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Performance Analysis of Biserial Servers Connected with a Main Server by Fuzzy Ordering Approach

Syamala. P [1], Ramesh. R [2*], Seenivasan. M [3] and Sakthivel. K [4]

[1] Department of Mathematics, AVVM Sri Pushpam College (Affiliated to Bharathidasan University, Tiruchirappalli), Poondi, Thanjavur, Tamilnadu, India, E-mail: syam.rethinam87@gmail.com

[2] Department of Mathematics, Arignar Anna Govt. Arts College, Musiri, Tamilnadu, India.

*Corresponding Author : rameshsanju123@gmail.com

[3] Mathematics Wing - CDOE, Annamalai University, Annamalai Nagar- 608002, Tamil Nadu, India.

E-mail : emseeni@yahoo.com

4 Department of Mathematics, Govt. Arts and Science College, Veppanthattai, Perambalur, Tamilnadu, India.

E mail : ksvgac@gmail.com

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Abstract:

This article presents some appraises for biserial servers connected with a main server by fuzzy ordering approach. Here we have three servers, so that three queues are formed in front of the servers. The patrons were presented along with poisson stream to the queues, and the rate of service was observed by exponential stream. The customers may get the services either from server 1(or 2), to server 2 (or 1), (biserial) then Main server or from the servers 1 or 2 to (biserial) Main server. So we get more possibilities from this situation. We consider the arrival, service and possibility rates as the fuzzy parameters. By virtue of our proposed ordering approach, we shift the fuzzy rates to crisp numerals. We detect the measures expected waiting duration in the system (E(W)) and the expected aggregate of customers in the system (E(N)) of biserial servers connected with a main server with fuzzy nature.

Keywords: Biserial Servers, Possibilities, Fuzzy Ordering Technique, Measures.

1. Introduction

Queueing theory refers to a study of the function, formation, and crowding of waiting queues by mathematically. Basically, a queue formation involves 2 parts. If something or someone that requests a service, usually called customer, request, or job. Something or someone that delivers or completes the services usually called the server. The theory of queues scrutinizes the entire waiting system, as well as the number of servers, customer arrival rate, number of customers, average service completion time, and capacity of the system.

Artalejo[1,2] analyzed some famous queues in his articles. Vandana Saini et. al [14] described Fuzzy Tandem queues. Suzuki [13] surveyed about biserial queue. Vinod et. al [16] analyzed the behavior of biserial queues. Depak Gupta and Renu Gupta [5] presented bulk biserial queuing system.

In the very first Zadeh [19,20] introduced Fuzzy set and its operations. Fuzzy subset theory was described by Kaufmann [7]. Klir, Yuvan [8] produced applications of Fuzzy logic. Venkatesh and Prakasam [15] suggested a new fuzzy distribution. Chen &Chen [3], R. R.Yager [18], Dat et al [4], Deng &Liu [6] and Wang & Lee [17] are illustrated the approach of Fuzzy ordering. Ramesh and Kumaraghuru [9] contributed an approach of Centroid grounded Fuzzy ordering to priority queueing system. Ramesh and Hari Ganesh [10,11] authorized Expansion and Wingspans Center Fuzzy ordering approach to the queueing system. Ramesh and Seenivasan [12] contributed Centroid of Centroids ordering approach in Interval Valued Type-2 Fuzzy Environment.

In this article, we used our proposed ordering technique towards the Biserial servers connected with a main server queuing model to catch the crisp numerals.

2. Preliminaries

Definition:

A fuzzy set is managed with the partnership function $\mu_{\tilde{A}}$, morphing from members of universe of discourse Z to [0,1]. (i,e) $\mu_{\tilde{A}}$: Z →[0,1] is a mapping talked to the degree of partnership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}$ (z)is talked to the partnership value of z Є Z in the fuzzy set $\tilde{A}$. (i,e) $\tilde{A}$ = {(z, $\mu_{\tilde{A}}$ (z)); z Є Z}

Definition:

A triangular fuzzy numeral $\tilde{A}$ (a1,a2,a3;1) is encompassed with partnership function

$$\mu(z) = \begin{cases} \dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2 \\ 1 \quad , & z = a_2 \\ \dfrac{a_3-z}{a_3-a_2}, & a_2 \le z \le a_3 \\ 0, otherwise \end{cases}$$

Definition:

A trapezoidal fuzzy numeral $\tilde{A}$ (a1,a2,a3,a4;1)is encompassed with partnership function

$$\mu(z) = \begin{cases} \dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2 \\ 1 \quad , & a_2 \le z \le a_3 \\ \dfrac{a_4-z}{a_4-a_3}, & a_3 \le z \le a_4 \\ 0, otherwise \end{cases}$$

3. Centroid of Incenters Fuzzy Ordering Method

Fuzzy ordering technique is a fantastic tool in optimization obstacles. Earlier, countless authors used centroid based strategies for ordering the fuzzy numerals. Our target is to detect the centroid of Incenters (G) which is the balancing point of every plane structures.

