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Introduction to RG-Closed Type Sets in Topological Ordered Spaces

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Abstract:

This research paper presents a novel concept in the field of topological ordered spaces, which is the introduction of a new class of sets called "rg-closed sets". This new class of sets is formed by generalizing closed sets using rg-open sets in topological ordered spaces. Notably, rg-closed sets strictly lie between the classes of closed sets and rg-closed collections in topological ordered spaces. Additionally, the article also covers a discussion on rg* closed sets.

Keywords: Topological ordered space, rg-closed set (irg, drg, brg-closed sets), r*g*-closed set (ir*g*, dr*g*, br*g*-closed sets).

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1. INTRODUCTION

The first research on topological ordered spaces was conducted by Leopoldo Nachbin [1]. In 1970, Levine [19] invented a superclass of sets known as rg-closed sets. Later, M. K. R. S. Veera Kumar introduced a novel category of sets [14], in the year 2014 G. Srinivasa Rao et.al [4-6 &19-25] studied and explained g-closed and g^*-closed sets in topological ordered space, which should be placed before the rg-closed sets and closed set classes. This new class of sets was not only different but also significant. In 2001, M. K. R. S. Veera Kumar presented research on i-closed, d-closed, and b-closed sets, which were introduced for the first time.

A topological ordered space is referred to as a triple $(X, \tau, \leq)$, where X is a non-empty set, τ is a topology on X and $\leq$ is a partial order on X.

Definition 1.1[5]: For any $a \epsilon X, \{b \epsilon X / a \leq b\}$ will be represented by $[a, \rightarrow]$. If P = i(P), where i(P) = $\bigcup_{a \epsilon P}[a, \rightarrow]$, then a subset P of a topological ordered space $(X, \tau, \leq)$ is known to be increasing.

Definition 1.2[5]: For any $a \epsilon X, \{b \epsilon X / b \leq a\}$ will be represented by $[\leftarrow, a]$. If P = d(P), where d(P) = $\bigcup_{a \epsilon P}[\leftarrow, a]$, then a subset P of a topological ordered space $(X, \tau, \leq)$ is known to be decreasing.

An increasing (resp. a decreasing) set is the complement of a decreasing (resp. an increasing) set. C(P) denotes the complement of ‘P’ in X.

dcl (P) = $\cap\{$F/F is a decreasing closed subset of X containing P with F = d(F)$\}$.

icl (P) = $\cap\{$F/F is an increasing closed subset of X containing P with F = i(F)$\}$.

bcl (P) = $\cap\{$F/F is a closed subset of X containing P with F=i(F)=d(F)$\}$.

IO(X) (resp. DO(X), BO(X)) represents the set of all decreasing (or increasing, both increasing and decreasing) open subsets of a topological ordered space$(X, \tau, \leq)$.

For a subset P of a space$(X,\ \tau,\leq)$, cl(P) (resp. icl(P), bcl(P)) denote the decreasing (resp. increasing, both increasing and decreasing) closure of P.

2. Topological ordered space with rg-closed sets

Definition 2.1: A topological space (X,τ) has a subset P is known as rg-closed [29] set, if cl(P) ⊆ R whenever P ⊆ R and R is regular open in (X,τ).

Definition 2.2: A topological space (X,τ)has a subset P is called r*g*-closed set [29], if rcl(P) ⊆ R whenever P ⊆ R and R is g-open in (X,τ).

Theorem 2.3: Every r*g*-closed set is a rg-closed set.

Proof: suppose P ⊆ R and R is regular open. Now, R is regular open ⟹ R is open.

W. k. t every closed set is a g-closed set. So, every open set is a g-open set. Since, R is open we have R is g-open. Therefore, rcl(P) ⊆ R, whenever P ⊆ R and R is g-open. Since P ⊆ R, there exist an open set G we have P ⊆ G ⊆ cl(G) ⊆ R. Since P ⊆ cl(G) ⇒ cl(P) ⊆ cl(cl(G)) = cl(G) ⊆ R. ⇒cl(P) ⊆ R. Therefore cl (P) ⊆ R, whenever P ⊆ R and R is regular open.

The following illustration demonstrates that a rg-closed set does not always have to be a r*g*-closed set.

Example 2.4: Let X = $\{p, q, r\}$,τ = $\{\emptyset, X, \{p\},\{q\},\{p, q\}\}$ and ≤ = $\{(p, p), (q, q),(r, r),(p, q),(q, r), (p, r)\}$. Clearly, a topological ordered space is $(X,\tau,\leq)$. r*g*-closed sets are $\emptyset$, X, $\{r\}$, $\{q, r\}$, $\{p, r\}$. rg-closed sets are $\emptyset$, X, $\{r\}$, $\{p, q\}$, $\{q, r\}$, $\{p, r\}$. Let P = $\{p, q\}$. Clearly, A is rg-closed set but not r*g*-closed set.

3. Results between i(r*g*), d(r*g*) and b(r*g*) closed type sets

Here are some definitions that we introduce:

Definition 3.1: If ircl(P) ⊆ R whenever P ⊆ R and R is g-open in (X,τ), then a subset P of $(X,\tau,\leq)$ is called i(r*g*)-closed set.

Definition 3.2: if drcl (P) ⊆ R whenever P ⊆ R and R is g-open in (X,τ), then a subset P of $(X,\tau,\leq)$ is called d(r*g*)-closed set.

Definition 3.3: if brcl (P) ⊆ R whenever P ⊆ R and R is g-open in (X,τ), then a subset P of $(X,\tau,\leq)$ is called b(r*g*)-closed set.

