

Numerical Analysis of Stability of Plant-Animal Interactions in Riparian Zones

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Abstract:

Ecological interactions of Plant, Animal Species, particularly in Riparian Zones have become a subject of intense interest in recent years. These Zones whether along a river bank or shore line, link dry and wet environments. These zones are biologically significant and study of Ecological models of Plant-Animal interactions in these zones help understand the myriad of factors at play there.

The authors of the present article, in their previous work have studied the interaction between a Plant and an Herbivorous mammal and the long term dynamics of their populations as Prey and Predator interactions. For this they closely followed the Lotka (1925)- Volterra (1931) model.

The interactions have been modeled as system of Ordinary Differential Equations from which the Equilibrium points are identified and the stability analysis of these points is carried out by Linear Stability analysis.

In this present work numerical estimates are presented for the stability analysis for the two species Eco system considered earlier. The equations of perturbation for each steady state are solved numerically using the MatLab command 'ode45' which employs Runge-Kutta method of 4th order for solving simultaneous differential equations. The numerical results have been given to MatLab command 'plot' to obtain a graphical representation of the same. Criteria for the asymptotic stability of the states have been noted.

Keywords: Plant-Animal Interactions, Riparian Zones.

1. Introduction

A riparian zone is a strip of land beside a river or stream. These areas, whether along a river bank or a shoreline, link wet and dry environments.

These areas are crucial for maintaining biodiversity and ecological stability. The intricate interdependencies between plants and animals in riparian zones may be better understood by exploring environmental models of plant-animal interactions. Riparian zones are biologically significant, and ecological models of plant-animal interactions help understand the myriad of factors at play there. Researchers and conservationists make more informed choices about how best to protect and restore riparian ecosystems by learning more about these mechanisms. The aforementioned ecological models accurately portray the dynamic interplay between plants and animals and their responses to environmental changes in riparian zones.

In the case of the Riparian zones, it can be stated here that this can possess a “diverse array of species” and environmental processes. For this reason, it can be stated here that ecological diversity is connected to different flood regimes. This is related to the “altitudinal climate shifts, geographically unique channel processes, as well as the upland influences on the fluvial corridor” (Singh *et al.*, 2021). Based on this content of the research, it can be noted that these stated processes include some of the accurate elements, such as the “surface and subsurface hydrology”, “carbon, nitrogen, and phosphorus transport, removal, and cycling”, “sediment transport and deposition,” as well as the vegetation growth.

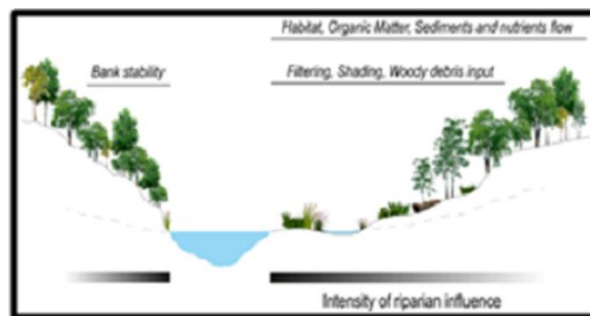


Figure 1. Stability Analysis of Riparian Zones (Source: Pedraza *et al.*, 2021)

In the case of the Case *et al.*, (2023), it can be pointed out that these stated aquatic ecosystems can interact with the riparian areas closely. However, the areas can be the “zones of lush, green, moisture-loving vegetation” covered with wetlands, streams, lakes, and rivers. Based on the view of Chamoli *et al.*, (2023), it may be quite significant to note that the riparian areas form “a transition zone between dry and aquatic upland habitats”. These concepts are pretty reliable for the collected content for this current research study (Gallitelli & Scalici, 2023). This mechanism and the research structure are pretty effective for solving the recent risk factors based on the Riparian zones.

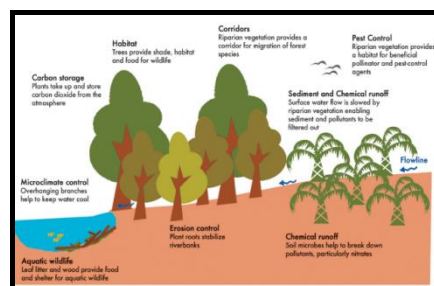


Figure 2. The Procedure of Riparian Reserves for Creating the Environmental Benefits to the River, Wider Landscape and Plantation (Source: Neu *et al.*, 2023)

In the case of Figure 2 above, the “mechanism of riparian reserves for creating the environmental benefits to the river, wider landscape and plantation” can be noted sustainably (Neu *et al.*, 2023). However, based on this research, it can be annotated here that the riparian reserves carry a significant “number of environmental benefits” for the present water systems. This water system includes filtering the water before it flows into the rivers, food protection, and stabilizing the river bank. On

the other hand, it can be stated here that the concept of these riparian reserves is quite beneficial for “biodiversity conservation” in waterways and on land (Veenma *et al.*, 2023). It can be pretty significant for Carbon storage.

