Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 7s (2024)

A Collaboration Graph of the Lilavati Prize Winners with Certain Domination Parameters

Renuka Lakshmi A[1], Vani M[2], Koteswaramma N [3]*, Ramprasad C[4*], Janardhana Rao K [5]

[1,2,3,4] Department of Science and Humanities, Vasireddy Venkatadri Institute of Technology, Nambur, Guntur (Dt),A.P, India.

[5]Research scholar and Lecturer in Mathematics, Government Degree College, Kovvur, East Godavari (Dt), A.P, India.

Email id: renukalakshmiavvari@gmail.com, ramprasadchegu1984@gmail.com, koteswaramma@vvit.net

Article History:

Received: *01-06-2024*

Revised: *03-07-2024*

Accepted: *29-07-2024*

Abstract:

The Lilavati Prize stands as a hallmark of excellence in the field of mathematics, recognizing outstanding contributions to the discipline. This study delves into the domination parameters exhibited by past Lilavati Prize winners, aiming to uncover the underlying factors contributing to their exceptional achievements. Through a meticulous analysis of their mathematical prowess, academic backgrounds, research methodologies, and societal impact, this research seeks to delineate the common traits and strategies employed by these laureates. Utilizing a combination of quantitative data analysis and qualitative assessments, we aim to provide insights into the intricate interplay between individual brilliance, collaborative endeavors, and institutional support in shaping mathematical excellence. By identifying key domination parameters among Lilavati Prize winners, this study not only offers valuable insights into the dynamics of mathematical achievement but also provides guidance for fostering future talent in the field.

In this present article, the collaboration graph of Lilavathi prize winners (2010-2022) was obtained with 14 nodes and 78 links. It will be quite time consuming to share a piece of information from a given node to any other node even though the whole network is connected. Moreover it demands more resources in terms of time, memory etc. To optimize the use of above resources the idea of computing the split domination, strong split domination, strong nonsplit domination, inverse domination, inverse non split domination, edge, outer connected, total connected, connected edge domination number and just excellent assumes significance. In this paper we aimed to find some particular domination parameters namely Split, Strong, Strong split, Inverse, connected, edge connected, outer connected , just excellent; domination & total domination number for the collaboration graph of Lilavathi prize winners.

Keywords: Dominating set, ErdösNumber, Lilavati award, collaborative graph, domination parameters

1.Introduction

In advanced era, graphs have grown up as the most effective leading tool of mathematics in various subjects. In the interim it has also turned out as a full-fledged substantial field of Mathematics. In graphs, the domination theory has extensive applications in different fields. Now days, it can be considered as the most basic concepts in the theory of graphs and its practical significance in web graphs, social networks, biological patterns etc., shows the increased curiosity towards the topic. In general the domination appears in problems like locating the facilities in which one aims to reduce the distance a person requires to reach the nearest facility when the facility is fixed. A similar kind of problem arises where the maximum distance to a particular facility remains constant and a person tries to see that everyone is facilitated with the minimum number of facilities. Also the concept of domination arises in the problems of obtaining a set of representatives, in electrical networks or in land survey etc.,

Claude Berge [2] and Oystein Ore[15] make known the concept of dominant sets in graphs as the first time in 1958 and 1962 respectively. After reading write-up of Ernie Cockayne and Stephen Hedetniemi, many academics became more interested in the subject of domination in graphs [3]. The dominating idea is also useful in problems with land surveys, establishing representative sets for communications or power grid monitoring, and facility placement. Peter J. Slater, Stephen T. Hedetniemi, T.W. Haynes, V.R. Kulli, and Stephen T. Hedetniemi are cited as well as their works, for more information on several dominant variations depicted on the graph. Many graph theorists Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Muddebihal, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, Nagaraja Rao, Vangipuram putforth many interesting concepts of domination theory and related topics. Some of them worked in introducing a new domination parameter and obtaining the boundaries of the defined variable in terms of the graph parameters. And some of them worked on the graph algorithms to study the complexity results of domination parameters. The combination of domination with other graph theoretical properties resulted in several domination parameters and many of them are defined by inflicting an added constraint on the dominating set.

