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Assessing Healthcare Service Quality in Andhra Pradesh Using Icosagonal Fuzzy Numbers: An Innovative Multi Criteria Decision Making Approach

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Abstract:

This study uses Icosagonal Fuzzy Numbers (IFNs) to assess the quality of healthcare services in Andhra Pradesh across four zones, we rank characteristics such as appointment scheduling, medical quality, hygiene, lab efficiency, care quality, safety, value for money, dependability, cost-effectiveness, facility quality, and recommendations. Data analysis suggests that hygiene is the most important factor in both zones. A comparison with other fuzzy number approaches (Pentagonal, Hexagonal, Heptagonal, and Hex-Decagonal) reveals that IFNs provide a more nuanced view of service quality. The findings provide insights for enhancing healthcare services in Andhra Pradesh.

Keywords: Fuzzy number, icosagonal fuzzy number, defuzzification, mean alpha-cut, hospital quality, different zones in Andhra Pradesh

1. Introduction:

Healthcare services are a critical component of modern society, ensuring the well-being and health of populations. These services encompass a wide range of activities, including preventive care, diagnosis, treatment, and rehabilitation. The effectiveness and efficiency of healthcare services can significantly influence the quality of life and overall health outcomes of individuals and communities.

In recent years, the application of advanced mathematical and computational techniques has become increasingly important in optimizing healthcare services. One such technique involves the use of fuzzy numbers, which allow for more flexible and realistic modeling of uncertainty and imprecision inherent in healthcare data. This article explores the innovative use of icosagonal fuzzy numbers in the context of healthcare services.

Healthcare service quality assessment is critical for improving patient outcomes and optimizing resource allocation. Traditional assessment methods often fall short due to the complexity and inherent uncertainties in healthcare data. Fuzzy set theory, particularly using icosagonal fuzzy numbers, offers a robust framework to address these challenges.

Fuzzy numbers, pioneered by L.A. Zadeh [1,17], offer a pragmatic approach to handling imprecise numerical values. Various types of fuzzy sets have been devised to address ambiguity in existing problems, permitting a gradual evaluation of element membership within a set on a scale from 0 to 1.

Interval arithmetic, initially proposed by Dwyer in 1951 and further developed through Zadeh's extension principle and Moore's contributions [19], extends standard arithmetic operations to encompass fuzzy numbers [18]. Triangular and trapezoidal fuzzy numbers have commonly been employed to represent imprecision in real-world scenarios. However, in this study, we advocate the utilization of Icosagonal fuzzy numbers to evaluate healthcare service quality across four zones in Andhra Pradesh.

Zadeh [1] pioneered a novel concept known as fuzzy set theory (FST). This foundational theory of uncertainty has been widely and successfully applied across various disciplines. The core concept of fuzzy sets and numbers was formulated by Chang and Zadeh [2]. Subsequently, mathematicians have explored numerous outcomes of this theory [3,4], leading to significant advancements and diverse applications of FST. As a result, the topic has garnered substantial interest and attention.

In our everyday life, we frequently encounter fuzziness in various decision-making scenarios. These situations often arise in contexts such as quality assessment, game theory, and other areas. Assessing the quality of healthcare services is of paramount importance, especially in diverse regions like Andhra Pradesh. With its varying demographics and healthcare infrastructure across different zones, ensuring high-quality healthcare delivery becomes crucial. Effective quality assessment mechanisms not only enhance patient outcomes but also contribute to the overall efficiency and reliability of healthcare systems. In Andhra Pradesh, where access to healthcare services may vary significantly between urban and rural areas, comprehensive quality assessment initiatives can help identify gaps, allocate resources effectively, and improve the overall standard of care. By addressing specific challenges and tailoring strategies to the unique needs of each zone, healthcare providers can strive towards achieving equitable, accessible, and high-quality healthcare services throughout the region.

The notion of fuzzy numbers has been described as a fuzzy subset of the real number line by Dubois and Prade [15]. A fuzzy number embodies a quantity whose values are nuanced rather than precise, diverging from the exactness of single-valued numbers [12, 5]. To navigate the nuances of real-life scenarios, numerous scholars have turned to triangular and trapezoidal fuzzy numbers [8, 10, 11, 14]. Moreover, the introduction of hexagonal, octagonal, and decagonal fuzzy numbers has aimed to dispel ambiguity [4, 6, 7, 9]. Many researchers have prioritized addressing uncertain linguistic terms within group decision-making processes [3, 11, 13]. In decision-making scenarios, experts may resort to uncertain linguistic terms to articulate opinions when faced with uncertainty and information gaps. Such terms are frequently employed as inputs in decision analysis endeavors. Traditionally, linguistic values have been represented using fuzzy numbers like triangular and trapezoidal shapes. However, confining membership functions to triangular or trapezoidal forms becomes intricate when vagueness extends across twenty distinct points. Thus, this paper embarks on exploring a novel concept: the Icosagonal Fuzzy Number (IFN), tailored for uncertain linguistic environments.

This paper's work is organised as follows: Section 2 provides a quick overview of the fundamental definitions of fuzzy numbers. Section 3 presents symmetric and asymmetric representations of Icosagonal fuzzy numbers with corresponding α-cuts. Section 4 discusses defuzzification algorithms for linear and Non-linear Icosagonal fuzzy numbers with symmetry. Also, this section proposes a ranking method based on the mean of $\alpha - cut$ method. Section 5 includes comparison of proposed defuzzification approaches for different zones of Andhra Pradesh. The conclusion of this paper is proceeded in Section 6.

2. Mathematical Preliminaries:

This section covers essential definitions and notations that will be utilized throughout the paper.

Definition 2.1: Let $\breve{X}$ be the universal set, then the fuzzy set denoted by $\tilde{A}$ is the family of ordered pairs $(\breve{x}, \mu_{\hat{A}}(\breve{x}))$, where the first element $\breve{x}$ is the member of universal set $\breve{X}$ and the second element represents the membership value corresponding to the element $\breve{x} \in \breve{X}$ in [0, 1] i.e. $\tilde{A}= \{((\breve{x}, \mu_{\hat{A}}(\breve{x}))| \breve{x} \in \breve{X}\}$ and the mapping $\mu_{\tilde{A}} : \breve{X} \longrightarrow [0, 1]$ is the membership function of $\tilde{A}$.

Definition 2.2: A fuzzy set $\tilde{A}$ in $\widetilde{R}$ is called a fuzzy number if it satisfies the following axioms:

(1) There exist at least one $\breve{x}_0 \in \widetilde{R}$ with $\mu_{\tilde{A}}(\breve{x}_0) = 1$.

(2) Membership function $\mu_{\tilde{A}}(\breve{x})$ is piecewise continuous.

(3) $\tilde{A}$ must be a convex fuzzy set.

Definition 2.3: The α - cut of a fuzzy set $\tilde{A}$ defined on the universal set $\breve{X}$ is a crisp set denoted by $\tilde{A}_\alpha$, which contains all those elements of $\breve{X}$, whose membership value is greater than or equal to $\alpha \in [0, 1]$ in $\tilde{A}$ i.e. $\tilde{A}_\alpha = \{\breve{x} \in \breve{X} | \mu_{\tilde{A}}(\breve{x}) \geq \alpha\}$ for $\alpha \in [0, 1]$.

3. Theory of Icosagonal Fuzzy Number and Its Different Representations:

In this section, we study different types of icosagonal fuzzy numbers.

Definition 3.1: A linear icosagonal fuzzy number with symmetry $\tilde{A}_{LS}$ given by Figure 1 is represented by

$$\hat{A}_{LS} = \{(\dot{s}_1^*, \dot{s}_2^*, \dot{s}_3^*, \dot{s}_4^*, \dot{s}_5^*, \dot{s}_6^*, \dot{s}_7^*, \dot{s}_8^*, \dot{s}_9^*, \dot{s}_{10}^*, \dot{s}_{11}^*, \dot{s}_{12}^*, \dot{s}_{13}^*, \dot{s}_{14}^*, \dot{s}_{15}^*, \dot{s}_{16}^*, \dot{s}_{17}^*, \dot{s}_{18}^*, \dot{s}_{19}^*, \dot{s}_{20}^*); (\ddot{\overline{\mathrm{p}}}_1', \ddot{\overline{\mathrm{p}}}_2', \ddot{\overline{\mathrm{p}}}_3', \ddot{\overline{\mathrm{p}}}_4') \}$$

and its membership function is defined as follows:

