n-3)(2^n-4)\left(\frac{|\det A(C(T))|}{\prod_{i=1}^{2^n-2} d_i}\right)^{\frac{2}{2^n-3}}}$$

Where $det\, A(C(T))$ denotes the determinant of adjacency matrix of $GRC(T)$.

Proof. Proceeding as in the above, we have $\sum_{i=1}^{2^n-2} \rho_i{}^2 = \frac{2^n-2}{\Delta(C(T))}$

Using arithmetic geometric mean inequality

$$2 \sum_{2\leq i<j\leq 2^n-2} |\rho_i||\rho_j| \geq (2^n-3)(2^n-4)\left(\prod_{i=2}^{2^n-2} |\rho_i|\right)^{\frac{2}{2^n-3}}$$

$$= (2^n-3)(2^n-4)(|\det \mathrm{RE}(C(T))\,|)^{\frac{2}{2^n-3}}$$

$$= (2^n-3)(2^n-4)\left(\frac{|\det A(C(T))|}{\prod_{i=1}^{2^n-2} d_i}\right)^{\frac{2}{2^n-3}} \quad (by\ lemma\ 2.2)$$

Now, $\left(\sum_{i=2}^{2^n-2} |\rho_i\,|\right)^2 = \sum_{i=2}^{2^n-2} \rho_i{}^2 + 2\sum_{2\leq i<j\leq 2^n-2} |\rho_i||\rho_j|$

$$\sum_{i=2}^{2^n-2} |\rho_i| \geq \sqrt{\frac{2^n-2}{\Delta(C(T))} - 1 + (2^n-3)(2^n-4)\left(\frac{|\det A(C(T))|}{\prod_{i=1}^{2^n-2} d_i}\right)^{\frac{2}{2^n-3}}}$$

$$RE(C(T)) = \sum_{i=1}^{2^n-2} |\rho_i|$$

$$RE(C(T)) \geq 1 + \sqrt{\frac{2^n-2}{\Delta(C(T))} - 1 + (2^n-3)(2^n-4)\left(\frac{|\det A(C(T))|}{\prod_{i=1}^{2^n-2} d_i}\right)^{\frac{2}{2^n-3}}}$$

Example 3.1. The following graph is corresponding to the approximation space $I = (U, R)$ where R induces 3 equivalence classes.

Let $U = (x_1, x_2, x_3, x_4, x_5, x_6)$

Here $E = \{X_1, X_2, X_3\}$ *where* $X_1 = \{x_1, x_3\}$, $X_2 = \{x_2, x_4, x_6\}$, $X_3 = \{x_5\}$

$Here\ V(C(T)) = \{ RS(X_1), RS(X_2), RS(X_3), RS(x_1), RS(x_2), RS(x_1 \cup X_2),$
$RS(X_1 \cup x_2), RS(x_1 \cup x_2), RS(X_1 \cup X_2), RS(x_1 \cup X_3), RS(X_1 \cup X_3),$
$RS(x_2 \cup X_3), RS(X_2 \cup X_3), RS(x_1 \cup X_2 \cup X_3), RS(X_1 \cup x_2 \cup X_3),$
$RS(x_1 \cup x_2 \cup X_3)\}$

The minimum dominating matrix $A_D(C(T))$ is given by

$A_D\big(C(T)\big) =$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

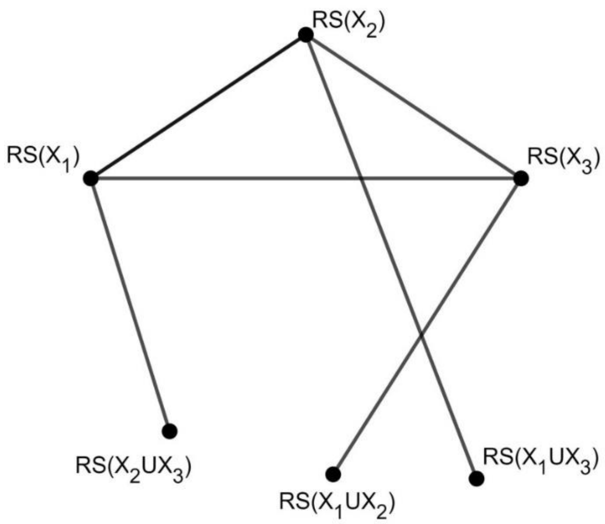

Figure 1: Rough complemented graph

The spectrum of minimum dominating energy are $\left[3.3028, 1^{(2)}, -0.3028, (-1)^{(2)}\right]$

