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On Primitive Ternary Γ-Semirings

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Abstract:

Subsequent to presenting the thoughts of primitive Ternary Γ-semi ring along with primitive ideal of a Γ-semi ring we concentrate on them through operator semi ring and get a few outcomes similar to those of semi ring hypothesis.

Keywords: Semi-irreducible, Irreducible, faithul ΓS-semimodules, primitive Ternary Γ-semi ring.

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1. Introduction :

Presenting the thought of Ternary Γ-semiring S-Semimodule entitled ΓS-semimodule alongside the thoughts of semi-irreducible, irreducible, faithul ΓS-semimodules with a goal to present the idea of primitive Γ-semiring in addition to prospect to present the idea of Jacobson radical. Now we concentrate on primitive TΓ-semiring by means of the operator semirings. We confirm that by [6] a right operator semiring R is primitive iff TΓ-semiring S is primitive. In conclusion, "primitive h-ideal of a TΓ-semiring S utilizing the connection amid the Annihilator of an irreducible TΓS-semimodule M in S with the aim of M in the right operator semiring R of TΓ-semiring S".

"Throughout this paper TΓ-semiring - TΓ-S,Ternary ΓS-semimodule denoted by TΓS-S, Ternary ΓS-subsemimodule denoted by TΓS-SS, irreducible TΓ-semimodule- ITΓ-S, irreducible TΓS-semimodule- ITΓS-S , Primitive Ideal-PI, right operator semiring-ROS
faithful irreducible TΓS-semimodule- FTΓS-S, faithful irreducible R-semimodule-FIR-S, irreducible R-semimodule-IR-S, faithful irreducible $R/P^{*\prime}$-semimodule- FI$(R/P^{*\prime})$-S, additive commutative monoid-ACM, TΓS-semiring- TΓSS"

Preliminaries

Definition 2.1: Let U, Γ be two additive commutative semigroups. Then U is entitled a ***ternary gamma semiring*** presented ∃ a mapping $U\times\Gamma\times U\times\Gamma\times U\rightarrow U$ suiting the next conditions:

(1) $x\eta y\varsigma(z\lambda p\psi q)=x\eta(y\varsigma z\lambda p)\psi q=(x\eta y\varsigma z)\lambda p\psi q$

(2) $[(p+q)\eta r\varsigma s]=[(p\eta r\varsigma s]+[q\eta r\varsigma s]$

(3) $[p\eta(q+r)\varsigma s]=[(p\eta q\varsigma s]+[p\eta r\varsigma s]$

(4) $[p\eta q\varsigma(r+s)]=[(p\eta q\varsigma r]+[p\eta q\varsigma s]\forall p,q,r,s\in U \,\&\, \eta,\varsigma,\psi,\lambda\in\Gamma.$

An ideal J in a TΓ-semiring S is called a k-ideal if $x_1+y_1\in J, x_1\in S, y_1\in J\Rightarrow x_1\in J.$

In a TΓ-S semiring S an ideal I is entitled an 'h-ideal' if $x_1+y_1+z_1=y_2+z_1, x_1,z_1\in S \,\&\, y_1,y_2\in I\Rightarrow x_1\in I.$ permit S be a TΓ-S & free additive commutative semigroup G generate by $\Gamma\times S\times\Gamma\times S.$ subsequently relation ρ on G, defined by If

$\sum_i[\tau_i,k_i]=\sum_j[\sigma_j,l_j]$ in R then $\sum_i s\tau_i t\lambda_i k_i=\sum_j s\sigma_j t\mu_j k_j\forall s,t\in S.$

Semi-irreducible, Irreducible, Faithful TΓ-Semimodules :

Definition 3.1: Permit S is a TΓ-S. An ACM 'M' is known as merely TΓS-S, if $\exists N\times\Gamma\times S\times\Gamma\times S\to N$ (Images to be denoted by $k\nu s\eta t: k\in N,\nu,\eta\in\Gamma$) satisfying the following conditions:

(i) $(k+l)\nu s\eta t=k\nu s\eta t+l\nu s\eta t$

(ii) $k\nu s\eta(t+u)=k\nu s\eta t+k\nu s\eta u$

(iii) $k\nu(s+t)\eta u=k\nu s\eta u+k\nu t\eta u$

(iv) $(k\nu s\eta t)\varepsilon u\pi v=k\nu(s\eta t\varepsilon u)\pi v+k\nu s\eta(t\varepsilon u\pi v)$

(v) $0_N\eta t\varepsilon u=0_N=k\varepsilon s\pi 0_S=k\varepsilon 0_S\pi s\forall k,l\in N,s,t,u,v\in S.$

In addition to the above conditions if $\sum k\varepsilon e\phi f=k\forall k\in N,$ where {e, f}is an identity element of S, then N is believed to be a unitary right ternary ΓS-semimodule.
A left TΓS-semimodule can be defined likewisely.

Example: 3.2: A TΓSS 'S', wherever the 'additive commutative semigroup' S of all 2×3 matrices above Q_0^+ & Γ is also 'additive commutative semigroup' of all 3×2 matrices over the identical set & $n\varepsilon k\eta l$ denote product of matrices of $n,\varepsilon,k,\eta,l; n\in N,k,l\in S \,\&\, \varepsilon,\eta\in\Gamma$ now the right unity of S is

$\sum_{i=1}^{3}[\gamma_i,e_i,\varepsilon_i,f_i]$ where

$$\gamma_1=\varepsilon_1=\begin{pmatrix}1&0\\0&1\\0&0\end{pmatrix},\gamma_2=\varepsilon_2=\begin{pmatrix}0&0\\0&1\\0&0\end{pmatrix},\gamma_3=\varepsilon_3=\begin{pmatrix}0&0\\1&0\\0&0\end{pmatrix},$$

$$f_1=e_1=\begin{pmatrix}1&0&0\\0&\frac{1}{3}&0\end{pmatrix}, f_2=e_2=\begin{pmatrix}0&0&1\\0&\frac{1}{3}&0\end{pmatrix}, f_3=e_3=\begin{pmatrix}0&\frac{1}{3}&0\\0&0&1\end{pmatrix}.$$

A subset $N(\neq\phi)$ of a TΓS-S N is known as a TΓS-S of N if (i) $k+l\in N$, (ii) $k\varepsilon s\eta t\in N\forall s,t\in S,\varepsilon,\eta\in\Gamma,k,l\in N$ holds the zero of N.

A TΓS-SS N of a TΓS-subsemimodule O is named KΓS-subsemimodule O if $k+l\in N,l\in N,k\in O\Rightarrow k\in N.$ Let N be a TΓS-SS of a TΓS-subsemimodule O. Subsequently kclosure of N, is described by $\overline{N} = \{ j \in O : j + k = l : k, l \in N \}$. A TΓS-SS 'N' of a TΓS-S O is named as ternary (hΓS-) subsemimodule of O if $x_1 + n_1 + z_1 = n_2 + z_1, n_1, n_2 \in N, x_1, z_1 \in O \Rightarrow x_1 \in N$.

Let N be a TΓS-SS of a TΓS-S 'O'. Afterward h-closure of N, is named by $\overline{N}$ = $\{ o \in N : o + n_1 + z = n_2 + z, n_1, n_2 \in N, z \in O \}$.

Proposition 3.03: Let N be a TΓS-SS 'N' of a TΓS-S "O". Afterward kΓS-(hΓS-) subsemimodule iff $\overline{\mathrm{N}} = \mathrm{N}$.

Proof: The confirmation involves routine check. A TΓS-S 'O' is known as cancellative if $k + l = k + m, k, l, m \in O \Rightarrow l = m$.

All through the remainder of the paper TΓS-S is 'cancellative'.

Definition 3.04: A TΓS-S $\{0\} \neq O$ is called as 'irreducible' iff random set $u, v \in O$ among $u \neq v$ for any $k \in N \exists f_i, g_i, h_j, i_j \in S, \sigma_i, \tau_i \in \Gamma (j = 1,2,3,4,...s \,\&\, i = 1,2,3,4,....r, s, r \in Z^+)$ such that $\ni k + \sum_i u\tau_i f_i \sigma_i g_i + \sum_j v \lambda_j h_j \mu_j i_j = \sum_j u \lambda_j h_j \mu_j i_j + \sum_i v \tau_i f_i \sigma_i g_i$. A TΓS-S 'O' is said to be semi-irreducible iff $O\Gamma S \Gamma S \neq \{0\}$ & O doesn't have any kΓS-subsemimodule except 0 & O.