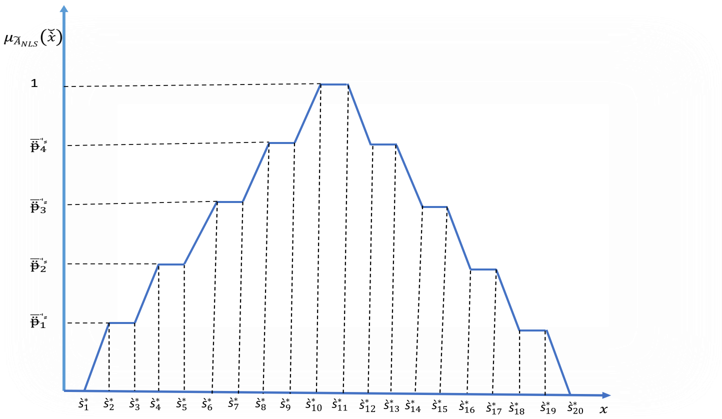

Figure 1: Linear Icosagonal fuzzy number with symmetry

$$\mu_{\tilde{A}_{LS}}(\breve{x}) = \begin{cases} 0; & \breve{x} \leq \grave{s}_1^* \\ \overline{\ddot{p}}_1 \left(\dfrac{\breve{x} - \grave{s}_1^*}{\grave{s}_2^* - \grave{s}_1^*}\right); & \grave{s}_1^* \leq \breve{x} \leq \grave{s}_2^* \\ \overline{\ddot{p}}_1; & \grave{s}_2^* \leq \breve{x} \leq \grave{s}_3^* \\ \overline{\ddot{p}}_1 + (\overline{\ddot{p}}_2 - \overline{\ddot{p}}_1)\left(\dfrac{\breve{x} - \grave{s}_3^*}{\grave{s}_4^* - \grave{s}_3^*}\right); & \grave{s}_3^* \leq \breve{x} \leq \grave{s}_4^* \\ \overline{\ddot{p}}_2; & \grave{s}_4^* \leq \breve{x} \leq \grave{s}_5^* \\ \overline{\ddot{p}}_2 + (\overline{\ddot{p}}_3 - \overline{\ddot{p}}_2)\left(\dfrac{\breve{x} - \grave{s}_5^*}{\grave{s}_6^* - \grave{s}_5^*}\right); & \grave{s}_5^* \leq \breve{x} \leq \grave{s}_6^* \\ \overline{\ddot{p}}_3; & \grave{s}_6^* \leq \breve{x} \leq \grave{s}_7^* \\ \overline{\ddot{p}}_3 + (\overline{\ddot{p}}_4 - \overline{\ddot{p}}_3)\left(\dfrac{\breve{x} - \grave{s}_7^*}{\grave{s}_8^* - \grave{s}_7^*}\right); & \grave{s}_7^* \leq \breve{x} \leq \grave{s}_8^* \\ \overline{\ddot{p}}_4; & \grave{s}_8^* \leq \breve{x} \leq \grave{s}_9^* \\ \overline{\ddot{p}}_4 + (1 - \overline{\ddot{p}}_4)\left(\dfrac{\breve{x} - \grave{s}_9^*}{\grave{s}_{10}^* - \grave{s}_9^*}\right); & \grave{s}_9^* \leq \breve{x} \leq \grave{s}_{10}^* \\ 1; & \grave{s}_{10}^* \leq \breve{x} \leq \grave{s}_{11}^* \\ \overline{\ddot{p}}_4 + (1 - \overline{\ddot{p}}_4)\left(\dfrac{\grave{s}_{12}^* - \breve{x}}{\grave{s}_{12}^* - \grave{s}_{11}^*}\right); & \grave{s}_{11}^* \leq \breve{x} \leq \grave{s}_{12}^* \\ \overline{\ddot{p}}_4; & \grave{s}_{12}^* \leq \breve{x} \leq \grave{s}_{13}^* \\ \overline{\ddot{p}}_3 + (\overline{\ddot{p}}_4 - \overline{\ddot{p}}_3)\left(\dfrac{\grave{s}_{14}^* - \breve{x}}{\grave{s}_{14}^* - \grave{s}_{13}^*}\right); & \grave{s}_{13}^* \leq \breve{x} \leq \grave{s}_{14}^* \\ \overline{\ddot{p}}_3; & \grave{s}_{14}^* \leq \breve{x} \leq \grave{s}_{15}^* \\ \overline{\ddot{p}}_2 + (\overline{\ddot{p}}_3 - \overline{\ddot{p}}_2)\left(\dfrac{\grave{s}_{16}^* - \breve{x}}{\grave{s}_{16}^* - \grave{s}_{15}^*}\right); & \grave{s}_{15}^* \leq \breve{x} \leq \grave{s}_{16}^* \\ \overline{\ddot{p}}_2; & \grave{s}_{16}^* \leq \breve{x} \leq \grave{s}_{17}^* \\ \overline{\ddot{p}}_1 + (\overline{\ddot{p}}_2 - \overline{\ddot{p}}_1)\left(\dfrac{\grave{s}_{18}^* - \breve{x}}{\grave{s}_{18}^* - \grave{s}_{17}^*}\right); & \grave{s}_{17}^* \leq \breve{x} \leq \grave{s}_{18}^* \\ \overline{\ddot{p}}_1; & \grave{s}_{18}^* \leq \breve{x} \leq \grave{s}_{19}^* \\ \overline{\ddot{p}}_1 \left(\dfrac{\grave{s}_{20}^* - \breve{x}}{\grave{s}_{20}^* - \grave{s}_{19}^*}\right); & \grave{s}_{19}^* \leq \breve{x} \leq \grave{s}_{20}^* \\ 0; & \breve{x} \geq \grave{s}_{20}^* \end{cases}$$

where $0 < \overline{\ddot{p}}_1 < \overline{\ddot{p}}_2 < \overline{\ddot{p}}_3 < \overline{\ddot{p}}_4 < 1$.

Definition 3.2: The α - cut of linear icosagonal fuzzy number with symmetry is the collection of all $\breve{x} \in \breve{X}$, whose membership function $\mu_{\tilde{A}_{LS}}(\breve{x})$ is greater than or equal to α i.e., $\left(\tilde{A}_{LS}\right)_\alpha = \left\{\breve{x} \in \breve{X} \mid \mu_{\tilde{A}_{LS}}(\breve{x}) \geq \alpha\right\}$ for $\alpha \in [0, 1]$ as

$$\left(\tilde{A}_{LS}\right)_\alpha = \left\{\begin{array}{ll}
\tilde{A}_{1L}(\alpha) = \dot{s}_1^* + \left(\dfrac{\alpha}{\bar{\ddot{p}}_1}\right)(\dot{s}_2^* - \dot{s}_1^*); & \alpha \in [0, \bar{\ddot{p}}_1] \\
\tilde{A}_{2L}(\alpha) = \dot{s}_3^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_1}{\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1}\right)(\dot{s}_4^* - \dot{s}_3^*); & \alpha \in [\bar{\ddot{p}}_1, \bar{\ddot{p}}_2] \\
\tilde{A}_{3L}(\alpha) = \dot{s}_5^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_2}{\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2}\right)(\dot{s}_6^* - \dot{s}_5^*); & \alpha \in [\bar{\ddot{p}}_2, \bar{\ddot{p}}_3] \\
\tilde{A}_{4L}(\alpha) = \dot{s}_7^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_3}{\bar{\ddot{p}}_4 - \bar{\ddot{p}}_3}\right)(\dot{s}_8^* - \dot{s}_7^*); & \alpha \in [\bar{\ddot{p}}_3, \bar{\ddot{p}}_4] \\
\tilde{A}_{5L}(\alpha) = \dot{s}_9^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)(\dot{s}_{10}^* - \dot{s}_9^*); & \alpha \in [\bar{\ddot{p}}_4, 1] \\
\tilde{A}_{5R}(\alpha) = \dot{s}_{12}^* - \left(\dfrac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)(\dot{s}_{12}^* - \dot{s}_{11}^*); & \alpha \in [\bar{\ddot{p}}_4, 1] \\
\tilde{A}_{4R}(\alpha) = \dot{s}_{14}^* - \left(\dfrac{\alpha - \bar{\ddot{p}}_3}{\bar{\ddot{p}}_4 - \bar{\ddot{p}}_3}\right)(\dot{s}_{14}^* - \dot{s}_{13}^*); & \alpha \in [\bar{\ddot{p}}_3, \bar{\ddot{p}}_4] \\
\tilde{A}_{3R}(\alpha) = \dot{s}_{16}^* - \left(\dfrac{\alpha - \bar{\ddot{p}}_2}{\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2}\right)(\dot{s}_{16}^* - \dot{s}_{15}^*); & \alpha \in [\bar{\ddot{p}}_2, \bar{\ddot{p}}_3] \\
\tilde{A}_{2R}(\alpha) = \dot{s}_{18}^* - \left(\dfrac{\alpha - \bar{\ddot{p}}_1}{\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1}\right)(\dot{s}_{18}^* - \dot{s}_{17}^*); & \alpha \in [\bar{\ddot{p}}_1, \bar{\ddot{p}}_2] \\
\tilde{A}_{1R}(\alpha) = \dot{s}_{20}^* - \left(\dfrac{\alpha}{\bar{\ddot{p}}_1}\right)(\dot{s}_{20}^* - \dot{s}_{19}^*); & \alpha \in [0, \bar{\ddot{p}}_1]
\end{array}\right\}$$

Where $\tilde{A}_{1L}(\alpha), \tilde{A}_{2L}(\alpha), \tilde{A}_{3L}(\alpha), \tilde{A}_{4L}(\alpha), \tilde{A}_{5L}(\alpha)$ *and* $\tilde{A}_{1R}(\alpha), \tilde{A}_{2R}(\alpha), \tilde{A}_{3R}(\alpha), \tilde{A}_{4R}(\alpha), \tilde{A}_{5R}(\alpha)$ are the increasing and decreasing functions of α correspondingly in the respective intervals.