Kulli and Janakiram make known the idea of split domination number[10]. By Sampath Kumar[16], the idea of a strong domination number was first presented. By Kulli and Janakiram [13], the idea of a strong split domination number was first presented. Inverse domination in graphs is a notion that was first developed by Kulli V.R[9]. Also V.Yegnanarayanan with Lokeswary and Renuka Lakshmi A. obtained the domination numbers for the colloboration graph of the Rolf Nevalinna prize winners [18 &19]. A note on inverse split and nonsplit domination in graphs was given by Ameenal bibi along with selvakumar[1].Yamuna in colloboration with sridharan[17] worked on the Just excellent graphs whereas Cockayne E.J, Hartnell B.L, Hedetniemi S.T, and Laskar R[4] has given an article on "Efficient domination in graphs". A brief study on 'The domatic number problem' was given by Gerald J.Chang[5] and Joanna Cyman[8] has given a discussion on "The outer connected domination number of a graph"

1.1 Statement of Problem

Attract collaboration graph of lilavati prize winners by considering all the winners from the date of establishment to the present (i.e, from 2010 to 2022) and also to determine its domination parameters such as split, strong split, strong, strong non split, inverse, inverse non split edge and connected edge.

1.2 A brief about Lilavati ward - LA

The Lilavati Prize is a unique international award that recognizes magnificent hype work in mathematics. This award was tremendously eulogized in the conference and mathematical media, the 'International Congress of Mathematicians' closing ceremony always includes an award ceremony, thus the IMU decided to make it a recrudescing prize that is given out once every four years. Instead of honoring mathematical research, the Lilavati Prize seeks to promote endeavors as far as possible.

1.3 Brief history of Lilavati award

An award for virtuoso contributions to poignant in the field of public mathematics is Lilavathi award. After the Indian mathematician Bhskara II (also known as Bhaskara Achrya), who wrote the algebra and arithmetic mathematics monograph "Lilavati" in the 12th century, the term "lilavathi" was coined. In this volume, the author poses a series of (basic) arithmetic problems to Lilavati (perhaps his daughter) in the form of verses, and proposes elucidations. This work appears to have evolved into the primary resource for studying algebra and arithmetic in medieval India. The work has been rewritten in Persian and is widely read in West Asia.

Inaugurated by the Executive Organising Committee (EOC) of the International Conference of Mathematics (ICM) with the authorization of the IMU Executive Committee (EC), the Lilavati Prize was presented for the first time at the closing ceremony of the ICM in Hyderabad, India, in 2010. It is sponsored by Infosys and includes a cash reward of 10 lakh rupees as well as a certificate.

1.4 Details about Lilavati Award (LA) winners (2010 to 2022)

S.No	Name of the prize winner	Year	Country
1	Simon singh	2010	Indian settled in Britain
2	Adrián Paenza	2014	Argentina
3	Ali Nesin	2018	Turkey
4	Nikolai andreev	2022	-

Table 1

2. Notations and Terminology

2.1. Collaboration graph:- By a collaboration graph G we mean agraph whose vertex elements denote all research scientists (either presently alive or no more and we introduce an edge between two vertex elements u_1 and $u_2$of $V(G)$, if they are joint authors of a published paper or a book.

2.2. Erdös number: By an Erdos number of a research scientist u ina collaboration graph, we mean the number $d(Erdös, u)$. (By $d(x, y)$ we mean the smallest distance between x and y)

Note 2.1 : Paul Erdös himself by virtue of the above definition earns the Erdös number zero. All co-authors directly related with Erdös has Erdös number 1. Interestingly there are 511 such people with Erdös number 1. Unfortunately this number has become a fixed number as Erdös is no more. The co-authors of co-authors of Erdös earns the Erdös number two and so on.

2.3. Domination number: If every element selected from $V(G) - A$ is connected by a path of distance one to some element in A, then we say that set A, whose elements come from the vertex set of a graph $G = (V, E)$, is a dominant set. We use the symbol $\gamma(G)$ to denote the domination number of G and $(G) = min \mid A \mid$.

2.4. Induced Subgraph*:* A subgraph formed by the subset Dvertices of a graph G whose edges are all the connecting edges of the respective vertices, denoted by $\langle D \rangle$.

2.5. Split Domination number: If the subgraph $(V\ (G)\ -\ D)$ is disconnected, the dominating set $D \subseteq V(G)$is a split dominating set. Split domination number in G, is denoted by γ_s(G) and γ_s(G) = min|D|.

2.6. Strong Domination number: A set $D \subseteq V(G)$is a strong dominating set of G if for each vertex $x \in V(G) - D$ there is a vertex $y \in D$ with $xy \in E(G)$ and $deg(x, G) < deg(y, G)$.Strong domination number in G, is denoted by $\gamma_{st}(G)$ and $\gamma_{st}(G)$ = min|D|.