The thoughts of both semi-irreducibility& irreducibility concur by the idea of irreducibility in a TΓ-ring S([7],[8],[9]).

Proposition 3.05: An ideal Q of a TΓ-semiring S & a TΓS-S 'N'

by $N\Gamma S \Gamma Q \neq \{0\}$. Then, at that point, the accompanying assertions are valid.

(1) If N is semi-irreducible & $n \in N$ afterward n=0 iff $n\tau s \varepsilon q = 0 \forall \tau, \varepsilon \in \Gamma \,\&\, \forall s \in S, q \in Q$ i.e., n=0 iff $N\Gamma S \Gamma Q = \{0\}$.

(2) If N is irreducible & $u, v \in N$ afterward u=v iff $\sum_{i=1}^{n} u\tau_i k_i \sigma_i l_i = \sum_{i=1}^{n} v \tau_i k_i \sigma_i l_i, \forall \tau_i, \sigma_i \in \Gamma, \forall k_i, l_i \in S, i = 1,2,....p;$ p is any positive integer.

Proof: (1) Given N is semi-irreducible & $n\tau s \varepsilon q = 0 \forall \tau, \varepsilon \in \Gamma \,\&\, \forall s \in S, q \in Q$. Let $N_0 = \{ y \in N : y\Gamma S \Gamma Q = \{0\} \}$. $\Rightarrow n \in N_{0.}$.

Let $k, l \in N_{0.}$. Then $(k + l)\Gamma S \Gamma Q \subseteq k\Gamma S \Gamma Q + l\Gamma S \Gamma Q = \{0\}$. Thus $(k + l) \in N_{0.}$

Thus $N_{0.}$ is a TΓS-subsemimodule of N. Let $(k + l), l \in N_{0.} \,\&\, k \in N$. Then $(k + l)\tau s \sigma q = 0 \,\&\, l\tau s \sigma q = 0 \forall \tau, \sigma \in \Gamma \,\&\, \forall q \in Q, s \in S$.

$\Rightarrow k\tau s \sigma q = k\tau s \sigma q + l\tau s \sigma q = (k + l)\tau s \sigma q \forall \tau, \sigma \in \Gamma \,\&\, \forall q \in Q, s \in S$ hence $k\Gamma S \Gamma Q = \{0\}$.

Hence $k \in N_0$ proving N_0 is a ternary K-subsemimodule of N. $\because N\Gamma S \Gamma Q \neq \{0\}, N_0 \neq N$. $\because$ S is semi-irreducible subsequently $N_0 = \{0\}$. $\therefore$ n=0. On the contrary, if n=0 then $n\tau s \varepsilon q = 0 \forall \tau, \varepsilon \in \Gamma \,\&\, \forall s \in S, q \in Q$.

(2) Let N be irreducible & $u, v \in N \ni u \neq v$. $\because N\Gamma S \Gamma Q \neq \{0\}, \exists n \in N, t \in S \,\&\, q \in Q \ni n\tau t \sigma q \neq 0$.

Again since N is irreducible, for this n,

$\exists f_i, g_i, h_j, i_j \in S(1 \le i \le r, 1 \le j \le s; r, s \in Z^+$ such that

$$n + \sum_{i=1}^{r} u\tau f_i \sigma g_i + \sum_{j=1}^{s} v\lambda h_j \mu i_j = \sum_{j=1}^{s} u\tau h_j \sigma i_j + \sum_{i=1}^{r} v\lambda f_i \mu g_i .$$

Hence $n\tau\delta t\varepsilon p + \sum_{i=1}^{r} u\tau f_i \sigma g_i \eta t\vartheta p + \sum_{j=1}^{s} v\lambda h_j \mu i_j \pi t\varsigma p = \sum_{j=1}^{s} u\tau h_j \sigma i_j t\vartheta p + \sum_{i=1}^{r} v\lambda f_i \mu g_i t\vartheta p$