Definition 3.3: A linear icosagonal fuzzy number with asymmetry $\tilde{A}_{LA}$ given by Figure 2 is represented by

$$\tilde{A}_{LA} = \{(\dot{s}_1^*, \dot{s}_2^*, \dot{s}_3^*, \dot{s}_4^*, \dot{s}_5^*, \dot{s}_6^*, \dot{s}_7^*, \dot{s}_8^*, \dot{s}_9^*, \dot{s}_{10}^*, \dot{s}_{11}^*, \dot{s}_{12}^*, \dot{s}_{13}^*, \dot{s}_{14}^*, \dot{s}_{15}^*, \dot{s}_{16}^*, \dot{s}_{17}^*, \dot{s}_{18}^*, \dot{s}_{19}^*, \dot{s}_{20}^*); (\bar{\ddot{p}}_1, \bar{\ddot{p}}_2, \bar{\ddot{p}}_3, \bar{\ddot{p}}_4, \bar{\ddot{l}}_1, \bar{\ddot{l}}_2, \bar{\ddot{l}}_3, \bar{\ddot{l}}_4)\}$$

and its membership function is defined as follows:

$$\mu_{\tilde{A}_{LA}}(\check{x}) = \begin{cases} 0; & \check{x} \le \grave{s}_1^* \\ \overline{\ddot{p}}_1 \left(\dfrac{\check{x} - \grave{s}_1^*}{\grave{s}_2^* - \grave{s}_1^*}\right); & \grave{s}_1^* \le \check{x} \le \grave{s}_2^* \\ \overline{\ddot{p}}_1; & \grave{s}_2^* \le \check{x} \le \grave{s}_3^* \\ \overline{\ddot{p}}_1 + (\overline{\ddot{p}}_2 - \overline{\ddot{p}}_1)\left(\dfrac{\check{x} - \grave{s}_3^*}{\grave{s}_4^* - \grave{s}_3^*}\right); & \grave{s}_3^* \le \check{x} \le \grave{s}_4^* \\ \overline{\ddot{p}}_2; & \grave{s}_4^* \le \check{x} \le \grave{s}_5^* \\ \overline{\ddot{p}}_2 + (\overline{\ddot{p}}_3 - \overline{\ddot{p}}_2)\left(\dfrac{\check{x} - \grave{s}_5^*}{\grave{s}_6^* - \grave{s}_5^*}\right); & \grave{s}_5^* \le \check{x} \le \grave{s}_6^* \\ \overline{\ddot{p}}_3; & \grave{s}_6^* \le \check{x} \le \grave{s}_7^* \\ \overline{\ddot{p}}_3 + (\overline{\ddot{p}}_4 - \overline{\ddot{p}}_3)\left(\dfrac{\check{x} - \grave{s}_7^*}{\grave{s}_8^* - \grave{s}_7^*}\right); & \grave{s}_7^* \le \check{x} \le \grave{s}_8^* \\ \overline{\ddot{p}}_4; & \grave{s}_8^* \le \check{x} \le \grave{s}_9^* \\ \overline{\ddot{p}}_4 + (1 - \overline{\ddot{p}}_4)\left(\dfrac{\check{x} - \grave{s}_9^*}{\grave{s}_{10}^* - \grave{s}_9^*}\right); & \grave{s}_9^* \le \check{x} \le \grave{s}_{10}^* \\ 1; & \grave{s}_{10}^* \le \check{x} \le \grave{s}_{11}^* \\ \overline{\ddot{p}}_4 + (1 - \overline{\ddot{p}}_4)\left(\dfrac{\grave{s}_{12}^* - \check{x}}{\grave{s}_{12}^* - \grave{s}_{11}^*}\right); & \grave{s}_{11}^* \le \check{x} \le \grave{s}_{12}^* \\ \overline{\ddot{p}}_4; & \grave{s}_{12}^* \le \check{x} \le \grave{s}_{13}^* \\ \overline{\ddot{p}}_3 + (\overline{\ddot{p}}_4 - \overline{\ddot{p}}_3)\left(\dfrac{\grave{s}_{14}^* - \check{x}}{\grave{s}_{14}^* - \grave{s}_{13}^*}\right); & \grave{s}_{13}^* \le \check{x} \le \grave{s}_{14}^* \\ \overline{\ddot{p}}_3; & \grave{s}_{14}^* \le \check{x} \le \grave{s}_{15}^* \\ \overline{\ddot{p}}_2 + (\overline{\ddot{p}}_3 - \overline{\ddot{p}}_2)\left(\dfrac{\grave{s}_{16}^* - \check{x}}{\grave{s}_{16}^* - \grave{s}_{15}^*}\right); & \grave{s}_{15}^* \le \check{x} \le \grave{s}_{16}^* \\ \overline{\ddot{p}}_2; & \grave{s}_{16}^* \le \check{x} \le \grave{s}_{17}^* \\ \overline{\ddot{p}}_1 + (\overline{\ddot{p}}_2 - \overline{\ddot{p}}_1)\left(\dfrac{\grave{s}_{18}^* - \check{x}}{\grave{s}_{18}^* - \grave{s}_{17}^*}\right); & \grave{s}_{17}^* \le \check{x} \le \grave{s}_{18}^* \\ \overline{\ddot{p}}_1; & \grave{s}_{18}^* \le \check{x} \le \grave{s}_{19}^* \\ \overline{\ddot{p}}_1 \left(\dfrac{\grave{s}_{20}^* - \check{x}}{\grave{s}_{20}^* - \grave{s}_{19}^*}\right); & \grave{s}_{19}^* \le \check{x} \le \grave{s}_{20}^* \\ 0; & \check{x} \ge \grave{s}_{20}^* \end{cases}$$

where $0 < \overline{\ddot{p}}_1 < \overline{\ddot{l}}_1 < \overline{\ddot{p}}_2 < \overline{\ddot{l}}_2 < \overline{\ddot{p}}_3 < \overline{\ddot{l}}_3 < \overline{\ddot{p}}_4 < \overline{\ddot{l}}_4 < 1$.

Definition 3.4: The α-cut of linear icosagonal fuzzy number with asymmetry is the collection of all $\check{x} \in \check{X}$, whose membership function $\mu_{\tilde{A}_{LA}}(\check{x})$ is greater than or equal to α i.e.

$$\left(\tilde{A}_{LA}\right)_\alpha = \left\{\check{x} \in \check{X} \mid \mu_{\tilde{A}_{LA}}(\check{x}) \ge \alpha\right\} \text{ for } \alpha \in [0,1] \text{ as}$$

$$\left(\tilde{A}_{LA}\right)_\alpha = \begin{cases} \tilde{A}_{1L}(\alpha) = \grave{s}_1^* + \left(\dfrac{\alpha}{\bar{\ddot{p}}_1^{*}}\right)(\grave{s}_2^* - \grave{s}_1^*); & \alpha \in [0, \bar{\ddot{p}}_1^{*}] \\ \tilde{A}_{2L}(\alpha) = \grave{s}_3^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_1^{*}}{\bar{\ddot{p}}_2^{*} - \bar{\ddot{p}}_1^{*}}\right)(\grave{s}_4^* - \grave{s}_3^*); & \alpha \in [\bar{\ddot{p}}_1^{*}, \bar{\ddot{p}}_2^{*}] \\ \tilde{A}_{3L}(\alpha) = \grave{s}_5^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_2^{*}}{\bar{\ddot{p}}_3^{*} - \bar{\ddot{p}}_2^{*}}\right)(\grave{s}_6^* - \grave{s}_5^*); & \alpha \in [\bar{\ddot{p}}_2^{*}, \bar{\ddot{p}}_3^{*}] \\ \tilde{A}_{4L}(\alpha) = \grave{s}_7^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_3^{*}}{\bar{\ddot{p}}_4^{*} - \bar{\ddot{p}}_3^{*}}\right)(\grave{s}_8^* - \grave{s}_7^*); & \alpha \in [\bar{\ddot{p}}_3^{*}, \bar{\ddot{p}}_4^{*}] \\ \tilde{A}_{5L}(\alpha) = \grave{s}_9^* + \left(\dfrac{\alpha - \bar{\ddot{p}}_4^{*}}{1 - \bar{\ddot{p}}_4^{*}}\right)(\grave{s}_{10}^* - \grave{s}_9^*); & \alpha \in [\bar{\ddot{p}}_4^{*}, 1] \\ \tilde{A}_{5R}(\alpha) = \grave{s}_{12}^* - \left(\dfrac{\alpha - \bar{\ddot{l}}_4^{*}}{1 - \bar{\ddot{l}}_4^{*}}\right)(\grave{s}_{12}^* - \grave{s}_{11}^*); & \alpha \in [\bar{\ddot{l}}_4^{*}, 1] \\ \tilde{A}_{4R}(\alpha) = \grave{s}_{14}^* - \left(\dfrac{\alpha - \bar{\ddot{l}}_3^{*}}{\bar{\ddot{l}}_4^{*} - \bar{\ddot{l}}_3^{*}}\right)(\grave{s}_{14}^* - \grave{s}_{13}^*); & \alpha \in [\bar{\ddot{l}}_3^{*}, \bar{\ddot{l}}_4^{*}] \\ \tilde{A}_{3R}(\alpha) = \grave{s}_{16}^* - \left(\dfrac{\alpha - \bar{\ddot{l}}_2^{*}}{\bar{\ddot{l}}_3^{*} - \bar{\ddot{l}}_2^{*}}\right)(\grave{s}_{16}^* - \grave{s}_{15}^*); & \alpha \in [\bar{\ddot{l}}_2^{*}, \bar{\ddot{l}}_3^{*}] \\ \tilde{A}_{2R}(\alpha) = \grave{s}_{18}^* - \left(\dfrac{\alpha - \bar{\ddot{l}}_1^{*}}{\bar{\ddot{l}}_2^{*} - \bar{\ddot{l}}_1^{*}}\right)(\grave{s}_{18}^* - \grave{s}_{17}^*); & \alpha \in [\bar{\ddot{l}}_1^{*}, \bar{\ddot{l}}_2^{*}] \\ \tilde{A}_{1R}(\alpha) = \grave{s}_{20}^* - \left(\dfrac{\alpha}{\bar{\ddot{l}}_1^{*}}\right)(\grave{s}_{18}^* - \grave{s}_{17}^*); & \alpha \in [0, \bar{\ddot{l}}_1^{*}] \end{cases}$$

where $\tilde{A}_{1L}(\alpha), \tilde{A}_{2L}(\alpha), \tilde{A}_{3L}(\alpha), \tilde{A}_{4L}(\alpha), \tilde{A}_{5L}(\alpha)$ *and* $\tilde{A}_{1R}(\alpha), \tilde{A}_{2R}(\alpha), \tilde{A}_{3R}(\alpha), \tilde{A}_{4R}(\alpha), \tilde{A}_{5R}(\alpha)$ are the increasing and decreasing functions of α correspondingly in the respective intervals.