The maximal independent and dominating matrix of $GRC(T)$ is given by

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial, spectrum and maximal independent domination energy are as follows.

$$\text{f}(GRC(T), \lambda) = \lambda(\lambda^2 - 2)(\lambda^3 - 3\lambda^2 - \lambda + 4)$$

Spec $(GRC(T))$ = $\begin{pmatrix} -1.4142 & -1.1149 & 0 & 1.2541 & 1.4142 & 2.8608 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

$\mathcal{E}_{ID}\big(C(T)\big) = 8.0582.$

Here the maximal independent and domination number $\left|ID\big(C(T)\big)\right| = 2^{n-1} - 1 = 3.$

4. Generation of various graph energies using Python for the Rough complemented graph

In this section, Python Code is provided for Siedel, Randic and Minimum dominating energies for the graph $GRC(T)$. Additionally, the energies are compared using bar diagram for various values of n.

Certain auxiliary functions used for displaying and increasing the visual appeal of the graphs and their energies are not included in the code presented. Libraries used to aid code reusability have been imported at the top of the first code section.

Graph generation code

```
import numpy as np
import networkx as nx
from itertools import chain, combinations
import matplotlib.pyplot as plt
import csv  # Utility Functions
def powerset(iterable):
   s = set(iterable)
   return set(chain.from_iterable(combinations(s, r) for r in range(len(s)+1)))
def EnergyOfMatrix(A):
   eigVals = np.linalg.eigvals(A)  # Compute eigenvalues of A
   sumAbsEigVals = sum(abs(eigVals))   # Compute sum of absolute values of
eigenvalues
   return sumAbsEigVals
def EnergyOfGraph(G):
   return EnergyOfMatrix(nx.adjacency_matrix(G).todense())
n = int(input("Enter the value of n: "))
n_nat = set(range(1, n + 1))
powerset_n_nat = powerset(n_nat)
powerset_n_nat_min_1 = powerset(range(2, n + 1))

i_d_set = set( # Independent Dominating Set
   (1, x)
   for x in powerset_n_nat_min_1.difference(set(range(2, n + 1)))
)
vertices = set( elem for elem in powerset(n_nat) if (elem not in [tuple(), tuple(n_nat)]))
edges = set(
```

```
   (vert1, vert2)
   for vert1 in vertices
   for vert2 in vertices
   if (
       (len(set(vert1).intersection(set(vert2))) == 0)
       or (vert1 == vert2 and (vert1 in i_d_set))
   )
) # %%
graph = nx.Graph()
graph.add_nodes_from(vertices)
graph.add_edges_from(edges)
graphdegrees = {node: val for (node, val) in graph.degree()}
siedel_edges = []  # Stores edges as
randic_edges = []  # (v1,v2,weight) triples
for vert1 in vertices:
   for vert2 in vertices:
       if set(vert1) == set(vert2):
          if len(vert1) == 1:
           pass
       elif len(set(vert1).intersection(set(vert2))) == 0:
           siedel_edges.append((vert1, vert2, -1)) # min_dom.append((vert1,vert2,1))
           randic_edges.append(
               (vert1, vert2, 1 / np.sqrt((graphdegrees[vert1] * graphdegrees[vert2])))
           )
       elif len(set(vert1).intersection(set(vert2))) != 0:# min_dom.append((vert1,vert2,0))
           siedel_edges.append((vert1, vert2, 1))
           randic_edges.append((vert1, vert2, 0))
seidel_graph = nx.Graph()
seidel_graph.add_weighted_edges_from(siedel_edges)
randic_graph = nx.Graph()
randic_graph.add_weighted_edges_from(randic_edges)
min_dom_adj = nx.adjacency_matrix(graph).todense() # GRC T
for i in range(len(min_dom_adj)):
   for j in range(len(min_dom_adj[0])):
       if i==j:
           if len(list(vertices)[i]) == 1:
               min_dom_adj[i][i] = 1
```