2.7. Strong split domination number: A dominating set If the induced subgraph $V(G) - D$ is completely disconnected with at least two vertices, then $D \subseteq V(G)$ is a strong split dominating set. Strong split domination number of G, is denoted by $\gamma_{ss}(G)$ and $\gamma_{ss}(G) = min|D|$.

2.8. Inverse Domination number: Assume that set D is the least domination set in G.The set D' is referred to as an inverse dominating set if, with regard to D, a set, let's say D' in $V(G)\ -\ D$, which is a dominating set in G, corresponds. The symbol for G's inverse domination number is $\gamma^{-1}(G)$. Inverse domination number of G, is denoted by $\gamma^{-1}(G)$ and $\gamma^{-1}(G) = min|D|$.

2.9. Strong non split domination number: A dominating set D of a graph $G\ =\ (V, E)$ is a strong non split dominating set if the induced sub graph $(V - D)$ is complete. Strong non Split domination number in G, is denoted by γ_{ns}(G) and γ_{ns}(G) = min|D|.

2.10. Inverse non split domination number: Let D be the smallest non-split dominating set in G. If a set D' in $V(G)\ -\ D$ corresponds to a non split dominating set in G, then the set D' is termed the inverse non split dominating set, and the inverse non split dominating number of G, is denoted by $\gamma_{ns}^{-1}(G)$ and $\gamma_{ns}^{-1}(G) = min|D|$.

2.11. Total Domination Set: By a total dominating set B of a graph Gwe mean a set with the ensuing properties: (a) B is a dominating set; (b) B is connected (By B we mean the subgraph induced by B). The property (b) can also be stated as follows: Every element of B must be connected with at least one element in B by a path length one. Total domination number of G is denoted by $\gamma_t(G)$ and $\gamma_t(G)\ =\ min|\ B|$.

2.12. Private neighborhood: Let $A\ \subseteq\ V(G)$&$v\ \in\ A$. Then byprivate neighbourhood of v with respect to A, denoted $Pr(v, A)$, we mean $Pr(v, A)\ =\ \{w\ \in\ V(G) :\ N\ (w)\ \cap\ A\ = \{v\}\}$.

Note 2.2: Easily we can see that

(i) if $w\ \in\ V(G) - A$ and w is adjacent to $v$in$A$ the $w\ \in\ Pr(v, A)$

(ii) if $w\ \in\ A$ and $w\ \neq\ v$ then w is not in $Pr(v, A)$;

(iii) if $w\ =\ v$ is not adjacent to a vertex of A then $w\ \in\ Pr(v, A)$.

2.13. Minimal dominating set: A dominating set A is called a minimal dominating set iff $\forall\ v\ \in\ A\ Pr(v, A)\ \neq\ \emptyset$.

2.14. Outer – connected dominating set: In a graph $G\ =\ (V, E)$, an outer-connected dominant set is a set $D \subseteq V$ in which each vertex that is not in D is adjacent to at least one vertex in D and the subgraph induced by $V \backslash D$ is connected. Outer connected domination number is denoted by $\gamma_{\tilde{c}}(G)$ and $\gamma_{\tilde{c}}(G) = min|\ D|$

2.15. Excellent graph: An excellent graph is one where every vertex of the graph G belongs to γ -set.

2.16. Domatic Number: The domatic number $d(G)$ of a graph G is the maximum number of elements in a partition of $V(G)$ into dominating sets.

2.17. Just Excellent: A excellent graph if for every v,there is precisely one $\gamma - set$ of G that contains v then G is said to be just exellent.

3.Materials and Methods

3.1 Construction of Collaboration graph [13]: By G we mean graph of collaboration graph for the prize winners from 2010 to 2022.

3.2 Vertex set of G: Firstly, notice the mammoth Erdös is just number three among all the winners and collaborators. This signifies that the partnership has progressed from Erdös at stage 0 through stage 3 and eventually to stage 4. By stage i, we refer to collaborators with Erdös number i. 14 vertices are produced for the vertex set of G by the procedure outlined below. Let $V(G) = \{w_1, w_2, w_3, \dots \dots \dots, w_{14}\}$. $w_i's$ are the following w_1 = Paul Erdos, w_2= Simon Singh, w_3 = Adrian paenza, w_4 = Ali nesin, w_5 = Nikolai Andreev, w_6 = peter komjath, w_7 =Gregory L. Cherlin, w_8 =Melvyn B.Nathanson, w_9 = Sergei Vladimirovich konyagin, w_{10} = Anand pillay, w_{11} = David William masser, w_{12} = Enrico Bombieri, w_{13} = D.R.Heath Brown, w_{14} = Angus J. Macntyre.