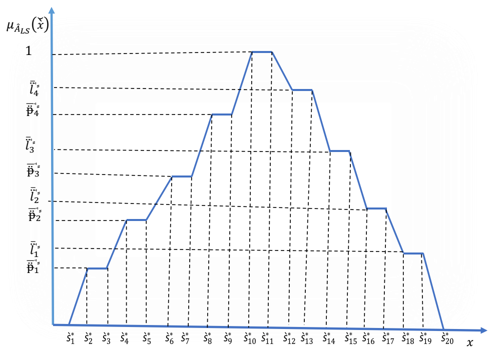

Figure 2: Linear Icosagonal fuzzy number with asymmetry

Definition 3.5: A non-linear icosagonal fuzzy number with symmetry $\tilde{A}_{NS}$ given by Figure 3 is represented by

$\tilde{A}_{NLS} = \{(\grave{s}_1^*, \grave{s}_2^*, \grave{s}_3^*, \grave{s}_4^*, \grave{s}_5^*, \grave{s}_6^*, \grave{s}_7^*, \grave{s}_8^*, \grave{s}_9^*, \grave{s}_{10}^*, \grave{s}_{11}^*, \grave{s}_{12}^*, \grave{s}_{13}^*, \grave{s}_{14}^*, \grave{s}_{15}^*, \grave{s}_{16}^*, \grave{s}_{17}^*, \grave{s}_{18}^*, \grave{s}_{19}^*, \grave{s}_{20}^*); (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5 ; \tilde{v}_1, \tilde{v}_2, , \tilde{v}_3, \tilde{v}_4, \tilde{v}_5; \bar{\ddot{p}}_1^{*}, \bar{\ddot{p}}_2^{*}, \bar{\ddot{p}}_3^{*}, \bar{\ddot{p}}_4^{*}) \}$

and its membership function is defined as follows:

$$\mu_{\tilde{A}_{NLS}}(\breve{x}) = \begin{cases} 0; & \breve{x} \le \grave{s}_1^* \\ \bar{\ddot{p}}_1 \left(\dfrac{\breve{x} - \grave{s}_1^*}{\grave{s}_2^* - \grave{s}_1^*} \right)^{\tilde{u}_1}; & \grave{s}_1^* \le \breve{x} \le \grave{s}_2^* \\ \bar{\ddot{p}}_1; & \grave{s}_2^* \le \breve{x} \le \grave{s}_3^* \\ \bar{\ddot{p}}_1 + (\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1) \left(\dfrac{\breve{x} - \grave{s}_3^*}{\grave{s}_4^* - \grave{s}_3^*} \right)^{\tilde{u}_2}; & \grave{s}_3^* \le \breve{x} \le \grave{s}_4^* \\ \bar{\ddot{p}}_2; & \grave{s}_4^* \le \breve{x} \le \grave{s}_5^* \\ \bar{\ddot{p}}_2 + (\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2) \left(\dfrac{\breve{x} - \grave{s}_5^*}{\grave{s}_6^* - \grave{s}_5^*} \right)^{\tilde{u}_3}; & \grave{s}_5^* \le \breve{x} \le \grave{s}_6^* \\ \bar{\ddot{p}}_3; & \grave{s}_6^* \le \breve{x} \le \grave{s}_7^* \\ \bar{\ddot{p}}_3 + (\bar{\ddot{p}}_4 - \bar{\ddot{p}}_3) \left(\dfrac{\breve{x} - \grave{s}_7^*}{\grave{s}_8^* - \grave{s}_7^*} \right)^{\tilde{u}_4}; & \grave{s}_7^* \le \breve{x} \le \grave{s}_8^* \\ \bar{\ddot{p}}_4; & \grave{s}_8^* \le \breve{x} \le \grave{s}_9^* \\ \bar{\ddot{p}}_4 + (1 - \bar{\ddot{p}}_4) \left(\dfrac{\breve{x} - \grave{s}_9^*}{\grave{s}_{10}^* - \grave{s}_9^*} \right)^{\tilde{u}_5}; & \grave{s}_9^* \le \breve{x} \le \grave{s}_{10}^* \\ 1; & \grave{s}_{10}^* \le \breve{x} \le \grave{s}_{11}^* \\ \bar{\ddot{p}}_4 + (1 - \bar{\ddot{p}}_4) \left(\dfrac{\grave{s}_{12}^* - \breve{x}}{\grave{s}_{12}^* - \grave{s}_{11}^*} \right)^{\tilde{v}_5}; & \grave{s}_{11}^* \le \breve{x} \le \grave{s}_{12}^* \\ \bar{\ddot{p}}_4; & \grave{s}_{12}^* \le \breve{x} \le \grave{s}_{13}^* \\ \bar{\ddot{p}}_3 + (\tilde{p}_4 - \bar{\ddot{p}}_3) \left(\dfrac{\grave{s}_{14}^* - \breve{x}}{\grave{s}_{14}^* - \grave{s}_{13}^*} \right)^{\tilde{v}_4}; & \grave{s}_{13}^* \le \breve{x} \le \grave{s}_{14}^* \\ \bar{\ddot{p}}_3; & \grave{s}_{14}^* \le \breve{x} \le \grave{s}_{15}^* \\ \bar{\ddot{p}}_2 + (\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2) \left(\dfrac{\grave{s}_{16}^* - \breve{x}}{\grave{s}_{16}^* - \grave{s}_{15}^*} \right)^{\tilde{v}_3}; & \grave{s}_{15}^* \le \breve{x} \le \grave{s}_{16}^* \\ \bar{\ddot{p}}_2; & \grave{s}_{16}^* \le \breve{x} \le \grave{s}_{17}^* \\ \bar{\ddot{p}}_1 + (\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1) \left(\dfrac{\grave{s}_{18}^* - \breve{x}}{\grave{s}_{18}^* - \grave{s}_{17}^*} \right)^{\tilde{v}_2}; & \grave{s}_{17}^* \le \breve{x} \le \grave{s}_{18}^* \\ \bar{\ddot{p}}_1; & \grave{s}_{18}^* \le \breve{x} \le \grave{s}_{19}^* \\ \bar{\ddot{p}}_1 \left(\dfrac{\grave{s}_{20}^* - \breve{x}}{\grave{s}_{20}^* - \grave{s}_{19}^*} \right)^{\tilde{v}_1}; & \grave{s}_{19}^* \le \breve{x} \le \grave{s}_{20}^* \\ 0; & \breve{x} \ge \grave{s}_{20}^* \end{cases}$$

where $0 < \bar{\ddot{p}}_1 < \bar{\ddot{p}}_2 < \bar{\ddot{p}}_3 < \bar{\ddot{p}}_4 < 1$.

Definition 3.6: The $\alpha -$ cut of non-linear icosagonal fuzzy number with symmetry is the collection of all $\breve{x} \in \breve{X}$, whose membership function $\mu_{\tilde{A}_{NLS}}(\breve{x})$ is greater than or equal to α i.e.

$$\left(\tilde{A}_{NLS}\right)_\alpha = \left\{\breve{x} \in \breve{X} \mid \mu_{\tilde{A}_{NLS}}(\breve{x}) \ge \alpha\right\} \text{ for } \alpha \in [0, 1] \text{ as}$$