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min_dom_graph = nx.relabel_nodes(nx.from_numpy_array(min_dom_adj),{i:vert for i,vert in enumerate(vertices)}) # mapping = {i:vert for i,vert in enumerate(vertices)}

Energy Calculation Code

Functions were written that would ease the process of calculating the Graph energies for any graph:

```
def EnergyOfMatrix(A):
  eigVals = np.linalg.eigvals(A)  # Compute eigenvalues of A
  sumAbsEigVals = sum(abs(eigVals))  # Compute sum of absolute values of eigenvalues
  return sumAbsEigVals
def EnergyOfGraph(G):
  return EnergyOfMatrix(nx.adjacency_matrix(G).todense())
```

Generated Graphs and Charts

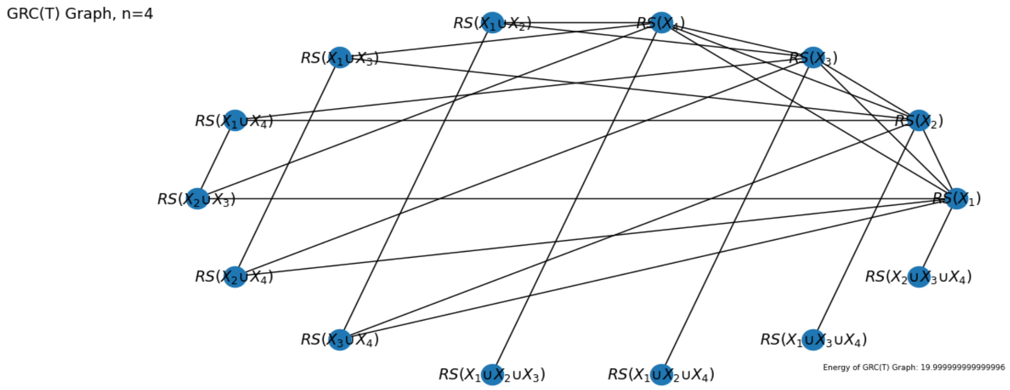

Fig 2: $GRC(T)$ graph when $n = 4$

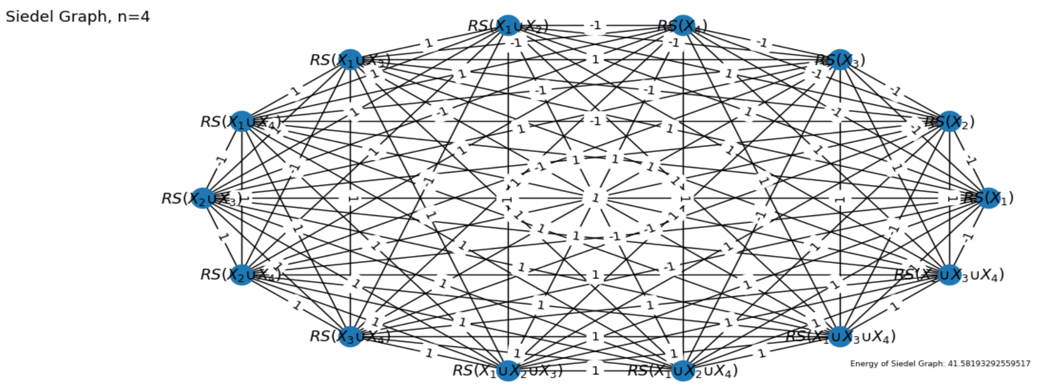

Fig 3: The Siedel Graph

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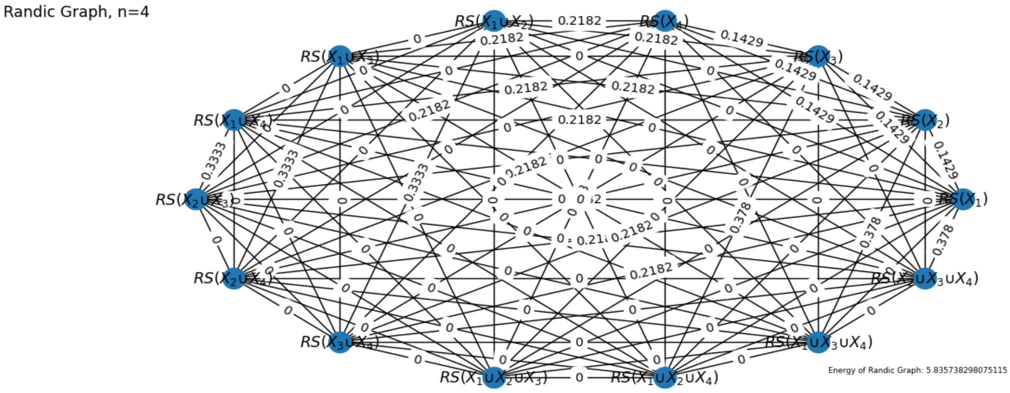