Right now, to obtain G for the trapezium APQD, we split APQD to ΔAPR, ΔRQD & ΔARD. The above mentioned triangles provided the incenters I_1, I_2 & I_3. Such I_1, I_2 & I_3 (non-collinear) created another $\Delta(I_1, I_2, I_3)$. From this triangle, we find the centroid G and this is equidistant to all incenters. These are all instructed in the Figure 1.

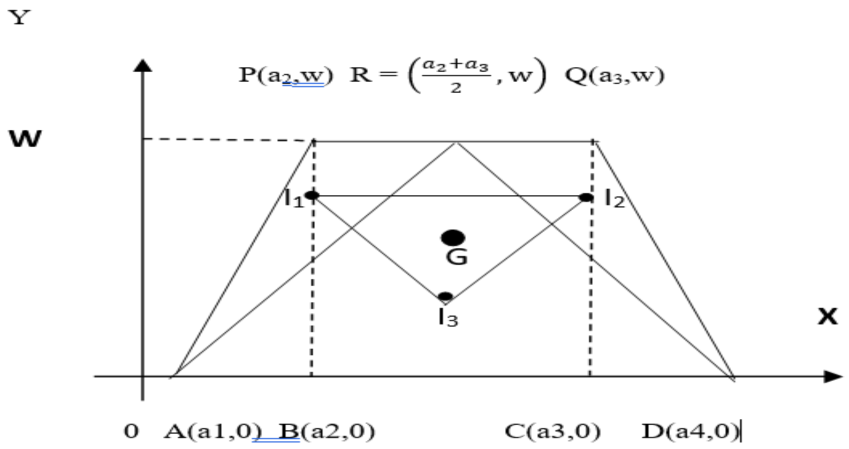

Fig 1. Centroid of Incenters

Take a Generalized Trapezoidal Fuzzy Number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$

Then the incenters

$$I_1(x_{i1},y_{i1})=\left(\frac{a_1l_1+a_2m_1+\frac{a_2+a_3}{2}n_1}{l_1+m_1+n_1},\frac{w(m_1+n_1)}{l_1+m_1+n_1}\right);$$

$$I_2(x_{i2},y_{i2})=\left(\frac{a_1l_2+\frac{a_2+a_3}{2}m_2+a_4n_2}{l_2+m_2+n_2},\frac{wm_2}{l_2+m_2+n_2}\right) \text{ and } I_3(x_{i3},y_{i3})=\left(\frac{\frac{a_2+a_3}{2}l_3+a_3m_3+a_4n_3}{l_3+m_3+n_3},\frac{w(l_3+m_3)}{l_3+m_3+n_3}\right).$$

Where

$l_1 = (a_3 - a_2)$, $m_1 = \frac{1}{2}\sqrt{(a_2 + a_3 - 2a_1)^2 + 4W^2}$, $n_1 = \sqrt{(a_2 - a_1)^2 + W^2}$, $l_2 = \frac{1}{2}\sqrt{(2a_4 - a_2 - a_3)^2 + 4W^2}$, $m_2 = (a_4 - a_1)$, $n_2 = \frac{1}{2}\sqrt{(a_2 + a_3 - 2a_1)^2 + 4W^2}$, $l_3 = \sqrt{(a_4 - a_3)^2 + W^2}$, $m_3 = \frac{1}{2}\sqrt{(2a_4 - a_2 - a_3)^2 + 4W^2}$, $n_3 = (a_3 - a_2)$.