$$\left(\tilde{A}_{NLS}\right)_{\alpha}=\begin{cases}\tilde{A}_{1L}(\alpha)=\dot{s}_1^*+\left(\frac{\alpha}{\bar{\ddot{p}}_1}\right)^{\frac{1}{\ddot{u}_1}}(\dot{s}_2^*-\dot{s}_1^*); & \alpha\in\left[0,\bar{\ddot{p}}_1\right]\\ \tilde{A}_{2L}(\alpha)=\dot{s}_3^*+\left(\frac{\alpha-\bar{\ddot{p}}_1}{\bar{\ddot{p}}_2-\bar{\ddot{p}}_1}\right)^{\frac{1}{\ddot{u}_2}}(\dot{s}_4^*-\dot{s}_3^*); & \alpha\in\left[\bar{\ddot{p}}_1,\bar{\ddot{p}}_2\right]\\ \tilde{A}_{3L}(\alpha)=\dot{s}_5^*+\left(\frac{\alpha-\bar{\ddot{p}}_2}{\bar{\ddot{p}}_3-\bar{\ddot{p}}_2}\right)^{\frac{1}{\ddot{u}_3}}(\dot{s}_6^*-\dot{s}_5^*); & \alpha\in\left[\bar{\ddot{p}}_2,\bar{\ddot{p}}_3\right]\\ \tilde{A}_{4L}(\acute{\alpha})=\dot{s}_7^*+\left(\frac{\alpha-\bar{\ddot{p}}_3}{\bar{\ddot{p}}_4-\bar{\ddot{p}}_3}\right)^{\frac{1}{\ddot{u}_4}}(\dot{s}_8^*-\dot{s}_7^*); & \alpha\in\left[\bar{\ddot{p}}_3,\bar{\ddot{p}}_4\right]\\ \tilde{A}_{5L}(\acute{\alpha})=\dot{s}_9^*+\left(\frac{\alpha-\bar{\ddot{p}}_4}{1-\bar{\ddot{p}}_4}\right)^{\frac{1}{\ddot{u}_5}}(\dot{s}_{10}^*-\dot{s}_9^*); & \alpha\in\left[\bar{\ddot{p}}_4,1\right]\\ \tilde{A}_{5R}(\alpha)=\dot{s}_{12}^*-\left(\frac{\alpha-\bar{\ddot{p}}_4}{1-\bar{\ddot{p}}_4}\right)^{\frac{1}{\ddot{v}_5}}(\dot{s}_{12}^*-\dot{s}_{11}^*); & \alpha\in\left[\bar{\ddot{p}}_4,1\right]\\ \tilde{A}_{4R}(\alpha)=\dot{s}_{14}^*-\left(\frac{\alpha-\bar{\ddot{p}}_3}{\bar{\ddot{p}}_4-\bar{\ddot{p}}_3}\right)^{\frac{1}{\ddot{v}_4}}(\dot{s}_{14}^*-\dot{s}_{13}^*); & \alpha\in\left[\bar{\ddot{p}}_3,\bar{\ddot{p}}_4\right]\\ \tilde{A}_{3R}(\alpha)=\dot{s}_{16}^*-\left(\frac{\alpha-\bar{\ddot{p}}_2}{\bar{\ddot{p}}_3-\bar{\ddot{p}}_2}\right)^{\frac{1}{\ddot{v}_3}}(\dot{s}_{16}^*-\dot{s}_{15}^*); & \alpha\in\left[\bar{\ddot{p}}_2,\bar{\ddot{p}}_3\right]\\ \tilde{A}_{2R}(\alpha)=\dot{s}_{18}^*-\left(\frac{\alpha-\bar{\ddot{p}}_1}{\bar{\ddot{p}}_2-\bar{\ddot{p}}_1}\right)^{\frac{1}{\ddot{v}_2}}(\dot{s}_{18}^*-\dot{s}_{17}^*); & \alpha\in\left[\bar{\ddot{p}}_1,\bar{\ddot{p}}_2\right]\\ \tilde{A}_{1R}(\alpha)=\dot{s}_{20}^*-\left(\frac{\alpha}{\bar{\ddot{p}}_1}\right)^{\frac{1}{\ddot{v}_1}}(\dot{s}_{20}^*-\dot{s}_{19}^*); & \alpha\in\left[0,\bar{\ddot{p}}_1\right]\end{cases}$$

where $\tilde{A}_{1L}(\alpha),\tilde{A}_{2L}(\alpha),\tilde{A}_{3L}(\alpha),\tilde{A}_{4L}(\alpha),\tilde{A}_{5L}(\alpha)\ and\ \tilde{A}_{1R}(\alpha),\tilde{A}_{2R}(\alpha),\tilde{A}_{3R}(\alpha),\tilde{A}_{4R}(\alpha),\tilde{A}_{5R}(\alpha)$ are the increasing and decreasing functions of α correspondingly in the respective intervals.

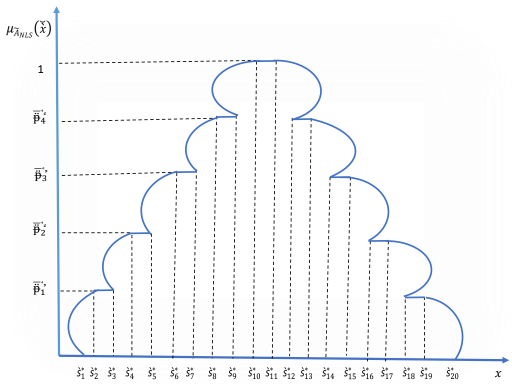

Figure 3: Non-Linear Isagonal fuzzy number with symmetry

4. Defuzzification:

Defuzzification plays a crucial role in converting fuzzy inference results into precise values. To achieve this, fuzzy logic utilizes various techniques such as the centroid method, the greatest of maxima, the smallest of maxima, the mean of maxima, the bisector of area, and the graded mean integral value. In this proposal, we outline four distinct methods for defuzzifying symmetric linear icosagonal fuzzy numbers. These techniques are based on several defuzzification strategies that can also be applied to non-linear symmetric icosagonal fuzzy numbers (NLS), among other scenarios. This approach provides a comprehensive and versatile perspective on the defuzzification process, tailored to the specific characteristics of the fuzzy numbers in question.

4.1. Defuzzification of LS Based on the Approach of Mean of α-cut Method for NLS:

In this section, we proposed a method to compute the defuzzification of linear icosagonal fuzzy number with symmetry as follows:

Since the left and right α-cuts of a non-linear icosagonal fuzzy number with symmetry are given by

$$L_1^*(\alpha) = \grave{s}_1^* + \left(\frac{\alpha}{\bar{\ddot{\mathfrak{p}}}_1}\right)^{\frac{1}{\tilde{u}_1}} (\grave{s}_2^* - \grave{s}_1^*); \qquad \alpha \in [0, \bar{\ddot{\mathfrak{p}}}_1]$$

$$R_1^*(\alpha) = \grave{s}_{20}^* - \left(\frac{\alpha}{\bar{\ddot{\mathfrak{p}}}_1}\right)^{\frac{1}{\tilde{v}_1}} (\grave{s}_{20}^* - \grave{s}_{19}^*); \qquad \alpha \in [0, \bar{\ddot{\mathfrak{p}}}_1]$$

$$L_1^*(\alpha) = \grave{s}_3^* + \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_1}{\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1}\right)^{\frac{1}{\tilde{u}_2}} (\grave{s}_4^* - \grave{s}_3^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_1, \bar{\ddot{\mathfrak{p}}}_2]$$

$$R_1^*(\alpha) = \grave{s}_{18}^* - \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_1}{\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1}\right)^{\frac{1}{\tilde{v}_2}} (\grave{s}_{18}^* - \grave{s}_{17}^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_1, \bar{\ddot{\mathfrak{p}}}_2]$$

$$L_1^*(\alpha) = \grave{s}_5^* + \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_2}{\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2}\right)^{\frac{1}{\tilde{u}_3}} (\grave{s}_6^* - \grave{s}_5^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_2, \bar{\ddot{\mathfrak{p}}}_3]$$

$$R_1^*(\alpha) = \grave{s}_{16}^* - \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_2}{\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2}\right)^{\frac{1}{\tilde{v}_3}} (\grave{s}_{16}^* - \grave{s}_{15}^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_2, \bar{\ddot{\mathfrak{p}}}_3]$$

$$L_1^*(\alpha) = \grave{s}_7^* + \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_3}{\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3}\right)^{\frac{1}{\tilde{u}_4}} (\grave{s}_8^* - \grave{s}_7^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_3, \bar{\ddot{\mathfrak{p}}}_4]$$

$$R_1^*(\alpha) = \grave{s}_{14}^* - \left(\frac{\alpha - \bar{\ddot{\mathfrak{p}}}_3}{\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3}\right)^{\frac{1}{\tilde{v}_4}} (\grave{s}_{14}^* - \grave{s}_{13}^*); \qquad \alpha \in [\bar{\ddot{\mathfrak{p}}}_3, \bar{\ddot{\mathfrak{p}}}_4]$$

$$L_1^*(\alpha) = \grave{s}_9^* + \left(\frac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)^{\frac{1}{\tilde{u}_5}} (\grave{s}_{10}^* - \grave{s}_9^*); \qquad \alpha \in [\bar{\ddot{p}}_4, 1]$$

$$R_1^*(\alpha) = \grave{s}_{12}^* - \left(\frac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)^{\frac{1}{\tilde{v}_5}} (\grave{s}_{12}^* - \grave{s}_{11}^*); \qquad \alpha \in [\bar{\ddot{p}}_4, 1]$$

Then the mean of $\alpha -$ cut of $\hat{A}_{NLS}$ is

$$R(\tilde{A}_{NLS}) = \int_{\alpha=0}^{1} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha$$

$$R(\tilde{A}_{NLS}) = \int_{0}^{\bar{\ddot{p}}_1} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha + \int_{\bar{\ddot{p}}_1}^{\bar{\ddot{p}}_2} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha + \int_{\bar{\ddot{p}}_2}^{\bar{\ddot{p}}_3} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha$$

$$+ \int_{\bar{\ddot{p}}_3}^{\bar{\ddot{p}}_4} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha + \int_{\bar{\ddot{p}}_4}^{1} \left(\frac{L_1^*(\alpha) + R_1^*(\alpha)}{2}\right) d\alpha$$

$$R(\tilde{A}_{NLS}) = \int_{0}^{\bar{\ddot{p}}_1} \left(\frac{\grave{s}_1^* + \left(\frac{\alpha}{\bar{\ddot{p}}_1}\right)^{\frac{1}{\tilde{u}_1}} (\grave{s}_2^* - \grave{s}_1^*) + \grave{s}_{20}^* - \left(\frac{\alpha}{\bar{\ddot{p}}_1}\right)^{\frac{1}{\tilde{v}_1}} (\grave{s}_{20}^* - \grave{s}_{19}^*)}{2}\right) d\alpha$$

$$+ \int_{\bar{\ddot{p}}_1}^{\bar{\ddot{p}}_2} \left(\frac{\grave{s}_3^* + \left(\frac{\alpha - \bar{\ddot{p}}_1}{\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1}\right)^{\frac{1}{\tilde{u}_2}} (\grave{s}_4^* - \grave{s}_3^*) + \grave{s}_{18}^* - \left(\frac{\alpha - \bar{\ddot{p}}_1}{\bar{\ddot{p}}_2 - \bar{\ddot{p}}_1}\right)^{\frac{1}{\tilde{v}_2}} (\grave{s}_{18}^* - \grave{s}_{17}^*)}{2}\right) d\alpha$$