3.3 Edge set of G: Let us represent the edge set of G by $E(G)$. The step by step procedure to be outlined below reveals that E(G) = $\{f_1, f_2, \dots, f_{78}\}$. The f_i's are described as follows: $f_1 = (w_1, w_2)$; $f_2 = (w_1,w_3)$; $f_3 = (w_1,w_4)$; $f_4 = (w_1,w_5)$; $f_5 = (w_1,w_7)$; $f_6 = (w_1,w_9)$; $f_7 = (w_1,w_{10})$; $f_8 = (w_1, w_{11})$; $f_9 = (w_1,w_{12})$; $f_{10} = (w_1,w_{13})$; $f_{11} = (w_1,w_{14})$; $f_{12} = (w_2,w_3)$; $f_{13} = (w_2,w_4)$; $f_{14} = (w_2,w_5)$; $f_{15} = (w_2,w_6)$; $f_{16} = (w_2,w_7)$; $f_{17} = (w_2,w_8)$; $f_{18} = (w_2,w_9)$; $f_{19} = (w_2,w_{10})$; $f_{20} = (w_2,w_{11})$; $f_{21} = (w_2,w_{12})$; $f_{22} = (w_2,w_{13})$; $f_{23} = (w_2,w_{14})$; $f_{24} = (w_3,w_4)$; $f_{25} = (w_3,w_5)$; $f_{26} = (w_3,w_6)$; $f_{27} = (w_3,w_7)$; $f_{28} = (w_3,w_8)$; $f_{29} = (w_3,w_9)$; $f_{30} = (w_3,w_{10})$; $f_{31} = (w_3,w_{11})$;$f_{32}=(w_3,w_{12})$;$f_{33} = (w_3,w_{13})$; $f_{34} = (w_3,w_{14})$; $f_{35} = (w_4, w_5)$;$f_{36} = (w_4, w_6)$;$f_{37} = (w_4, w_8)$; $f_{38} = (w_4, w_9)$;$f_{39} = (w_4, w_{11})$;$f_{40} = (w_4, w_{12})$; $f_{41} = (w_4, w_{13})$;$f_{42} = (w_4, w_{14})$; $f_{43} = (w_5, w_6)$; $f_{44} = (w_5, w_7)$; $f_{45} = (w_5, w_8)$; $f_{46} = (w_5, w_{10})$; $f_{47} = (w_5, w_{11})$; $f_{48} = (w_5, w_{13})$; $f_{49} = (w_5, w_{14})$; $f_{50}= (w_6,w_8)$; $f_{51} = (w_6,w_9)$; $f_{52} = (w_6,w_{10})$; $f_{53} = (w_6,w_{11})$; $f_{54} = (w_6,w_{12})$; $f_{55} = (w_6,w_{13})$; $f_{56} = (w_6,w_{14})$; $f_{57} = (w_7,w_8)$; $f_{58} = (w_7,w_9)$; $f_{59} = (w_7,w_{10})$; $f_{60} = (w_7,w_{11})$; $f_{61} = (w_7,w_{12})$; $f_{62} = (w_7,w_{13})$; $f_{63} = (w_8,w_{10})$; $f_{64} = (w_8,w_{11})$; $f_{65} = (w_8,w_{12})$; $f_{66} = (w_8,w_{13})$; $f_{67} = (w_8,w_{14})$; $f_{68} = (w_9,w_{10})$; $f_{69} = (w_9,w_{11})$; $f_{70}=(w_9,w_{12})$;$f_{71}=(w_9,w_{14})$;$f_{72} = (w_{10}, w_{12})$; $f_{73} = (w_{10}, w_{13})$;$f_{74} = (w_{10}, w_{14})$; $f_{75} = (w_{11}, w_{13})$;$f_{76} = (w_{11}, w_{14})$; $f_{77} = (w_{12},w_{13})$; $f_{78} = (w_{12},w_{14})$;

3.4 Number of prize winners at each level: Note that at level 0 Erdös sits alone. At level 3 two mathematicians are there, they are w_4 and w_5

3.5 The LAPW's at various levels: Obviously there is none at level 0.Interestingly, none of the direct collaborators of Paul Erdös have won leelavati award and hence at level 1 it is 0.At level 3, we have two prize winners w_4 = Ali Nesin and w_5 = Nikolai Andreev**.**

3.6 Procedure to obtain Collaboration graph:

Step 1:Click on the link http://www.ams.org/mathscinet/collaborationDistance.html. It will

direct us to the following screen from MathsciNet.