We detected the centroid $G_{\tilde{A}}(\widetilde{x_0},\widetilde{y_0})$ for $\tilde{A} = (a_1, a_2, a_3, a_4; w)$ as $G_{\tilde{A}}(\widetilde{x_0},\widetilde{y_0}) =$

$$\left(\frac{\frac{a_1l_1+a_2m_1+\frac{a_2+a_3}{2}n_1}{l_1+m_1+n_1}+\frac{a_1l_2+\frac{a_2+a_3}{2}m_2+a_4n_2}{l_2+m_2+n_2}+\frac{\frac{a_2+a_3}{2}l_3+a_3m_3+a_4n_3}{l_3+m_3+n_3}}{3}, \frac{\frac{w(m_1+n_1)}{l_1+m_1+n_1}+\frac{wm_2}{l_2+m_2+n_2}+\frac{w(l_3+m_3)}{l_3+m_3+n_3}}{3}\right)$$

As a diplomatic immunity, for a Generalized Triangular Fuzzy Number $\tilde{A} = (a_1, a_2, a_4; w)$ (ie, $a_2 = a_3$), the incenters enhance $I_1(x_{i1}, y_{i1}) = \left(\frac{a_1 l_1 + a_2(m_1+n_1)}{l_1+m_1+n_1}, \frac{w(m_1+n_1)}{l_1+m_1+n_1}\right)$; $I_2(x_{i2}, y_{i2}) = \left(\frac{a_1 l_2 + a_2 m_2 + a_4 n_2}{l_2+m_2+n_2}, \frac{w m_2}{l_2+m_2+n_2}\right)$ and $I_3(x_{i3}, y_{i3}) = \left(\frac{a_2(l_3+m_3)+a_4 n_3}{l_3+m_3+n_3}, \frac{w(l_3+m_3)}{l_3+m_3+n_3}\right)$. Where

$l_1 = 0$, $m_1 = \sqrt{(a_2-a_1)^2 + W^2}$, $n_1 = \sqrt{(a_2-a_1)^2+W^2}$, $l_2 = \sqrt{(a_4-a_2)^2+W^2}$, $m_2 = (a_4 - a_1)$, $n_2 = \sqrt{(a_2-a_1)^2+W^2}$, $l_3 = \sqrt{(a_4-a_2)^2+W^2}$, $m_3 = \sqrt{(a_4-a_2)^2+W^2}$ and $n_3 = 0$.

Then the centroid $G_{\tilde{A}}(\widetilde{x_0}, \widetilde{y_0})$ with incenters I_1, I_2 and I_3,

$$G_{\tilde{A}}(\widetilde{x_0}, \widetilde{y_0}) = \left(\frac{\frac{a_1 l_1 + a_2(m_1+n_1)}{l_1+m_1+n_1} + \frac{a_1 l_2 + a_2 m_2 + a_4 n_2}{l_2+m_2+n_2} + \frac{a_2(l_3+m_3)+a_4 n_3}{l_3+m_3+n_3}}{3}, \frac{\frac{w(m_1+n_1)}{l_1+m_1+n_1} + \frac{w m_2}{l_2+m_2+n_2} + \frac{w(l_3+m_3)}{l_3+m_3+n_3}}{3}\right).$$

Then the ordering function of $\tilde{A}$ is $R(\tilde{A}) = \widetilde{\mathbf{x_0}} \times \widetilde{\mathbf{y_0}}$.

4. Biserial servers connected with a main server

Biserial servers connected with a main server means, the customers may get the services either from server 1(or 2), to server 2 (or 1), (biserial) then Main server or from the server 1 or 2 directly to (biserial) the Main server. So, we get more possibilities from this situation. Here we have three servers, so that three queues are formed in front of the servers. The customers are presented to the server 1 (S_1) and server 2 (S_2) with the Fuzzy arrival rates $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ by the poisson stream, and the Fuzzy service rates $\tilde{\mu}_1$ and $\tilde{\mu}_2$ for S_1 and S_2 are followed by the exponential distribution. The Fuzzy service rate for the Main server is $\tilde{\mu}$. The general structure of Biserial servers connected with a main server as shown in the figure 2