$$+ \int_{\bar{\ddot{p}}_2}^{\bar{\ddot{p}}_3} \left(\frac{\grave{s}_5^* + \left(\frac{\alpha - \bar{\ddot{p}}_2}{\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2}\right)^{\frac{1}{\tilde{u}_3}} (\grave{s}_6^* - \grave{s}_5^*) + \grave{s}_{16}^* - \left(\frac{\alpha - \bar{\ddot{p}}_2}{\bar{\ddot{p}}_3 - \bar{\ddot{p}}_2}\right)^{\frac{1}{\tilde{v}_3}} (\grave{s}_{16}^* - \grave{s}_{15}^*)}{2}\right) d\alpha$$

$$+ \int_{\bar{\ddot{p}}_3}^{\bar{\ddot{p}}_4} \left(\frac{\grave{s}_7^* + \left(\frac{\alpha - \bar{\ddot{p}}_3}{\bar{\ddot{p}}_4 - \bar{\ddot{p}}_3}\right)^{\frac{1}{\tilde{u}_4}} (\grave{s}_8^* - \grave{s}_7^*) + \grave{s}_{14}^* - \left(\frac{\alpha - \bar{\ddot{p}}_3}{\bar{\ddot{p}}_4 - \bar{\ddot{p}}_3}\right)^{\frac{1}{\tilde{v}_4}} (\grave{s}_{14}^* - \grave{s}_{13}^*)}{2}\right) d\alpha$$

$$+ \int_{\bar{\ddot{p}}_4}^{1} \left(\frac{\grave{s}_9^* + \left(\frac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)^{\frac{1}{\tilde{u}_5}} (\grave{s}_{10}^* - \grave{s}_9^*) + \grave{s}_{12}^* - \left(\frac{\alpha - \bar{\ddot{p}}_4}{1 - \bar{\ddot{p}}_4}\right)^{\frac{1}{\tilde{v}_5}} (\grave{s}_{12}^* - \grave{s}_{11}^*)}{2}\right) d\alpha$$

$$R(\tilde{A}_{NLS}) = \frac{\bar{\ddot{\mathfrak{p}}}_1(\grave{s}_1^* + \grave{s}_{20}^*) + \frac{\bar{\ddot{\mathfrak{p}}}_1(\grave{s}_2^* - \grave{s}_1^*)}{\left(\frac{1}{\tilde{u}_1}+1\right)} - \frac{\bar{\ddot{\mathfrak{p}}}_1(\grave{s}_{20}^* - \grave{s}_{19}^*)}{\left(\frac{1}{\tilde{v}_1}+1\right)}}{2}$$

$$+ \frac{\left(\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1\right)(\grave{s}_3^* + \grave{s}_{18}^*) + \frac{\left(\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1\right)(\grave{s}_4^* - \grave{s}_3^*)}{\left(\frac{1}{\tilde{u}_2}+1\right)} - \frac{\left(\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1\right)(\grave{s}_{18}^* - \grave{s}_{17}^*)}{\left(\frac{1}{\tilde{v}_2}+1\right)}}{2}$$

$$+ \frac{\left(\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2\right)(\grave{s}_5^* + \grave{s}_{16}^*) + \frac{\left(\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2\right)(\grave{s}_6^* - \grave{s}_5^*)}{(\frac{1}{\tilde{u}_3}+1)} - \frac{\left(\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2\right)(\grave{s}_{16}^* - \grave{s}_{15}^*)}{\left(\frac{1}{\tilde{v}_3}+1\right)}}{2}$$

$$+ \frac{\left(\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3\right)(\grave{s}_7^* + \grave{s}_{14}^*) + \frac{\left(\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3\right)(\grave{s}_8^* - \grave{s}_7^*)}{(\frac{1}{\tilde{u}_4}+1)} - \frac{\left(\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3\right)(\grave{s}_{12}^* - \grave{s}_{11}^*)}{\left(\frac{1}{\tilde{v}_4}+1\right)}}{2}$$

$$+ \frac{\left(1 - \bar{\ddot{\mathfrak{p}}}_4\right)(\grave{s}_9^* + \grave{s}_{12}^*) + \frac{\left(1 - \bar{\ddot{\mathfrak{p}}}_4\right)(\grave{s}_{10}^* - \grave{s}_9^*)}{(\frac{1}{\tilde{u}_5}+1)} - \frac{\left(1 - \bar{\ddot{\mathfrak{p}}}_4\right)(\grave{s}_{12}^* - \grave{s}_{11}^*)}{\left(\frac{1}{\tilde{v}_5}+1\right)}}{2}$$

If we set all the non-linear parameters as unity i.e., $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4, \tilde{u}_5, \tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4, \tilde{v}_5 = 1$, then the proposed defuzzification technique applies for the linear icosagonal fuzzy number with symmetry as follows;

$$R(\tilde{A}_{NLS}) = \frac{\bar{\ddot{\mathfrak{p}}}_1(\grave{s}_1^* + \grave{s}_2^* + \grave{s}_{19}^* + \grave{s}_{20}^*) + \left(\bar{\ddot{\mathfrak{p}}}_2 - \bar{\ddot{\mathfrak{p}}}_1\right)(\grave{s}_3^* + \grave{s}_4^* + \grave{s}_{17}^* + \grave{s}_{18}^*) + \left(\bar{\ddot{\mathfrak{p}}}_3 - \bar{\ddot{\mathfrak{p}}}_2\right)(\grave{s}_5^* + \grave{s}_6^* + \grave{s}_{15}^* + \grave{s}_{16}^*)}{4}$$

$$+ \frac{\left(\bar{\ddot{\mathfrak{p}}}_4 - \bar{\ddot{\mathfrak{p}}}_3\right)(\grave{s}_7^* + \grave{s}_8^* + \grave{s}_{13}^* + \grave{s}_{14}^*) + \left(1 - \bar{\ddot{\mathfrak{p}}}_4\right)(\grave{s}_9^* + \grave{s}_{10}^* + \grave{s}_{11}^* + \grave{s}_{12}^*)}{4}$$

In particular, for $\bar{\ddot{\mathfrak{p}}}_1 = 0.20$, $\bar{\ddot{\mathfrak{p}}}_2 = 0.40$, $\bar{\ddot{\mathfrak{p}}}_3 = 0.60$ and $\bar{\ddot{\mathfrak{p}}}_3 = 0.80$ we have

$$R(\tilde{A}_{NLS}) = \frac{(\grave{s}_1^* + \grave{s}_2^* + \grave{s}_3^* + \grave{s}_4^* + \grave{s}_5^* + \grave{s}_6^* + \grave{s}_7^* + \grave{s}_8^* + \grave{s}_9^* + \grave{s}_{10}^* + \grave{s}_{11}^* + \grave{s}_{12}^* + \grave{s}_{13}^* + \grave{s}_{14}^* + \grave{s}_{15}^* + \grave{s}_{16}^* + \grave{s}_{17}^* + \grave{s}_{18}^* + \grave{s}_{19}^* + \grave{s}_{20}^*)}{20}$$

4.2. Calculation of fuzzified values:

In this section, employing a systematic approach with linguistic variables crafted to encapsulate and scrutinize the complexity within the collected responses, we precisely computed the fuzzified values using linear icosagonal fuzzy numbers. This involved a detailed procedure where the gathered data was rigorously examined using a series of linguistic factors designed to uncover and capture the intricate nuances embedded in the provided responses.

Linguistic variables	Icosagonal Fuzzy Number (IFN)
1. Very dissatisfied	(1.0, 2.5, 4.0, 5.5, 7.0, 8.5, 10.0, 11.5, 13.0, 14.5, 16.0, 17.5, 19.0, 20.5, 22.0, 23.5, 25.0, 26.5, 28.0, 29.5)
2. Dissatisfied	(2.5, 4.0, 5.5, 7.0, 8.5, 10.0, 11.5, 13.0, 14.5, 16.0, 17.5, 19.0, 20.5, 22.0, 23.5, 25.0, 26.5, 28.0, 29.5, 31.0)
3. Neutral	(4.0, 5.5, 7.0, 8.5, 10.0, 11.5, 13.0, 14.5, 16.0, 17.5, 19.0, 20.5, 22.0, 23.5, 25.0, 26.5, 28.0, 29.5, 31.0, 32.5)
4. Satisfied	(5.5, 7.0, 8.5, 10.0, 11.5, 13.0, 14.5, 16.0, 17.5, 19.0, 20.5, 22.0, 23.5, 25.0, 26.5, 28.0, 29.5, 31.0, 32.5, 34.0)
5. Very Satisfied	(7.0, 8.5, 10.0, 11.5, 13.0, 14.5, 16.0, 17.5, 19.0, 20.5, 22.0, 23.5, 25.0, 26.5, 28.0, 29.5, 31.0, 32.5, 34.0, 35.5)

5. Comparative Study of proposed different zones in Andhra Pradesh:

In this section, we compared all the proposed different zones in Andhra Pradesh using defuzzification technique.