$\forall t \in S \;\&\; p \in P, \delta, \varepsilon, \tau, \sigma, \eta, \mu, \vartheta, \varsigma, \lambda \in \Gamma.$

$$\Rightarrow n\delta t\varepsilon p + \sum_{i=1}^{r} u\tau f_i \sigma g_i' + \sum_{j=1}^{s} v\lambda h_j \mu i_j' = \sum_{j=1}^{s} u\tau h_j \sigma i_j' + \sum_{i=1}^{r} v\lambda f_i \mu g_i'$$

Where $g_i' = g_i \eta t\vartheta p \in P \;\&\; i_j' = i_j \pi t\varsigma p \in P.$

Since N is cancellative and $n\delta t\varepsilon p \ne 0$ so atleast one of $\sum_{i=1}^{r} u\tau f_i \sigma g_i' \ne \sum_{i=1}^{r} v\tau f_i \sigma g_i' \;\&\; \sum_{j=1}^{s} u\lambda h_j \mu i_j' \ne \sum_{j=1}^{s} v\lambda h_j \mu i_j'$ holds. The converse part follows easily.

Proposition 3.6: Let N be a TΓS-S & $N \ne \{0\}$. Subsequently N is semi-irreducible iff for every non-zero $n \in N, \overline{N\Gamma S\Gamma S} = N$ i.e. for any $k \in N \exists k_i, l_i \in S, \sigma_i, \tau_i \in \Gamma(j = 1,2,3,4...s \;\&\; i = 1,2,3,4....r, s, r \in Z^+)$ $\ni k + \sum_i n\tau_i f_i \sigma_i g_i = \sum_j n\lambda_j h_j \mu_j i_j$

Proof: Let $N \ne 0$ be semi-irreducible. Then $N\Gamma S\Gamma S \ne \{0\}$. Let $n \in N \ni n \ne 0.$

Hence by Proposition 3.5, $N\Gamma S\Gamma S \ne \{0\}$; so $\overline{N\Gamma S\Gamma S} \ne \{0\}$.

Since $\overline{N\Gamma S\Gamma S}$ is a ternary $k\Gamma S$-subsemimodule of N, $\overline{N\Gamma S\Gamma S} \ne N$. Hence for any $\overline{N\Gamma S\Gamma S} \ne \{0\}$. Hence for any $k \in N \exists k_i, l_i \in S, \sigma_i, \tau_i \in \Gamma(j = 1,2,3,4...s \;\&\; i = 1,2,3,4....r, s, r \in Z^+)$ such that

$$k + \sum_i n\tau_i f_i \sigma_i g_i = \sum_j n\lambda_j h_j \mu_j i_j$$

In opposition, assume for any $n(\ne 0) \in N, \overline{n\Gamma S\Gamma S} = N$. Let $\{0\} \ne$ N is ternary $k\Gamma S$-subsemimodule of N. $\exists m \in M \ni n \ne 0.$ So, by the given condition $\overline{N\Gamma S\Gamma S} = N.$ Hence for any for any $k \in N \exists f_i, g_i, h_j, i_j \in S, \sigma_i, \tau_i \in \Gamma(i = 1,2,3,....r \;\&\; j = 1,2,3,...s, r, s \in Z^+)$ such that $k + \sum_i n\tau_i f_i \sigma_i g_i = \sum_j n\lambda_j h_j \mu_j i_j$. Since M is a $k\Gamma S$-subsemimodule of N & $\sum_i n\tau_i f_i \sigma_i g_i, \sum_j n\lambda_j h_j \mu_j i_j \in M, k \in N.$ Hence M=N. Now if $N\Gamma S\Gamma S = \{0\}$ then $n\Gamma S\Gamma S = \{0\} \forall n \in N.$

In particular, $N\Gamma S\Gamma S = \{0\}$ for any nonzero $n \in N.$ Hence $\overline{n\Gamma S\Gamma S} = \{0\}$ for any $n(\ne 0) \in N.$ $\Rightarrow$ N=0- a disagreement.

Consequently N is ‘semi-irreducible’.

Corollary 3.07: If a TΓ-S N is ‘irreducible’, subsequently $\overline{n\Gamma S\Gamma S} = N$ & semi-irreducible.