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 7s (2024)

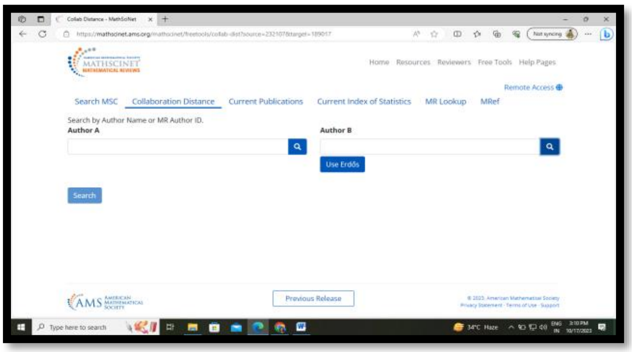

Figure 1: Collaboration distance description page

Step 2: Enter the name of the author and then enter the names of other authors or press the "Use Erdös" icon. For example, if one of the authors is Ali Nesin and the other is Paul Erdos, the outcome of our action is depicted in Figure 2.

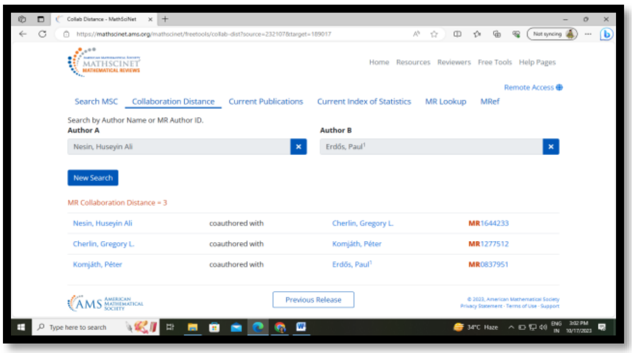

Figure 2: Respective author's collaboration details with Erdös number

Step 3: To acquaint with further details regarding the collaborative work of the above said authors in step b just act on the appropriate MR number blue link. It will take us to Figure.3

Figure 3: Data screen of required MathsciNet bibliography.

Step 4: Repeat steps 1 to 3 until all three prize winners collaboration details are acquired one after the other.

Step 5: Stop if steps 1 to 5 are well implemented.

3.7 Collaboration graph of Lilavati prize winners:

We create the collaborative graph using the program graphtea because the number of vertices and edges is constrained.

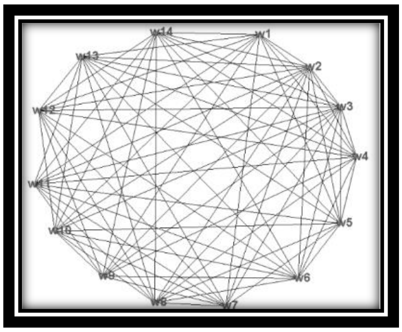

Figure 4: collaboration graph

3.8 Some Observations: As the count of LA winners is a single digit, the foregoing technique can be handled manually. But alternative strategies have to be worked out to construct the collaboration graphs of other popular prizes like the Nobel Prize; Fields medal etc where the number of prize winners is more than 50. If there is no co-author correlation between two arbitrary mathematicians x & y, our implementation of step b will return "No Path Found."

4. Results and Discussion

Theorem 4.1: $\gamma_{ss}(G) = 12$

Proof: The lilavati award winners and collaborators of prize winners are represented by the vertex set, which has 14 elements and is provided by $V(G) = \{w_1, w_2, \dots \dots, w_{14}\}$. Notice that w_2 & w_3 are the highest degree vertices in graph and $N(w_2) \cap N(w_3) \neq \emptyset$ gains entry as elements to the dominating set D. D is not a strong split since the subgraph that the remaining vertices trace is apical. As the subgraph traced by the remaining vertices is connected and D is not a strong split. To make D a strong split, let us look forward for other vertices with next highest degrees such vertices are $w_1$andw_{13} the subgraph traced by remaining vertices turns a regular graph with degree eight. Hence the dominating set with elements $\{w_1, w_2, w_3, w_{13}\}$ are again not a strong split. Again to make D a strong split dominating set the choice of vertices $\{w_4, w_6, w_9, w_{11}\}$ along with D leads to a regular graph with degree five. So, the cardinality of D must be altered continuously to get a disconnected graph with atleast two vertices. Finally this yielded to a dominating set becomes strong split with 12 elements given by $D = \{w_1, w_2, w_3, w_4, w_5, w_6, w_8, w_9, w_{10}, w_{11}, w_{12}, w_{13}\}$. Since, the cardinality of D forms strong split domination number. Therefore $\gamma_{ss}(G) = 12$. □