The composition of zones and districts in each zone of Andhra Pradesh is as hereunder:

Zone-I: Srikakulam, Vizianagaram and Visakhapatnam

Zone-II: East Godavari, West Godavari and Krishna

Zone-III: Guntur, Prakasam and Nellore

Zone-IV: Chittor, Kadapa, Anantapur and Kurnool

ZONE – I			
S. No	**Criteria**	**Defuzzified value**	**Rank (Icosagonal FN)**
1	Booking an appointment	19.8269	2
2	Quality of medicals	19.5962	4
3	Hygiene	19.9038	1
4	Time taken by labs	19.0962	8
5	Care provided by medical professionals & nurses	19.6346	3
6	Safety measures	19.4423	6
7	Value for money	19.2885	7

8	Reliability and recovery	19.5962	4
9	Cost effectiveness	18.9808	9
10	Quality of food, lift facility & security	18.5962	10
11	Recommendation	16.9038	11

Table 1: Computation for ranking of various parameters (health services) in Zone-I using defuzzified values.

ZONE – I											
S.no	**Criteria**	**Penta-gonal D.Value**	**Rank (P)**	**Hexagonal D.Value**	**Rank (H)**	**Hepta-gonal D.Value**	**Rank (Hepta)**	**Hex-Decagonal D.Value**	**Rank (Hex. D)**	**Icosagonal D. Value**	**Rank (I)**
1	Booking an appointment	16.1538	2	14.8782	2	13.1026	2	14.1891	2	19.8269	2
2	Quality of medicals	15.6923	4	14.4936	4	12.7949	4	13.9968	4	19.5962	4
3	Hygiene	16.3077	1	15.0064	1	13.2051	1	14.2532	1	19.9038	1
4	Time taken by labs	14.6923	8	13.6603	8	12.1282	8	13.5801	8	19.0962	8
5	Care provided by medical professionals & nurses	15.7692	3	14.5577	3	12.8462	3	14.0288	3	19.6346	3
6	Safety measures	15.3846	6	14.2372	6	12.5897	6	13.8686	6	19.4423	6
7	Value for money	15.0769	7	13.9808	7	12.3846	7	13.7404	7	19.2885	7
8	Reliability and recovery	15.6923	4	14.4936	4	12.7949	4	13.9968	4	19.5962	4
9	Cost effectiveness	14.4615	9	13.4679	9	11.9744	9	13.4840	9	18.9808	9
10	Quality of food, lift facility & security	13.6923	10	12.8269	10	11.4615	10	13.1635	10	18.5962	10
11	Recommendation	10.3077	11	10.0064	11	9.2051	11	11.7532	11	16.9038	11

Table 2: Defuzzified Values and Rankings of Healthcare Service Quality Criteria.

5.1. Comparative Analysis of Icosagonal Fuzzy Number Rankings with Proposed and Existing Rankings

ZONE – I							
		Defuzzified values					
S. No	**Criteria**	**Penta-gonal (R_1)**	**Hexagonal (R_2)**	**Hepta-gonal (R_3)**	**Hex-Decagonal (R_4)**	**Icosagonal (R_5)**	**Relation between R_1, R_2, R_3, R_4 & R_5**
1	Booking an appointment	16.1538	14.8782	13.1026	14.1891	19.8269	$R_3 < R_4 < R_2 < R_1 < R_5$
2	Quality of medicals	15.6923	14.4936	12.7949	13.9968	19.5962	$R_3 < R_4 < R_2 < R_1 < R_5$
3	Hygiene	16.3077	15.0064	13.2051	14.2532	19.9038	$R_3 < R_4 < R_2 < R_1 < R_5$
4	Time taken by labs	14.6923	13.6603	12.1282	13.5801	19.0962	$R_3 < R_4 < R_2 < R_1 < R_5$
5	Care provided by medical professionals & nurses	15.7692	14.5577	12.8462	14.0288	19.6346	$R_3 < R_4 < R_2 < R_1 < R_5$
6	Safety measures	15.3846	14.2372	12.5897	13.8686	19.4423	$R_3 < R_4 < R_2 < R_1 < R_5$
7	Value for money	15.0769	13.9808	12.3846	13.7404	19.2885	$R_3 < R_4 < R_2 < R_1 < R_5$
8	Reliability and recovery	15.6923	14.4936	12.7949	13.9968	19.5962	$R_3 < R_4 < R_2 < R_1 < R_5$
9	Cost effectiveness	14.4615	13.4679	11.9744	13.4840	18.9808	$R_3 < R_2 < R_4 < R_1 < R_5$
10	Quality of food, lift facility & security	13.6923	12.8269	11.4615	13.1635	18.5962	$R_3 < R_2 < R_4 < R_1 < R_5$
11	Recommendation	10.3077	10.0064	9.2051	11.7532	16.9038	$R_3 < R_2 <$

							$R_1 < R_4 < R_5$

Table 3: Order of a Icositetragon Fuzzy Number with the proposal and existing rankings

*R1, R2, R3, R4, R5 denote the ranking of Pentagonal, Hexagonal, Heptagonal, Hex Decagonal, and Icosagonal Fuzzy Numbers respectively.

ZONE – II			
S. No	**Criteria**	**Defuzzified value**	**Rank (Icosagonal FN)**
1	Booking an appointment	19.69	5
2	Quality of medicals	20.02	1
3	Hygiene	19.63	6
4	Time taken by labs	19.51	7
5	Care provided by medical professionals & nurses	19.72	3
6	Safety measures	19.81	2
7	Value for money	19.12	10
8	Reliability and recovery	19.72	3
9	Cost effectiveness	19.18	8
10	Quality of food, lift facility & security	19.15	9
11	Recommendation	16.72	11

Table 4: Computation for ranking of various parameters (health services) in Zone-II using defuzzified values.

ZONE – II											
S. no	**Criteria**	**Penta-gonal D.Value**	**Rank (P)**	**Hexagonal D.Value**	**Rank (H)**	**Hepta-gonal D.Value**	**Rank (Hepta)**	**Hex-Decagonal D.Value**	**Rank (Hex. D)**	**Icosagonal D. Value**	**Rank (I)**
1	Booking an appointment	15.88	5	14.65	5	12.92	5	14.08	5	19.69	5
2	Quality of medicals	16.54	1	15.20	1	13.36	1	14.35	1	20.02	1
3	Hygiene	15.76	6	14.55	6	12.84	6	14.03	6	19.63	6
4	Time taken by labs	15.52	7	14.35	7	12.68	7	13.93	7	19.51	7
5	Care provided by medical professionals & nurses	15.94	3	14.70	3	12.96	3	14.10	3	19.72	3

6	Safety measures	16.12	2	14.85	2	13.08	2	14.18	2	19.81	2
7	Value for money	14.74	10	13.70	10	12.16	10	13.60	10	19.12	10
8	Reliability and recovery	15.94	3	14.70	3	12.96	3	14.10	3	19.72	3
9	Cost effectivenes s	14.86	8	13.80	8	12.24	8	13.65	8	19.18	8
1 0	Quality of food, lift facility & security	14.80	9	13.75	9	12.2	9	13.63	9	19.15	9
1 1	Recommend ation	9.94	11	9.70	11	8.96	11	11.60	11	16.72	11

Table 5: Defuzzified Values and Rankings of Healthcare Service Quality Criteria.

5.2. Comparative Analysis of Icosagonal Fuzzy Number Rankings with Proposed and Existing Rankings

ZONE – II							
		Defuzzified values					
S. No	**Criteria**	**Penta-gonal (R_1)**	**Hexagonal (R_2)**	**Hepta-gonal (R_3)**	**Hex-Decagonal (R_4)**	**Icosagonal (R_5)**	**Relation between R_1, R_2, R_3, R_4 & R_5**
1	Booking an appointment	15.88	14.65	12.92	14.08	19.69	$R_3 < R_4 < R_2 < R_1 < R_5$
2	Quality of medicals	16.54	15.20	13.36	14.35	20.02	$R_3 < R_4 < R_2 < R_1 < R_5$
3	Hygiene	15.76	14.55	12.84	14.03	19.63	$R_3 < R_4 < R_2 < R_1 < R_5$
4	Time taken by labs	15.52	14.35	12.68	13.93	19.51	$R_3 < R_4 < R_2 < R_1 < R_5$
5	Care provided by medical professionals & nurses	15.94	14.70	12.96	14.10	19.72	$R_3 < R_4 < R_2 < R_1 < R_5$
6	Safety measures	16.12	14.85	13.08	14.18	19.81	$R_3 < R_4 < R_2 < R_1 < R_5$
7	Value for money	14.74	13.70	12.16	13.60	19.12	$R_3 < R_4 < R_2 < R_1 < R_5$
8	Reliability and recovery	15.94	14.70	12.96	14.10	19.72	$R_3 < R_4 < R_2 < R_1 < R_5$
9	Cost effectiveness	14.86	13.80	12.24	13.65	19.18	$R_3 < R_4 < R_2 < R_1 < R_5$
10	Quality of food, lift facility & security	14.80	13.75	12.2	13.63	19.15	$R_3 < R_4 < R_2 < R_1 < R_5$

11	Recommendation	9.94	9.70	8.96	11.60	16.72	$R_3 < R_2 < R_1 < R_4 < R_5$

Table 6: Order of a Icositetragon Fuzzy Number with the proposal and existing rankings

*R1, R2, R3, R4, R5 denote the ranking of Pentagonal, Hexagonal, Heptagonal, Hex Decagonal, and Icosagonal Fuzzy Numbers respectively

ZONE – III			
S. No	**Criteria**	**Defuzzified value**	**Rank (Icosagonal FN)**
1	Booking an appointment	19.6636	5
2	Quality of medicals	19.8364	2
3	Hygiene	19.7107	3
4	Time taken by labs	19.2866	7
5	Care provided by medical professionals & nurses	19.6322	6
6	Safety measures	19.8835	1
7	Value for money	18.9725	10
8	Reliability and recovery	19.6872	4
9	Cost effectiveness	19.0982	9
10	Quality of food, lift facility & security	19.1139	8
11	Recommendation	16.5301	11

Table 7: Computation for ranking of various parameters (health services) in Zone-III using defuzzified values.