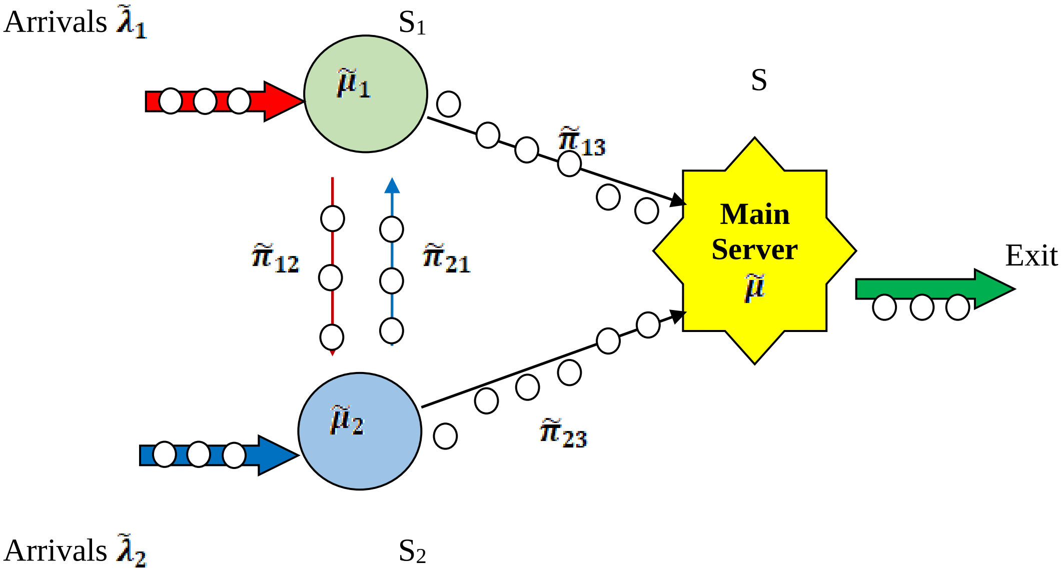

Fig 2. Biserial servers connected with a main server

Here we have four possibilities to reach the main server. Those are

1. $S_1 \rightarrow S_2 \rightarrow S$
2. $S_1 \rightarrow S$
3. $S_2 \rightarrow S_1 \rightarrow S$
4. $S_2 \rightarrow S$

Also we have the Fuzzy possibility rates $\tilde{\pi}_{12}$, $\tilde{\pi}_{23}$, $\tilde{\pi}_{21}$, and $\tilde{\pi}_{13}$.

By the basic Queueing theory concepts, the Fuzzy traffic intensities

1. $\widetilde{\rho_1} = \frac{(\tilde{\lambda}_1 + \tilde{\lambda}_2\tilde{\pi}_{21})}{\widetilde{\mu_1}(1-\tilde{\pi}_{12}\tilde{\pi}_{21})} < 1$
2. $\widetilde{\rho_2} = \frac{(\tilde{\lambda}_2 + \tilde{\lambda}_1\tilde{\pi}_{12})}{\widetilde{\mu_2}(1-\tilde{\pi}_{12}\tilde{\pi}_{21})} < 1$
3. $\widetilde{\rho_3} = \frac{\tilde{\pi}_{13}(\tilde{\lambda}_1 + \tilde{\lambda}_2\tilde{\pi}_{21}) + \tilde{\pi}_{23}(\tilde{\lambda}_2 + \tilde{\lambda}_1\tilde{\pi}_{12})}{\tilde{\mu}(1-\tilde{\pi}_{12}\tilde{\pi}_{21})} < 1$

By these appraises,

The expected aggregate of customers in the system (E(N)) = $E(N_1) + E(N_2) + E(N_3)$,

Where $E(N_1) = \frac{\widetilde{\rho_1}}{1-\widetilde{\rho_1}}$, $E(N_2) = \frac{\widetilde{\rho_2}}{1-\widetilde{\rho_2}}$, and $E(N_3) = \frac{\widetilde{\rho_3}}{1-\widetilde{\rho_3}}$.

The expected waiting duration in the system (E(W)) = $\frac{E(N)}{\tilde{\lambda}_1 + \tilde{\lambda}_2}$.

5. Numerical Example

In a particular clinic, it has a doctor and two departments. One is registration department and the other is Preliminary testing department. Here we consider the registration department is Sever 1 (S_1), the testing department is Server 2 (S_2), and the Doctor is the main server (S). The patients may get the services either from server 1(or 2), to server 2 (or 1), (biserial) then meet the Doctor or from the server 1 or 2 directly to meet the Doctor. In this situation, we can use our proposed formulas and get the measures such are the (E(N)) and (E(W)).