Proof: Let N be an ITΓ-S. $\Rightarrow N \ne \{0\}$. $\therefore \exists n(\ne 0) \in N.$

any $k \in N \exists f_i, g_i, h_j, i_j \in S, \sigma_i, \tau_i \in \Gamma(j = 1,2,3,...s \;\&\; i = 1,2,3,....r, s, r \in Z^+)$ $\ni k + \sum_i n\tau_i f_i \sigma_i g_i = \sum_j n\lambda_j h_j \mu_j i_j$. N is semi-irreducible TΓ-semimodule, by proposition (3.6), Subsequently $N\Gamma S\Gamma S \neq \{0\} \Rightarrow \overline{n\Gamma S\Gamma S} = \{0\}$. Since $\overline{n\Gamma S\Gamma S}$ is a ternary $k\Gamma S$-subsemimodule of N, $\overline{n\Gamma S\Gamma S} = N$.

Proposition 3.08: In a TΓ-S 'S', R exist its ROS. Next N is an ITΓS-S iff N is an IR-S.

Proof: Let N be an ITΓS-S. Characterize R-action on M as go behind: meant for $f, g \in N, \sum_i [\tau_i, k_i] \in R, f, g\sum_i [\tau_i, k_i] = \sum_i f\tau_i g\lambda_i k_i$. If $\sum_i [\tau_i, k_i] = \sum_j [\sigma_j, l_j]$ in R then $\sum_i s\tau_i t\lambda_i k_i = \sum_j s\sigma_j t\mu_j k_j \forall s, t \in S$. Since N is an irreducible TΓS-semimodule, $\overline{n\Gamma S\Gamma S} = N$. (Corollary 3.7). Then for $n \in N \exists f_k, g_k, h_t, i_t \in N, \tau_k, \sigma_k \in \Gamma, s_k, v_t \in S (t = 1, 2, ...q \ \& \ k = 1, 2, ...p; q, p \in Z^+)$

$\ni n + \sum_k f_k \tau_k g_k \sigma_k v_k = \sum_t h_t \lambda_t i_t \mu_t v_t$. So,

$\sum_i n\chi_i k_i \delta_i l_i + \sum_{k,i} f_k \tau_k g_k \sigma_k s_k \varepsilon_k k_i \gamma_i l_i = \sum_{t,i} h_t \lambda_t k_i \gamma_i l_i$.

$\Rightarrow (1) \sum_i n\chi_i k_i \delta_i l_i + \sum_{k,j} f_k \tau_k g_k \sigma_k s_k \varepsilon_k k_j \gamma_j l_j = \sum_{t,j} h_t \lambda_t i_t \varsigma_t v_t k_j \zeta_j l_j$.

Again

$\Rightarrow (2) \sum_j n\phi_j o_j \eta_j p_j + \sum_{k,j} f_k \tau_k g_k \sigma_k s_k \varepsilon_k k_j \gamma_j l_j = \sum_{t,j} h_t \lambda_t i_t \varsigma_t v_t k_j \zeta_j l_j$.

$\because$ From (1) and (2)N is cancellative.

$\sum_i n\chi_i k_i \delta_i l_i = \sum_j n\phi_j o_j \eta_j p_j$.

Accordingly the R-action define on N is well defined. Currently validate that N as R-semimodule. Let $u, v \in N, u \neq v$. $k \in N \exists f_i, g_i, h_j, i_j \in S, \tau_i, \sigma_i \in \Gamma \ni k + \sum_i u\tau_i f_i \sigma_i g_i + \sum_j v\lambda_j h_j \mu_j i_j = \sum_j u\lambda_j h_j \mu_j i_j + \sum_i v\tau_i f_i \sigma_i g_i$ (using irreducibility of N as TΓS-semimodule).

$\Rightarrow k + u\sum_i [\tau_i, f_i, \sigma_i, g_i] + v\sum_j [\lambda_j, h_j, \mu_j, i_j] + v\sum_i [\tau_i, f_i, \sigma_i, g_i]$ where $\sum_i [\tau_i, f_i, \sigma_i, g_i], \sum_j [\lambda_j, h_j, \mu_j, i_j] \in R$.

Consequently N is an IR-S ([6]).