Observation 4.1: Observed from graph that $\gamma_s(G) = \gamma_{ss}(G)$

Theorem 4.2: $\gamma_{sns}(G) = 7$

Proof: From the collaboration graph the vertex set with 14 elements is given by $V(G) = \{w_1, w_2, \dots \dots, w_{14}\}$ which denotes the lilavati prize winners and collaborators of prize winners. To make vertex set a strong non split dominating set, first we begin our argument with the dominating set of vertices in a collaboration network $\{w_1, w_6\}$however, the subgraph formed by these vertices does not form a complete graph. To make the subgraph complete the cardinality of D must alter continuously. Finally this give rise to strong non split dominating set D with seven elements $\{w_1, w_6, w_7, w_9, w_{10}, w_{12}, w_{14}\}$. Since, $|D|$ forms a $\gamma_{sns}(G)$- set. Therefore $\gamma_{sns}(G) = 7$□

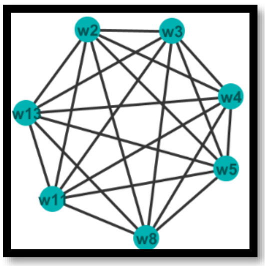

Figure 5: Complete graph

Theorem 4.3: $\gamma_{ns}^{-1}(G) = 1$

Proof: We begin with a non-split dominating set $D = \{w_2\}$, where each and every vertex in $V - D$ is adjacent to a vertex in D. If a set, say $D' = \{w_3\}$, corresponds to D, then $V - D$ forms a non split dominating set of G. The inverse non split domination number has a cardinality one. As a result, $\gamma_{ns}^{-1}(G) = 1$. □

Observation 4.2: For a collaboration graph $\gamma^{-1}(G) = \gamma_{ns}^{-1}(G)$

Theorem 4.3: $\gamma_s(G) = 1$

Proof: We begin our proof by stating that D is a minimal dominant set.Close examination of the vertex's surroundings w_2 reveals that N(w_2) $\cap V = \{w_3\}$. We conclude that D is a least dominating set. The cardinality should be least therefore either w_2 or w_3 be a dominating set. Let $D = \{w_2\} \subset V(G)$ that dominates the every vertex in $V - D$. If e is an edge with vertices x and y at both ends, the degree of $w_2$is larger than or equal to the degree of all other vertices in $V(G)$. i.e, $deg(x, G) \leq$ deg (y, G). Then we can say that w_2 strongly dominates. Since each vertex $V - D$ is strongly dominated by the vertex w_2 , $D = \{w_2\}$ becomes a strong dominating set of G. ∴ min$|\gamma_s(G)| = 1$ and so $\gamma_s(G) = 1$. □

Theorem 4.4: $\gamma'_s(G) = \gamma'_{cs}(G) = 6$

Proof: From the construction of the collaboration graph of the Lilavathi prize winners, we have $|V(G)| = 14$ and $|E(G)| = 78$ as the edge set E(G) = $\{f_1, f_2, ..., f_{78}\}$. To arrive at an edge dominating set for the graph, we start our assertion with an edge $f_1 = (w_1, w_2) \in E(G)$. If the considered edge f_1 covers all the edges of the graph then the set $D = \{f_1\}$ becomes an edge dominating set for G. But f_1 covers only 23 of the edges of . So the set D must be altered. Let us consider another edge f_{24}= (w_3,w_4). If the set D along with f_{24} covers all the edges of G then the set $D = \{f_1, f_{24}\}$ becomes an edge dominating set for G. But D covers only a partial part of edges of G. Continuously altering the set D by including the required number of edges, finally we arrive at a stage where the set D becomes an edge dominating set given by $D = \{f_1, f_{24}, f_{43}, f_{57}, f_{68}, f_{77}\}$. Therefore $\gamma'_s(G) = 6$. D also induces a apical subgraph. As a result, set D is also a connected edge dominating set. Therefore we can conclude that $\gamma'_s(G) = \gamma'_{cs}(G) = 6$. □

Theorem 4.5: The following conditions must apply for each vertex $v \in D$ in order for an outer connected dominating set D of G to be minimal:

a) $Pr[v, D] \neq \emptyset$

b) In the graph that D induced, v is a isolated vertex.

c) $N(v) \cap (V \setminus D) = \emptyset$

Proof: Firstly, prove sufficiency. If none of the three conditions are true under the assumption of the necessity part, then there is a vertex $v \in D$in order to$D' = D \setminus \{v\}$ is the dominating set of G.The $(V \setminus D')$induced graph is connected when $N(v) \cap (V \setminus D') \neq \emptyset$. This implies D' is an outer connected dominating set of G with inconsiderable number of elements than D, which is a paradox. Next, it is easy to prove the necessary condition under the assumption of the sufficiency condition. □

Theorem 4.6: For a graph,$\gamma_c^{\sim}(G) \leq n - w(G) + 1$where $w(G)$, clique number.