For Trapezoidal Fuzzy Number

Presuppose the arrival, service and possibility rates are trapezoidal fuzzy numbers modeled by $\tilde{\lambda}_1$ = [0,1,2,3], $\tilde{\lambda}_2$ = [1,2,3,4], $\tilde{\mu}_1$= [5,6,7,8], $\tilde{\mu}_2$= [4,5,6,7], $\tilde{\mu}$ = [2,3,4,5], $\tilde{\pi}_{12}$ = [0.5,0.6,0.7,0.8], $\tilde{\pi}_{21}$= [0.4,0.5,0.6,0.7], $\tilde{\pi}_{23}$ = [0.6,0.5,0.4,0.3], and $\tilde{\pi}_{13}$= [0.5,0.4,0.3,0.2], respectively. By using our proposed approach, we got the expected aggregate of patients in the system (E(N)) and the expected waiting duration of the patients in the system (E(W)) per unit time. Also, by changing the values of $\tilde{\lambda}_1$ and $\tilde{\mu}_1$ and fix all the other rates, then we reach the expected performance measures. We have tabulated all the performance measures as follows.

E(N)	$\tilde{\mu}_1$=[5,6,7,8]	$\tilde{\mu}_1$=[6,7,8,9]	$\tilde{\mu}_1$=[7,8,9,10]
$\tilde{\lambda}_1$= [0,1,2,3]	3.93	3.76	3.66
$\tilde{\lambda}_1$= [1,2,3,4]	7.00	6.54	6.32
$\tilde{\lambda}_1$= [2,3,4,5]	14.43	12.44	11.77

Table 1: E(N) per unit time $\tilde{\lambda}_1$ Vs $\tilde{\mu}_1$

Table 1 instructs that, if we increase the rates of arrivals of the patients then the expected aggregate of patients in the system (E(N)) is increased, and if we increase the rates of service then the expected aggregate of patients in the system (E(N)) is decreased.

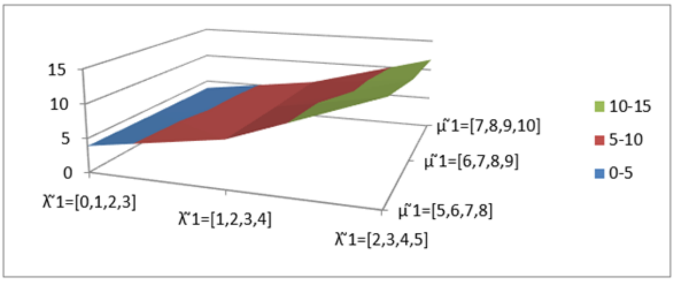

Fig 3: E(N) per unit time $\tilde{\boldsymbol{\lambda}}_1$ Vs $\tilde{\boldsymbol{\mu}}_1$

The figure 3 is the graphical representation for the values of Table 1.

E(W)	$\tilde{\mu}_1$=[5,6,7,8]	$\tilde{\mu}_1$=[6,7,8,9]	$\tilde{\mu}_1$=[7,8,9,10]
$\tilde{\lambda}_1$=[0,1,2,3]	1.48	1.42	1.39
$\tilde{\lambda}_1$=[1,2,3,4]	2.12	1.98	1.91
$\tilde{\lambda}_1$=[2,3,4,5]	3.63	3.14	2.97

Table 2: E(W) per unit time $\tilde{\boldsymbol{\lambda}}_1$ Vs $\tilde{\boldsymbol{\mu}}_1$

Table 2 instructs that, if we increase the rates of arrivals of the patients, then (E(W)) is increased, and if we increase the rates of service, then (E(W)) is decreased.

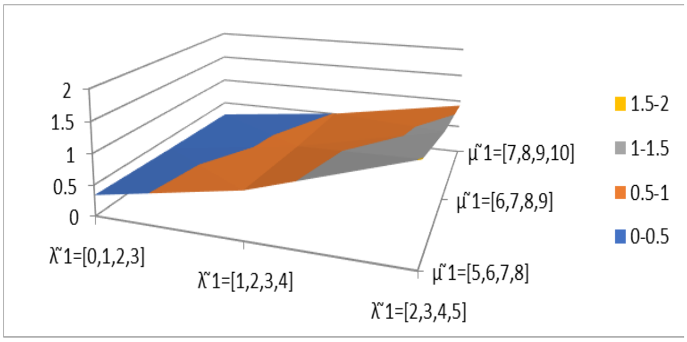

Fig 4: E(W) per unit time $\tilde{\boldsymbol{\lambda}}_1$ Vs $\tilde{\boldsymbol{\mu}}_1$

The figure 4 is the graphical representation for the values of Table 2.