2. Key species selection

Studying trophic interactions requires focal species from many trophic levels. It encompasses everything from plants to herbivores to carnivores. Examples include the interactions between a common plant (like the willow), an herbivorous insect (like the leaf beetle), and a predator (like a bird).

3. Mathematical model:

The study constructs an ecological model of the population dynamics of the species using differential equations. The model has to include things like trophic relationships, competition, and the complexity of habitats. In this regard, the ecological system, including those focusing on plant-animal interactions in Riparian zones, often uses the ‘*Lotka-Volterra Model*’ (Akjouj *et al.*, 2022). Understanding the dynamics of ecological systems, especially in the context of plant-animal interactions in Riparian zones, requires Mathematical modelling (Corenblit *et al.*, 2023). This recent research looks at the relationship between a plant and an herbivorous mammal in a basic ecological model. The long-term dynamics of these populations may be understood by the stability analysis of such a model (Beyer *et al.*, 2023).

The dynamics of the plant populations and herbivore populations are described by the following system of simultaneous differential equations

$$\frac{dP}{dt} = rP - aPA \quad (1)$$

$$\frac{dA}{dt} = baPA - dA \quad (2)$$

Here,

$P(t)$ is the population of plants at time t .

$A(t)$ is the population of herbivores at time t .

r is the intrinsic growth rate of plants.

a is the herbivores’ capture rate in terms of their consuming plants.

b is the conversion rate of herbivores consuming plants into the population growth.

d is the death rate of herbivores.

In order to illustrate this, this research uses a Lotka-Volterra Model, a commonly used framework for ecological modelling.

Based on the view of Chamoli *et al.*, (2023), the model equations (1) and (2) can be rewritten as follows:

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right) - aPA \quad (3)$$

$$\frac{dA}{dt} = -\delta A + \beta P A \quad (4)$$

Here,

r is the intrinsic growth rate of the plant population

k is the carrying capacity of the environment for the plants

α is the predation rate of herbivores on plants

δ represents the death rate of herbivores and

β is the efficiency of converting plant biomass into herbivore biomass.

4. Methodology

In order to analyze the stability of this ecological model, one can start by finding the equilibrium points, where

$$\frac{dP}{dt} = 0 \text{ and } \frac{dA}{dt} = 0 \quad (5)$$

These equilibrium points correspond to the stable population sizes for both plants and herbivores in the long run.

Considering the above equations (5), we get three Equilibrium states given by:

1. $P^* = 0, A^* = 0$
2. $P^* = K, A^* = 0$
3. $P^* = \frac{\delta}{\beta}, A^* = \frac{r(\beta k - \delta)}{\alpha \beta K}$

The numerical stability analysis of these Equilibrium states is carried out by deriving the perturbation equations corresponding to equations (3) and (4) and solving them numerically using the MatLab routine 'ode45' which employs the Runge-Kutta 4th order method and the solution curves are plotted using the 'plot' command (D. Ravi kiran *et.al.* 2011).

The perturbation equations for the system of differential equations (3) and (4) are obtained by substituting

$$P = P^* + U_P \text{ and } A = A^* + U_A \quad (6)$$

where U_P and U_A are deviations from the Equilibrium point which are so small that their products and higher powers are negligible.

5. Numerical stability analysis

Equilibrium State 1 (Fully washed out state): $P^* = 0, A^* = 0$

In this state, the populations of both the species are zero. The perturbation equations for this state are given by

$$\frac{dU_P}{dt} = r U_P \quad (7)$$

$$\text{and } \frac{dU_A}{dt} = -\delta U_A \quad (8)$$

Solving (7) and (8) numerically we get the following results under various cases.

Case 1.1: $U_{P0} < U_{A0}$ and $r < \delta$

Let $U_{P0} = 1, U_{A0} = 2, r = 0.5, \delta = 0.7$

Here, the initial value of the perturbation of Plant species is assumed to be less than that of Animal (Herbivore) species and the intrinsic growth rate of Plants is assumed to be less than the death rate of Herbivores.