On the contrary, presume N is an IR-S. Name Γ-action of S on N as go after: for $f \in N, \varpi, \varsigma \in \Gamma \ \& \ s_1, s_2 \in S, f\varpi S\varsigma S = f[\varpi, s_1, \varsigma, s_2]$. $\therefore$ N is a TΓS-S.

$\Rightarrow u, v \in N, u \neq v \ \& \ k \in N, \exists \sum_i [\tau_i, f_i, \sigma_i, g_i], \sum_j [\lambda_j, h_j, \mu_j, i_j] \in R \ni$

$\Rightarrow k + u\sum_i [\tau_i, f_i, \sigma_i, g_i] + v\sum_j [\lambda_j, h_j, \mu_j, i_j] = u\sum_j [\lambda_j, h_j, \mu_j, i_j] + v\sum_i [\tau_i, f_i, \sigma_i, g_i]$. S.

$\therefore k + \sum_i u\tau_i f_i \sigma_i g_i + \sum_j v\lambda_j h_j \mu_j i_j = \sum_j u\lambda_j h_j \mu_j i_j + \sum_i v\tau_i f_i \sigma_i g_i$.

Consequently by description N is an ITΓS-S.

Let S be a TΓ-S. Zeroed of S, is identified as $Z(S)=\{y_1\in S: y_1+z_1=z_1; z_1\in S\}$. evidently, 0 is a associate of Z(S) of a TΓ-S 'S' among zero (0) component. Z(S) of a TΓ-S 'S' is an 'h-ideal'. Permit O is a TΓS-S. $\Rightarrow$ (0: O) $=\{y\in S: o\Gamma y\Gamma s=\{0\}\,\&\, o\Gamma s\Gamma y=0 \forall o\in O, \forall s\in S\}$ where $O\Gamma y\Gamma s=\{\sum_{i=1}^{k} o_i\varpi_i y_i\vartheta_i s_i : o_i\in O, \varpi_i, \vartheta_i\in\Gamma, y_i, s_i\in S, k\in Z^+\}$. We call (0: O) the annihilator of O in S. Designate it by $A_S(O)$. A TΓS-S 'O' is held elect faithful if $A_S(O)=Z(S)$.

Proposition 3.09: A TΓS-S 'O'. Afterward $A_S(O)$ is an 'h-ideal' of S. Likewise, O is faithful $\Gamma(S/A_S(O))$- semi module.

Proof: Obviously $A_S(O)$ is an additive subsemigroup of S. Currently $y\in A_S(O), \tau, \vartheta\in\Gamma, s\in S$. Next $O\Gamma(y\vartheta S\tau S)=(O\Gamma y)\vartheta S\tau S=\{0\}$. Hence $y\vartheta S\tau S\in A_S(O)$ verifying as R- ideal. Correspondingly we confirm $A_S(O)$ is a L/La ideal of S.

After that $y+s_1+z=s_2+z$ wherever $y,z\in S, s_1, s_2\in A_S(O)$. Afterward $\forall y_1, y_2\in A_S(O), o\eta t\varpi y_1=o\eta y_1\varpi t=0\,\&\, o\eta t\varpi y_2=o\eta y_2\varpi t=0\forall o\in O\,\&\,\forall t\in S$.

Now $y+s_1+z=s_2+z\Rightarrow o\eta t\varpi x+o\eta t\varpi y_1+o\eta t\varpi z=o\eta t\varpi y_2+o\eta t\varpi z$.

This leads to $o\eta t\varpi y=0 \because o\eta t\varpi y_1=o\eta t\varpi y_2=0$ & O is additively cancellative.

Similarly, we can show that $o\eta y\varpi t=0\forall o\in O\,\&\,\forall y,t\in S$.

Thus $x\in A_S(O)$ and hence $A_S(O)$ is an 'h-ideal' of S.

Currently describe a Γ-action of $S/A_S(O)$ on O as below: $o\varepsilon(s/A_S(O))\varsigma(t/A_S(O))=o\varepsilon(S\varsigma S): s,t\in S, \varepsilon,\varsigma\in\Gamma, s/A_S(O)\in S/A_S(O)$.