Fig 4: The Randic Graph

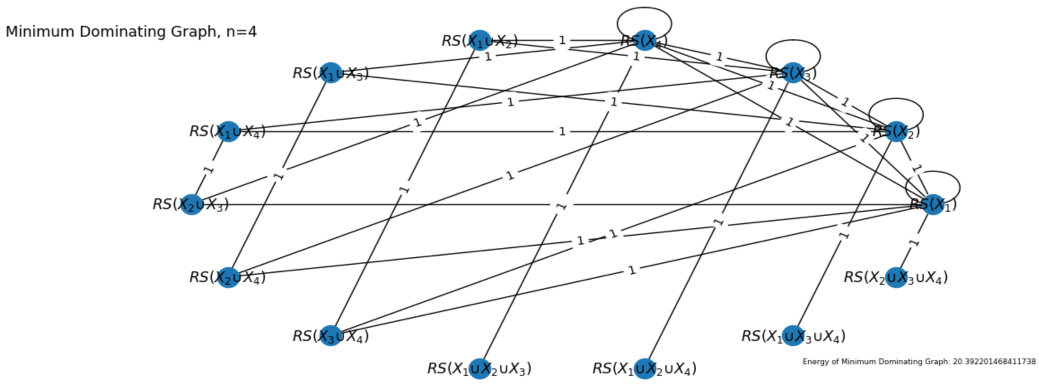

Fig 5: Minimum dominating graph

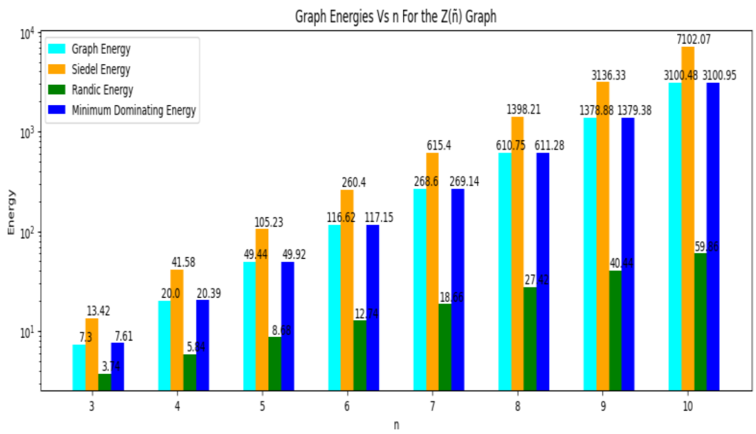

Fig 6: A comparison of the Adjacency Energy, Siedel Energy, Randic Energy and Minimum Dominating Energy for $GRC(T)$ graph for varying values of n.

5. Conclusion

In this study, the Rough complemented graph of the Rough semiring is defined using the equivalence classes. Also the maximal independent and dominating set of $GRC(T)$ graph is established. Additionally, the minimum dominating, Siedel and Randic energies of the $GRC(T)$ graph are defined and the lower and upper bounds are derived. All the above mentioned energies can be found using Python and several values of n are compared. Each notion is illustrated and supported by an example.

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