It can be seen that the perturbation curve corresponding to the Plant species diverge away from Equilibrium while that of the Animal species converges.

So this Equilibrium state is unstable under this case.

The results in tabular form and in graphical form are given below in Table 1.1.1 and Figure 1.1.1:

t	U_P	U_A	t	U_P	U_A
0	1.00	2.00	3.0	4.4817	0.24491
0.5	1.2840	1.4094	3.5	5.7546	0.17259
1.0	1.6487	0.99317	4.0	7.3891	0.12162
1.5	2.1170	0.69988	4.5	9.4877	0.085705
2.0	2.7183	0.49319	5.0	1.2182	0.060395
2.5	3.4903	0.34755			

Table 1.1.1: $U_{P0} <$

U_{A0} and $r < \delta$

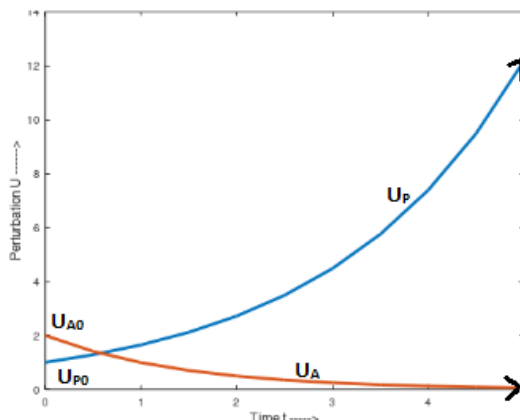


Fig. 1.1.1: $U_{P0} < U_{A0}$ and $r < \delta$

Case 1.2: $U_{P0} > U_{A0}$ and $r < \delta$

Let $U_{P0} = 2, U_{A0} = 1, r = 0.5, \delta = 0.7$

Here, the initial value of the perturbation of Plant species is more than that of Animal (Herbivore) species. And the intrinsic growth rate of Plants is less than the death rate of Herbivores.

Under this case also, the perturbation curve corresponding to the Plant species diverge away from Equilibrium where as that of the Animal species converges.

So this Equilibrium state is unstable under this case too.

The results in tabular form and in graphical form are given below in Table 1.2.1 and Figure 1.2.1:

t	U_P	U_A	t	U_P	U_A
0	2.00	1.00	3.0	8.9634	0.12246
0.5	2.5681	0.70469	3.5	11.509	0.086294
1.0	3.2974	0.49658	4.0	14.778	0.060810
1.5	4.2340	0.34994	4.5	18.975	0.042852
2.0	5.4366	0.24660	5.0	24.365	0.030198
2.5	6.9807	0.17377			

Table 1.2.1: $U_{P0} > U_{A0}$ and $r < \delta$

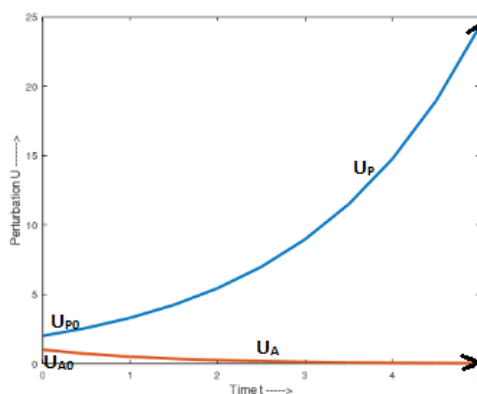


Fig. 1.2.1: $U_{P0} > U_{A0}$ and $r < \delta$

Equilibrium State 2 (Herbivore washed out state): $P^* = k, A^* = 0$

In this state, The Herbivore population is zero and the plant population is equal to k .

The Perturbation equations for this state are given by

$$\frac{dU_P}{dt} = -rU_P - \alpha k U_A \quad (9)$$

And
$$\frac{dU_A}{dt} = (\beta k - \delta)U_A \quad (10)$$

Solving (9) and (10) numerically we get the following results under various cases.

Case 2.1: $U_{P0} > U_{A0}$ and $\beta k - \delta > 0$

Let $U_{P0} = 2, U_{A0} = 1, r = 0.5, \alpha = 0.5,$

$$k = 1, \beta = 0.8, \delta = 0.5$$

Here, the initial value of the perturbation of Plant species is more than that of Animal (Herbivore) species. And the efficiency of converting Plant biomass into Herbivore biomass (β) is greater than the death rate of Herbivores (δ), keeping the carrying capacity of nature for Plants (k) fixed at 1.

Under this assumption, the current state is found to be Unstable.