If $t/A_S(O)=t'/A_S(O)$ then $t+i_1+z_1=t'+i_2+z_1\forall i_1, i_2\in A_S(O)\,\&\, z_1\in S$.

$\because i_1, i_2\in A_S(O)$, we have $o\nu s\varsigma i_1=o\nu s\varsigma i_2=0$.

$$t+i_1+z_1=t'+i_2+z_1$$
Now $\Rightarrow o\varsigma s\upsilon t+o\varsigma s\upsilon i_1+o\varsigma s\upsilon z_1=o\varsigma s\upsilon t'+o\varsigma s\upsilon i_2+o\varsigma s\upsilon z_1\forall o\in O, s\in S\,\&\,\varsigma,\upsilon\in\Gamma$
$$\Rightarrow o\varsigma s\upsilon t=o\varsigma s\upsilon t'\forall o\in O, \varsigma,\upsilon\in\Gamma.$$

$\therefore$ Γ-action of $S/A_S(O)$ on O is well defined.

Presently seeing that is O is a $\Gamma(S/A_S(O))$-semimodule.

It stays to confirm that $A_{S/A_S(O)}(O)=Z(S/A_S(O))$.

Clearly $Z(S/A_S(O))\subseteq A_{S/A_S(O)}(O)$.

$x/A_S(O) \in A_{S/A_S(O)}(O) \Rightarrow o\Gamma(t/A_S(O))\Gamma(x/A_S(O)) = 0$

Now let $\& o\Gamma(x/A_S(O))\Gamma(t/A_S(O)) = 0$

$i.e. ovt\eta x = 0 \,\&\, ovx\eta t = 0 \forall o \in O \,\&\, t \in S.$

Thus $x \in A_S(O)$ and hence

Consequently, $x/A_S(O) \in Z(S/A_S(O))$

Thus $A_{S/A_S(O)}(O) \subseteq Z(S/A_S(O)).$

Hence $A_{S/A_S(O)}(O) \subseteq Z(S/A_S(O)).$

Proposition3.10: A TΓSS ‘O’& R exist as ROS. Afterward

(i) $A_S(O)^{*\prime} = A_R(O) \,\&\, A_R(O)^* = A_S(O);$ Anywhere M is an ITΓS-S

(ii) $Z(S) = Z(R)^*, Z(S)^{*\prime} = Z(R).$

$$A_S(O)^{*\prime} = \{\sum_i [\theta_i, y_i] \in R : S(\sum_i [\theta_i, y_i]) \subseteq A_S(O)\}$$
$$= \{\sum_i [\theta_i, y_i] \in R : O\Gamma S\Gamma T(\sum_i [\theta_i, y_i]) \subseteq A_S(O)\}$$

Proof: $= \{\sum_i [\theta_i, y_i] \in R : M(\sum_i [\theta_i, y_i]) = \{0\}\} = A_R(O).$

$$A_R(O)^* = \{y \in S : [\Gamma, t] \subseteq A_R(o)\}$$
$$= \{y \in S : O[\Gamma, t] = \{0\}\}$$
$$= \{y \in S : O\Gamma T\Gamma y = \{0\}\} = A_S(O).$$

(ii) Via known 6.14-1proposition & zeroid is an ‘h-ideal’, $Z(S) = (Z(S)^*)^* \& Z(R) = (Z(R)^*)^{*\prime}$. Accordingly demonstrating one of 2 relations is adequate. Permit $x \in Z(R)^*$. Subsequently $Z(R) \supseteq [\Gamma, x]$ $\therefore S\Gamma S\Gamma x \subseteq SZ(R) \subseteq Z(S)$. $\because$ S have ‘left unity’, $x \in Z(S)$. $\Rightarrow Z(R)^* \subseteq Z(S)$. Now let $\sum_{i=1}^{m} [\theta_i, x_i] \in [\Gamma, Z(S)]$ wherever $\forall i = 1,2,3,4,....m, x_i \in Z(S)$. $\Rightarrow x_i + z_i = z_i$ for some $z_i \in S$. $[\theta_i, x_i] + [\theta_i, z_i] = [\theta_i, z_i] \forall i = 1,2,3,4....m.$

$\Rightarrow \sum_{i=1}^{m} [\theta_i, x_i] + \sum_{i=1}^{m} [\theta_i, z_i] = \sum_{i=1}^{m} [\theta_i, z_i]$ $\sum_{i=1}^{m} [\theta_i, z_i] \in Z(R)$ $\&[\Gamma, Z(S)] \subseteq Z(R).$ $\Rightarrow Z(R)^* \supseteq Z(S)$ $where \sum_{i=1}^{m} [\theta_i, z_i] \in R.$

$\therefore Z(S) = Z(R)^*.$

Proposition3.11: Let S be a TΓ-S moreover R exist its ROS. Afterward O is a FTΓS-S iff O is a FI R-S.