ZONE – III											
S. no	**Criteria**	**Penta-gonal D.Value**	**Rank (P)**	**Hexagonal D.Value**	**Rank (H)**	**Hepta-gonal D.Value**	**Rank (Hepta)**	**Hex-Decagonal D.Value**	**Rank (Hex. D)**	**Icosagonal D. Value**	**Rank (I)**
1	Booking an appointment	15.8272	5	14.6060	5	12.8848	5	14.0530	5	19.6636	5
2	Quality of medicals	16.1728	2	14.8940	2	13.1152	2	14.1970	2	19.8364	2
3	Hygiene	15.9215	3	14.6846	3	12.9476	3	14.0923	3	19.7107	3
4	Time taken by labs	15.0733	7	13.9777	7	12.3822	7	13.7389	7	19.2866	7
5	Care provided by medical professional	15.7644	6	14.5537	6	12.8429	6	14.0268	6	19.6322	6

	s & nurses										
6	Safety measures	16.2670	1	14.9725	1	13.1780	1	14.2363	1	19.8835	1
7	Value for money	14.4450	10	13.4542	10	11.9634	10	13.4771	10	18.9725	10
8	Reliability and recovery	15.8743	4	14.6453	4	12.9162	4	14.0726	4	19.6872	4
9	Cost effectiveness	14.6963	9	13.6636	9	12.1309	9	13.5818	9	19.0982	9
10	Quality of food, lift facility & security	14.7277	8	13.6898	8	12.1518	8	13.5949	8	19.1139	8
11	Recommendation	9.5602	11	9.3835	11	8.7068	11	11.4418	11	16.5301	11

Table 8: Defuzzified Values and Rankings of Healthcare Service Quality Criteria.

5.3. Comparative Analysis of Icosagonal Fuzzy Number Rankings with Proposed and Existing Rankings

ZONE – III							
		Defuzzified values					
S. No	**Criteria**	**Penta-gonal (R_1)**	**Hexagonal (R_2)**	**Hepta-gonal (R_3)**	**Hex-Decagonal (R_4)**	**Icosagonal (R_5)**	**Relation between R_1, R_2, R_3, R_4 & R_5**
1	Booking an appointment	15.8272	14.6060	12.8848	14.0530	19.6636	$R_3 < R_4 < R_2 < R_1 < R_5$
2	Quality of medicals	16.1728	14.8940	13.1152	14.1970	19.8364	$R_3 < R_4 < R_2 < R_1 < R_5$
3	Hygiene	15.9215	14.6846	12.9476	14.0923	19.7107	$R_3 < R_4 < R_2 < R_1 < R_5$
4	Time taken by labs	15.0733	13.9777	12.3822	13.7389	19.2866	$R_3 < R_4 < R_2 < R_1 <$

							R_5
5	Care provided by medical professionals & nurses	15.7644	14.5537	12.8429	14.0268	19.6322	$R_3 < R_4 < R_2 < R_1 < R_5$
6	Safety measures	16.2670	14.9725	13.1780	14.2363	19.8835	$R_3 < R_4 < R_2 < R_1 < R_5$
7	Value for money	14.4450	13.4542	11.9634	13.4771	18.9725	$R_3 < R_2 < R_4 < R_1 < R_5$
8	Reliability and recovery	15.8743	14.6453	12.9162	14.0726	19.6872	$R_3 < R_4 < R_2 < R_1 < R_5$
9	Cost effectiveness	14.6963	13.6636	12.1309	13.5818	19.0982	$R_3 < R_4 < R_2 < R_1 < R_5$
10	Quality of food, lift facility & security	14.7277	13.6898	12.1518	13.5949	19.1139	$R_3 < R_4 < R_2 < R_1 < R_5$
11	Recommendation	9.5602	9.3835	8.7068	11.4418	16.5301	$R_3 < R_2 < R_1 < R_4 < R_5$

Table 9: Order of a Icositetragon Fuzzy Number with the proposal and existing rankings

*R1, R2, R3, R4, R5 denote the ranking of Pentagonal, Hexagonal, Heptagonal, Hex Decagonal, and Icosagonal Fuzzy Numbers respectively.

ZONE – IV			
S. No	**Criteria**	**Defuzzified value**	**Rank (Icosagonal FN)**
1	Booking an appointment	19.9579	2
2	Quality of medicals	19.9134	3
3	Hygiene	19.8540	4
4	Time taken by labs	19.5866	7
5	Care provided by medical professionals & nurses	19.7500	5
6	Safety measures	20.0025	1
7	Value for money	19.2153	8
8	Reliability and recovery	19.6609	6
9	Cost effectiveness	18.8441	10
10	Quality of food, lift facility & security	19.1262	9
11	Recommendation	16.4233	11

Table 10: Computation for ranking of various parameters (health services) in Zone-IV using defuzzified values.

ZONE – IV											
S.no	**Criteria**	**Penta-gonal D.Value**	**Rank (P)**	**Hexagonal D.Value**	**Rank (H)**	**Hepta-gonal D.Value**	**Rank (Hepta)**	**Hex-Decagonal D.Value**	**Rank (Hex. D)**	**Icosagonal D. Value**	**Rank (I)**
1	Booking an appointment	16.3939	2	15.0965	2	13.2772	2	14.2983	2	19.9579	2
2	Quality of medicals	16.3267	3	15.0223	3	13.2178	3	14.2611	3	19.9134	3
3	Hygiene	16.2079	4	14.9233	4	13.1386	4	14.2116	4	19.8540	4
4	Time taken by labs	15.6733	7	14.4777	7	12.7822	7	13.9889	7	19.5866	7
5	Care provided by medical professionals & nurses	16.0000	5	14.7500	5	13.0000	5	14.1250	5	19.7500	5
6	Safety measures	16.5050	1	15.1708	1	13.3366	1	14.3354	1	20.0025	1
7	Value for money	14.9307	8	13.8589	8	12.2871	8	13.6795	8	19.2153	8
8	Reliability and recovery	15.8218	6	14.6015	6	12.8812	6	14.0507	6	19.6609	6
9	Cost effectiveness	14.1881	10	13.2401	10	11.7921	10	13.3700	10	18.8441	10
10	Quality of food, lift facility & security	14.7525	9	13.7104	9	12.1683	9	13.6052	9	19.1262	9
11	Recommendation	9.3465	11	9.2054	11	8.5644	11	11.3527	11	16.4233	11

Table 11: Defuzzified Values and Rankings of Healthcare Service Quality Criteria.

5.4. Comparative Analysis of Icosagonal Fuzzy Number Rankings with Proposed and Existing Rankings

ZONE – IV			
		Defuzzified values	

S. No	Criteria	Penta-gonal (R_1)	Hexagonal (R_2)	Hepta-gonal (R_3)	Hex-Decagonal (R_4)	Icosagonal (R_5)	Relation between R_1, R_2, R_3, R_4 & R_5
1	Booking an appointment	16.3939	15.0965	13.2772	14.2983	19.9579	$R_3 < R_4 < R_2 < R_1 < R_5$
2	Quality of medicals	16.3267	15.0223	13.2178	14.2611	19.9134	$R_3 < R_4 < R_2 < R_1 < R_5$
3	Hygiene	16.2079	14.9233	13.1386	14.2116	19.8540	$R_3 < R_4 < R_2 < R_1 < R_5$
4	Time taken by labs	15.6733	14.4777	12.7822	13.9889	19.5866	$R_3 < R_4 < R_2 < R_1 < R_5$
5	Care provided by medical professionals & nurses	16.0000	14.7500	13.0000	14.1250	19.7500	$R_3 < R_4 < R_2 < R_1 < R_5$
6	Safety measures	16.5050	15.1708	13.3366	14.3354	20.0025	$R_3 < R_4 < R_2 < R_1 < R_5$
7	Value for money	14.9307	13.8589	12.2871	13.6795	19.2153	$R_3 < R_4 < R_2 < R_1 < R_5$
8	Reliability and recovery	15.8218	14.6015	12.8812	14.0507	19.6609	$R_3 < R_4 < R_2 < R_1 < R_5$
9	Cost effectiveness	14.1881	13.2401	11.7921	13.3700	18.8441	$R_3 < R_2 < R_4 < R_1 < R_5$
10	Quality of food, lift facility & security	14.7525	13.7104	12.1683	13.6052	19.1262	$R_3 < R_4 < R_2 < R_1 < R_5$
11	Recommendation	9.3465	9.2054	8.5644	11.3527	16.4233	$R_3 < R_2 < R_1 < R_4 < R_5$

Table 12: Order of a Icositetragon Fuzzy Number with the proposal and existing rankings

*R1, R2, R3, R4, R5 denote the ranking of Pentagonal, Hexagonal, Heptagonal, Hex Decagonal, and Icosagonal Fuzzy Numbers respectively.

6. Conclusion:

This paper presents the idea of Icosagonal Fuzzy Numbers (IFNs) and how defuzzification methods use them. We have shown the efficacy of IFNs in handling complexity in linguistic data by contrasting defuzzification techniques among various zones in Andhra Pradesh. Compared to conventional shapes like trapezoidal or triangular fuzzy numbers, the suggested solutions offer a more nuanced depiction of fuzzy numbers.
Our results demonstrate the benefits of applying IFNs to healthcare service quality assessment, providing more accurate and thorough assessments. This study adds to the body of knowledge in fuzzy set theory by creating new directions for investigation and real-world applications in situations involving decision-making when ambiguity and uncertainty are common.

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