The results in tabular form and in graphical form are given below in Table 2.1.1 and Figure 2.1.1:

t	U_P	U_A	t	U_P	U_A
0	2.000	1.000	3.0	--	0.5488
0.5	1.3475	0.9048	3.5	--	0.4965
1.0	0.8593	0.8187	4.0	--	0.4493
1.5	0.4973	0.7408	4.5	--	0.4065
2.0	0.2316	0.6703	5.0	--	0.3678
2.5	0.0396	0.6065			

Table 2.1.1: $U_{P0} > U_{A0}$ and $\beta k - \delta > 0$

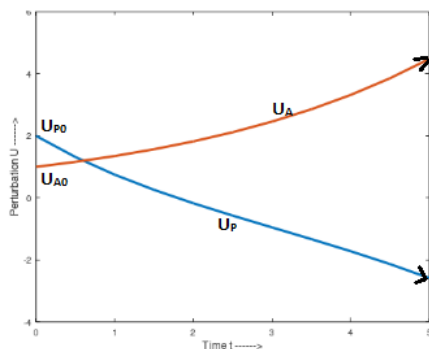


Fig. 2.1.1: $U_{P0} > U_{A0}$ and $\beta k - \delta > 0$

Case 2.2: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

Let $U_{P0} = 2$, $U_{A0} = 1$, $r = 0.5$, $\alpha = 0.5$,
 $k = 1$, $\beta = 0.5$, $\delta = 0.7$

Here, the condition of Case 2.1 is reversed.

That is, the efficiency of converting Plant biomass into Herbivore biomass (β) is less than the death rate of Herbivores (δ), keeping the carrying capacity of nature for Plants (k) fixed at 1.

Under this condition, both the perturbation curves converge to the equilibrium point.

Hence the current state is stable.

The results are illustrated in Table 2.2.1 and Fig. 2.2.1

t	U_P	U_A	t	U_P	U_A
0	2.000	1.000	3.0	--	2.4596
0.5	1.3182	1.1618	3.5	--	2.8577
1.0	0.7485	1.3499	4.0	--	3.3201
1.5	0.2598	1.5683	4.5	--	3.8574
2.0	--	1.8221	5.0	--	4.4817
2.5	--	2.1170			

Table 2.2.1: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

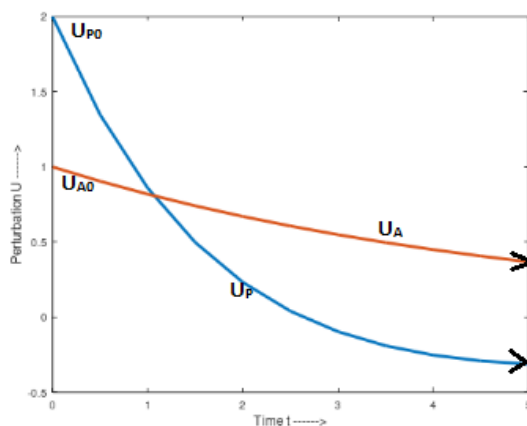


Fig. 2.2.1: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

Equilibrium State 3 (The Co-existing state): $P^* = \frac{\delta}{\beta}; A^* = \frac{r(\beta k - \delta)}{\alpha \beta k}$

In this state, both the Plant and Herbivore species are co-existing.

The Perturbation equations for this state are given by

$$\frac{dU_P}{dt} = -\left(\frac{r\delta}{k\beta}U_P + \frac{\alpha\delta}{\beta}U_A\right) \quad (11)$$

$$\frac{dU_A}{dt} = \frac{r}{\alpha k}(\beta k - \delta)U_P \quad (12)$$

Solving (11) and (12) numerically we get the following results under various cases.

Case 3.1: $U_{P0} < U_{A0}$ and $\beta k - \delta > 0$

Let $U_{P0} = 1, U_{A0} = 2, r = 0.7, \alpha = 0.6,$

$k = 1, \beta = 0.2, \delta = 0.1$

Here, the initial value of the perturbation of Plant species is less than that of Animal (Herbivore) species. And the efficiency of converting Plant biomass into Herbivore biomass (β) is greater than the death rate of Herbivores (δ), keeping the carrying capacity of nature for Plants (k) fixed at 1.

Under this assumption, the perturbation curve corresponding to the Herbivore species diverge from the Equilibrium point where as that of the Plant species converges to the Equilibrium point.

Hence this state is unstable under the present assumption.