Proof: Assume vertex set A is such that $|A| = w(G)$ and the subgraph A is complete. Now, since $(V \setminus A) \cup \{u\}$is an outer connected dominating set of G, for any $u \in A$. Thus result follows.

Theorem 4.7: If $L(G) > \beta_0(G)$, then $\gamma_c^{\sim}(G) \geq \gamma(G)$ where L(G) is the vertex connectivity of G and $\beta_0(G)$is vertex independence number of G

Proof: Let dominating set be Din G. Since$\gamma(G) \leq \beta_0(G) \leq L(G)$................(1) it follows that $V \setminus D$ is connected.

We have $\gamma_c^{\sim}(G) \leq L(G)$...............(2).

Subtracting (2) from (1) $\gamma(G) - \gamma_c^{\sim}(G) \leq 0$.

Thus $\gamma_c^{\sim}(G) \geq \gamma(G)$. □

Theorem 4.8: For each graph$\mathcal{G}$, $\gamma_c^{\sim}(G) \leq |V(G)| - diam(G) + s + 1$ where s is a $\gamma_c^{\sim}$- set D of 's minimal number of vertices.

Proof: Let $diam(G) = t$. Then there are three possibilities. First, it may occur that $x, y \in V \backslash D$. Then note that in this case, $V \backslash D$ has atleast $t + 1$ vertices. Second it may be the case that $x \in D$ & $y \in V \backslash D$. If there is a vertex $x_1 \in V \backslash D$ such that x_1 is connected to x by way of the vertices in D. Then this implies that $d(x, y) \geq k - (t + 1)$ and hence $V \backslash D$ has atleast $t - s$ vertices. This is because in the opposite case, there is an adjacent vertex z to each $x_1 \in V \backslash D$ such that $d(x, z) = d(x, y) + d(y, x_1) + d(x_1, z) \geq t + 1$, which is a contradiction. In this incident, we have $V \backslash D = \{y\}$. Finally, it may be an instance of $x, y \in D$. Assume that there is $x_1, y_1 \in V \backslash D$ such that x is connected to x_1 and y is connected to y_1 by way of the vertices of D. Then $d(x_1, y_1)$ is atleast $t - (s + 2)$ and hence $V \backslash D$ has atleast $(t - s - 1)$ vertices. This is because, otherwise there is exactly one vertex $x_1 \in V \backslash D$ that is adjacent to both x and y and $v \backslash D$={x_1} and as a consequence it has $G \cong K_{1,3}$. So in all three instances $V \backslash D$ has at least $(t - s - 1)$ vertices.

Theorem 4.9: If G is just excellent, then $\delta(v) \geq \frac{n}{\gamma(G)} - 1$

Proof: Let $V = S_1 \cup S_2 \cup \dots \dots \dots .\cup S_m$ be the phyletic of V into $\gamma - set$ of G. Set up a vertex $u \in V$. Assume that $u \in S_j$ since each S_i is a $-set$, u is adjacent to not less than a vertex of S_i,$i \neq j$. Hence $\delta(v) \geq m - 1 \geq \frac{n}{\gamma(G)}$

Theorem 4.10:$\gamma(G) = 1$

Proof: We begin our assertion with a claim that a dominating set of G must have one element. Let us deem that it is not so. This implies that G has a dominating set A with atleast two elements. Notice that w_2 and w_3 has the highest degree vertices in that order automatically gains entry as the elements of D and degree of w_2 and w_3 are same. Moreover $N(w_2) \cap N(w_3) \neq \emptyset$ is the reason for having either w_2 or w_3 in D as compulsory elements. In order to explore the possibility of having any other indispensable elements in A. We analysed the carefully the degree distribution of other vertices of G. We found it wise to look at the least degree vertices of G. Such vertices include the following two w_5 and w_6. Out of these vertices w_5 and w_6 are taken by w_2 itself involving of any other vertices in D leads to a contradiction to the definition. Therefore $\gamma(G) = 1$ □

Theorem 4.11:$\gamma_t(G)$ = 2

Proof: Let $B \subseteq V(G)$ be any total dominating set. We assert that$B \geq 1$. Since$B$ is a dominating set by Theorem 3.2.7 it follows that $B \geq 1$. Further we establishedin Theorem 3.2.7 guided by the structure of Gthat any dominating set of$G$mustcompulsorily contain the elements $w_2 \in B$. Let $B = \{w_1, w_2\}$ be any dominating set of G. Now B must be adjacent to at least one element of the set, finally B become a total dominating set.