For Triangular Fuzzy Number

Presuppose the arrival, service and possibility rates are triangular fuzzy numbers modelled by $\tilde{\lambda}_1$ = [0,1,3], $\tilde{\lambda}_2$ = [1,2,4], $\tilde{\mu}_1$= [5,6,8], $\tilde{\mu}_2$= [4,5,7], $\tilde{\mu}$ = [2,3,5], $\tilde{\pi}_{12}$ = [0.5,0.6,0.8], $\tilde{\pi}_{21}$= [0.4,0.5,0.7], $\tilde{\pi}_{23}$ = [0.6,0.5,0.3], and $\tilde{\pi}_{13}$= [0.5,0.4,0.2], respectively. By using our proposed approach, we got the expected aggregate of patients in the system (E(N)) and the expected waiting duration of the patients in the system (E(W)) per unit time. Also by changing the values of $\tilde{\lambda}_1$ and $\tilde{\mu}_1$ and fix all the other rates, then we reach the expected performance measures. We have tabulated all the performance measures as follows.

E(N)	$\tilde{\mu}_1$=[5,6,8]	$\tilde{\mu}_1$=[6,7,9]	$\tilde{\mu}_1$=[7,8,10]
$\tilde{\lambda}_1$=[0,1,3]	3.47	3.32	3.21
$\tilde{\lambda}_1$=[1,2,4]	7.06	6.51	6.28
$\tilde{\lambda}_1$=[2,3,5]	19.00	16.44	15.73

Table 3: E(N) per unit time $\tilde{\lambda}_1$ Vs $\tilde{\mu}_1$

Table 3 instructs that, if we increase the rates of arrivals of the patients then the expected aggregate of patients in the system (E(N)) is increased, and if we increase the rates of service then the expected aggregate of patients in the system (E(N)) is decreased.

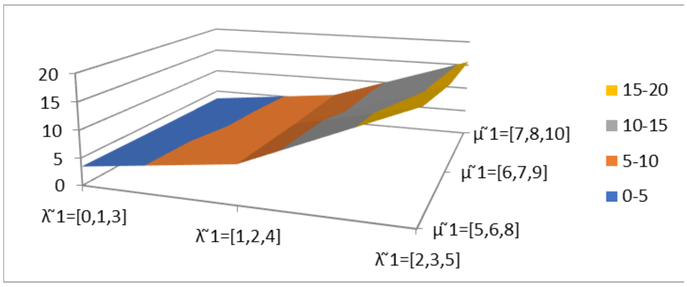

Fig 5: E(N) per unit time $\tilde{\lambda}_1$ Vs $\tilde{\mu}_1$

The figure 5 is the graphical representation for the values of Table 3.

E(W)	$\tilde{\mu}_1$=[5,6,8]	$\tilde{\mu}_1$=[6,7,9]	$\tilde{\mu}_1$=[7,8,10]
$\tilde{\lambda}_1$=[0,1,3]	1.33	1.28	1.23
$\tilde{\lambda}_1$=[1,2,4]	2.08	1.91	1.85
$\tilde{\lambda}_1$=[2,3,5]	4.51	3.92	3.75

Table 4: E(W) per unit time $\tilde{\lambda}_1$ Vs $\tilde{\mu}_1$

Table 4 instructs that, if we increase the rates of arrivals of the patients, then (E(W)) is increased, and if we increase the rates of service, then (E(W)) is decreased.

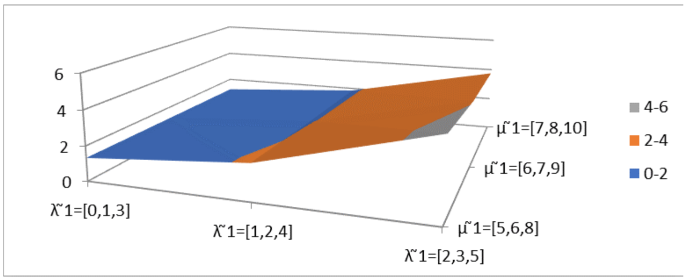

Fig 6: E(W) per unit time $\tilde{\lambda}_1$ Vs $\tilde{\mu}_1$

The figure 6 is the graphical representation for the values of Table 4.

Conclusion

In this article, we have analyzed the measures of the Biserial servers connected with a main server by Fuzzy ordering approach. Biserial queuing system is manipulated in operations and service mechanism for estimating the queue performances. We detect the performance measures E(N) and

E(W) are as classical numerals, the analyzer can clutch the foremost and future determinations. We halt that the culmination of fuzzy problems can be captured by the fuzzy ordering approach very beneficially. This approach can help to take supreme verdicts for future analyzer and researchers.

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