These results are illustrated in Table 3.1.1 and Fig. 3.1.1

t	U_P	U_A	t	U_P	U_A
0	1.0000	2.0000	3.0	--	2.8381
0.5	0.68517	2.1201	3.5	--	3.0086
1.0	0.35404	2.2475	4.0	--	3.1893
1.5	0.00562	2.3825	4.5	--	3.3809
2.0	--	2.5256	5.0	--	3.5840
2.5	--	2.6773			

Table 3.1.1: $U_{P0} < U_{A0}$ and $\beta k - \delta > 0$

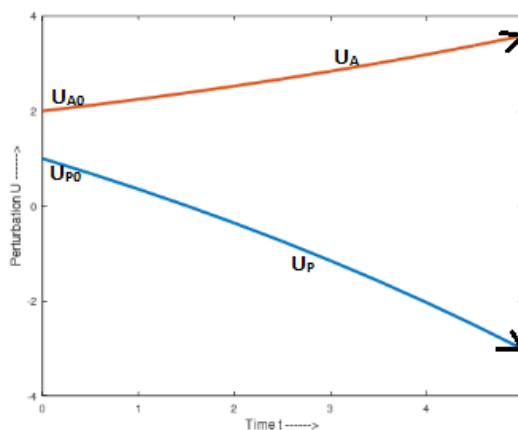


Fig. 3.1.1: $U_{P0} < U_{A0}$ and $\beta k - \delta > 0$

Case 3.2: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

Let $U_{P0} = 2, U_{A0} = 1, r = 0.7, \alpha = 0.6,$

$$k = 1, \beta = 0.3, \delta = 0.8$$

Here, the initial value of the perturbation of Plant species is greater than that of Animal (Herbivore) species.

And the efficiency of converting Plant biomass into Herbivore biomass (β) is less than the death rate of Herbivores (δ), keeping the carrying capacity of nature for Plants (k) fixed at 1.

Under these conditions, the perturbation curves corresponding to both Plant and Animal species converge to the Equilibrium position asymptotically.

Therefore, the current state is stable under this case.

These results are illustrated in Table 3.2.1 and Figure 3.2.1 below.

t	U_P	U_A	t	U_P	U_A
0	2.0000	1.0000	3.0	--	0.1737
0.5	1.1746	0.7470	3.5	--	0.1298
1.0	0.5838	0.5580	4.0	--	0.0969
1.5	0.1661	0.4168	4.5	--	0.0724
2.0	--	0.3114	5.0	--	0.0541
2.5	--	0.2326			

Table 3.2.1: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

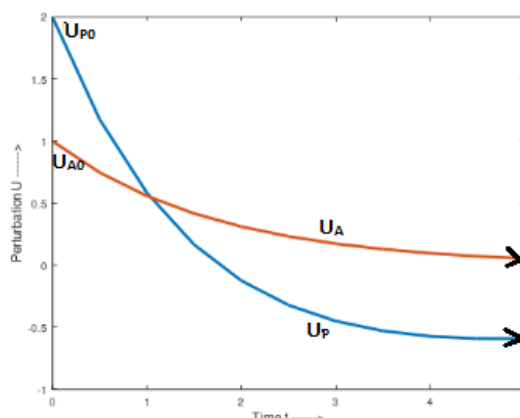


Fig. 3.2.1: $U_{P0} > U_{A0}$ and $\beta k - \delta < 0$

6. Results and Discussion

This research sheds light on the complex dynamics of riparian plant-animal interactions and their potential impact on ecosystem stability. It is reported that predator-prey dynamics drive periodic fluctuations in field populations. The result analyses the vegetation dynamics of Riparian zones to measure the broader perspective of riparian ecosystem interface. It also reports the temporal evolution of plant abundance concerning riparian ecosystems' adaptation and variability strategies.

In this research, to examine the stability of the dynamics of the Plant-Animal interactions along the riparian zones, numerical methods are used. These methods which are used to solve the systems of differential equations numerically are well proven in terms of their robustness and accuracy. The results thus obtained clearly establish and illustrate the conditions for stability.

As observed from various cases of analysis, it can be inferred that the Equilibrium state 1 is Unstable under all the cases and Equilibrium states 2 and 3 are asymptotically stable when $\beta k - \delta < 0$ and unstable when $\beta k - \delta > 0$.

That is, as long as the environment is conducive for the Herbivores to flourish by feeding themselves on plants, their population increases much more compared to that of the plants and thus the system becomes unstable. This is indicated by the fact that when the efficiency of converting Plant biomass into Herbivore biomass (β) is less than the death rate of Herbivores (δ), the system becomes asymptotically stable whereas, when this condition is reversed, the system becomes unstable.

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