Proof: Let O be a FTΓS-S. Subsequently via Proposition 3.8, O is an ITΓS-S. Over $A_S(O) = Z(S)$. $\therefore A_S(O)^{*\prime} = Z(S)^{*\prime}$. Via 3.10-proposition, $A_R(O) = Z(R)$. Therefore O is a FITΓS-S. Speak follows by switching the above contention.

Definition3.12: A TΓ-S S is known to be primitive if it has a FTΓS-S. P is an 'ideal' of S is entitled Primitive if the Bourne factor TΓS (S/P) is primitive. So a TΓS S is 'primitive' if {0} is a PI.

Lemma3.13: A TΓ-S 'S' & R is its ROS 'Q' be a proper ideal of S. Subsequently $R(S/Q), R/Q^{*\prime}$ are isomorphic, anywhere $R(S/Q)$ is the ROS of the Bourne factor TΓS S/Q.

Proposition3.14: Let S be a TΓ-S & R be its ROS. If P is a PI of S followed by a PI $P^{*\prime}$ of R.

Proof: Permit P be a PI of S. Afterward S/P is a 'primitive' TΓS.

$\exists$ an irreducible faithfuly $\Gamma(S/P)$-semimodule O. O is a faithful irreducible $R(S/Q)$- semimodule via 3.11-proposition wherever $R(S/Q)$ is the ROS of (S/P). $\because R/P^{*\prime}, R(S/P)$ are isomorphic(3.13-lemma), O is a FI $(R/P^{*\prime})$-S. Accordingly, $R/P^{*\prime}$ is a 'primitive semiring' ([6]), ie., $P^{*\prime}$ is PI.

Proposition 3.15 A TΓ-S 'S' & R exist its ROS. If U is a PI of R afterward $U^{*\prime}$ is a PI of S.

Proof: Presume U is a PI of R. Subsequently R/U is a 'primitive ternary semiring'. Accordingly, $\exists$ a FI R/U - S 'O'. O is a faithful irreducible $\Gamma(S/U^*)$-semimodule via 3.11-proposition, So S/U^* is a primitive TΓS, where U* is a PI of the TΓS S.

From the over two recommendations & Hypothesis 6.6([1]) the accompanying hypothesis follows without any problem:

Theorem3.17: Let S be a TΓ-S & R be its ROS. $\exists$ a bijection all PI's of S & the set of all PI's of R using the mapping $\mathrm{P} \to \mathrm{P}^{*\prime}$, wherever an ideal P of S.

Theorem3.18: A TΓ-S 'S' is 'primitive' iff its ROS 'R' is 'primitive'.

Proof: Permit S be a 'primitive' TΓS. $\exists$ FTΓS-S 'O'.

Subsequently, via 3.11-proposition, O is a FIR-S. Accordingly, R is a primitive semiring -[6]. Contrary follows via reversing the above argument.

Ultimately, the accompanying portrayal of 'primitive h-ideal' of a TΓS closely resembles that of a PI.

Theorem 3.19: A 'h-ideal' P of a TΓ-S S is 'primitive' iff $P = A_S(O)$ for some ITΓS-S O.

Proof: Let the 'h-ideal' P of the TΓS S be primitive. Via proposition 3.14, proposition-6.11([1]) $P^{*\prime}$ is a 'primitive h-ideal' of R $\therefore P^{*\prime} = A_R(O)$([6]), wherever O is an 'irreducible R-semimodule' (Proposition 3.8). Thus $(P^{*\prime})^* = A_R(O)^* \Rightarrow P = A_S(O)$ (3.10-proposition). By reversing the above argument converse follows.

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