Concluding remarks and open problems

The collaboration graph of Lilavathi prize winners from 2010-2022 was obtained which is a huge network with 14 nodes and 78 links. As the number of vertices and edges is limited in number, a software graph tea is used to construct the collaboration graph. Now to improve the more connectedness among the winners in a connected network the concept of domination in graphs will be more helpful. So, a few prominent variants of domination in graphs like split domination, strong split domination, strong non split domination, inverse domination, inverse non split domination, edge, outer connected, total connected, connected edge domination number and just excellent were calculated to serve the above purpose. The calculation of the domination parameters of the collaboration graph helps to reach very member of the graph with minimum connected ness. So the idea of computing the split domination, strong split domination, strong non split domination, inverse domination, inverse non split domination, edge, outer connected, total connected, connected edge domination number and just excellent assumes significance.

Open problem

Obtain Erdös centred collaboration graph for Nobel Prize, Fields Medal, Steel's Prize and Abel Prize etc and compute its dominating and total dominating numbers.

References

[1] Ameenal Bibi K., Selvakumar R., "The inverse split and nonsplit domination in graphs", International Journal of Computer Applications, Volume 8 – No.7 October 2010,pp:21-29.

[2] Berge C. (1962), Theory of graphs and its Applications, Methuen, London.

[3] Cockayne E.J., and Hedetniemi S.T. Towards a theory of domination in graphs, Networks, (1977)

[4] Cockayne E.J, Hartnell B.L, Hedetniemi S.T, and Laskar R, "Efficient domination in graphs", Clemson University Dept.of Mathematical Sciences, Technical report 1998, 558.

[5] Gerald J.Chang, The domatic number problem, Discrete Mathematics, volume 125, Issue 1-3, Feb 1994, pp:115-122.

[6] Haynes T.W., Hedetniemi S.T. and Slater P.J., Fundamentals of domination in graph, New York, Marcel Dekker, Inc.,1998.
[7] Haynes T.W., Hedetniemi S.T. and Slater P.J., Domination in graph, Advanced Topics, New York, Marcel Dekker, Inc.,1998.
[8] Joanna Cyman, "the outer connected domination number of a graph", Australisian Journal of combinatorics, Volume 38, 2007, pp: 35-46.
[9] Kulli V.R.and Sigarkanti S.C., Inverse domination in graphs, Research Gate, N.A.Sci.Letters, 1991
[10] Kulli V.R. and Janakiram B., "The split domination number of a graph", Graph Theory notes of New York, New York Academy of Sciences, XXXII, 1997.
[11] Kulli V.R. and Janakiram, " The nonsplit domination number of a graph",Indian Journal of Pure & applied Mathemtics, 31 2000,pp:545-550
[12] Kulli V.R. and Janakiram B., " The strong nonsplit domination number of a graph", International journal of Management and Systems, Vol 19 No-2, May- august 2003, pp:145-156.
[13] Kulli V.R. and Janakiram B., " the strong split domination number of a graph, Acta ciencia Indica, Vol XXXII M, No.2, 715(2006).
[14] Kulli V.R., Theory of Domination in Graphs, Vishwa Inter. Publ. Gulbarga (2010), India.
[15] Ore O.,Theory of Graphs, Amer, Math,Soc,Colloq,Publ, 1962.
[16] Vaidya S.K.& Karkar S.H., " On Strong domination number of Graphs", Applications and Applied Mathematics:An international journal , ISSN: 1932-9466, vol 12 , issue I pp: 604-612, 2017.
[17] Yamuna M. And Sridharan N., Just Excellent graphs, "International Journal of Engineering Science, Advanced Computing and Bio-Technology" Vol. 1, No. 3, July –September 2010, pp. 129-136.
[18] Yegnanarayanan, V. & Lokeshwary, B., On Graph domination numbers, International Journal of pure and applied mathematics, (2015), 105(4), 751-761.
[19] Yegnanarayanan, V. & Renuka Lakshmi, A., On domination and total Domination number of a collaboration graph, International Journal of Advanced Intelligence Paradigms, 2017, 9(5/6), 541-557.