Theorem 3.4: Every i(r*g*)-closed set is an i(rg)-closed set.

Proof: As far as we know, every r*g*-closed set is a rg-closed set. Therefore, every i(r*g*)-closed set is a i(rg)-closed set. In general, the following illustration demonstrates that, a i(rg)-closed set need not be an i(r*g*)-closed set.

Example 3.5: Let X = $\{p, q, r\}$, τ = $\{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$ and ≤ = $\{(p, p), (q, q), (r, r), (p, q), (q, r)\}$. Clearly $(X,\tau,\leq)$ is a topological ordered space. i(r*g*)-closed sets are $\emptyset$, X, $\{q, r\}$. i(rg)-closed sets are $\emptyset$, X, $\{r\}$, $\{q, r\}$. Let P = $\{r\}$. Clearly P is i(rg)-closed set but not i(r*g*)-closed set.

Theorem 3.6: Every d(r*g*)-closed set is an d(rg)-closed set.

Proof: We know, every r*g*-closed set is an rg-closed set. Thus, every d(r*g*)-closed set is an d(rg)-closed set. The following illustration demonstrates that, a d(rg)-closed set need not always be an d(r*g*)-closed set.

Example 3.7: Let X = $\{p, q, r\}$, τ = $\{\emptyset, X, \{p, q\}\}$ and ≤ = $\{(p, p), (q, q), (r, r), (p, q), (p, r)\}$. Clearly, a topological ordered space is $(X,\tau,\leq)$. $\emptyset$, X, $\{p, r\}$ are d(r*g*)-closed sets. $\emptyset$, X, $\{p\}$, $\{p,$

$q\}$,$\{p, r\}$ are d(rg)-closed sets. Let P = $\{p\}$. Clearly, P is a d(rg)-closed set but not a d(r*g*)-closed set.

Theorem 3.8: Every b(r*g*)-closed set is a b(rg)-closed set.

Proof: As far as we know, every r*g*-closed set is a rg-closed set. Thus, every b(r*g*)-closed set is a b(rg)-closed set. The following illustration demonstrates that, a b(rg)-closed set does not always have to be a b(r*g*)-closed set.

Example 3.9: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q), (r, q)\}$. Clearly, $(X, \tau, \leq)$ is a topological ordered space. $\emptyset$, X are b(r*g*)-closed sets. $\emptyset$, X, $\{p, q\}$ are b(rg)-closed sets. Let P = $\{p, q\}$. Clearly, P is a b(rg)-closed set but not a b(r*g*)-closed set.

Theorem 3.10: Every b(r*g*)-closed set is i(r*g*)-closed set.

Proof: As far as we know, a balanced set is always an increasing set. Then, all b(r*g*)-closed sets are i(r*g*)-closed sets. The following illustration demonstrates that, an i(r*g*)-closed set need not always be a b(r*g*)-closed set.

Example 3.11: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (q, p), (r, q), (r, p)\}$. Clearly, a topological ordered space is $(X, \tau, \leq)$. b(r*g*)-closed sets are $\emptyset$, X. i(r*g*)-closed sets are $\emptyset$, X, $\{p, q\}$. Let P = $\{p, q\}$. Clearly, P is i(r*g*)-closed set but not be a b(r*g*)-closed set.

Theorem 3.12: Every b(r*g*)-closed set is a d(r*g*)-closed set.

Proof: As we know, every balanced set is a decreasing set. Any set that is b(r*g*)-closed is also be an d(r*g*)-closed. The following illustration demonstrates that, a d(r*g*)-closed set does not always have to be a b(r*g*)-closed set.

Example 3.13: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (q, p), (r, q), (r, p)\}$. Clearly, $(X, \tau, \leq)$ is a topological ordered space. b(r*g*)-closed sets are $\emptyset$, X. d(r*g*)-closed sets are $\emptyset$, X, $\{q, r\}$. Let P = $\{q, r\}$. Clearly, P is a d(r*g*)-closed set but not a b(r*g*)-closed set.

Theorem 3.14: i(r*g*)-closed and d(r*g*)-closed are independent notions. The following example will demonstrate this.

Example 3.15: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}$. Clearly, $(X, \tau, \leq)$ is a topological ordered space. i(r*g*)-closed sets are $\emptyset$, X. d(r*g*)-closed sets are $\emptyset$, X, $\{q, r\}$. Let P = $\{q, r\}$. Clearly, P is a d(r*g*)-closed set but not be an i(r*g*)-closed set.

Example 3.16: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (q, p), (r, q), (r, p)\}$. Clearly, a topological ordered space is $(X, \tau, \leq)$. i(r*g*)-closed sets are $\emptyset$, X, $\{q, r\}$. d(r*g*)-closed sets are $\emptyset$, X. Le t P = $\{q, r\}$. Clearly, P is i(r*g*)-closed set but not be a d(r*g*)-closed set.

Theorem 3.17: Every b(rg)-closed set is an i(rg)-closed set.

Proof: As we know, a balanced set is always an increasing set. Hence, all b(rg)-closed sets are i(rg)-closed sets. Generally the next example demonstrates that, i(rg)-closed sets do not have to be b(rg)-closed sets.

Example 3.18: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q), (r, q)\}$. Clearly, $(X, \tau, \leq)$ is a topological ordered space. b(rg)-closed sets are $\emptyset$, X, $\{p, q\}$. i(rg)-closed sets are $\emptyset$, X, $\{q\}$, $\{p, q\}$, $\{q, r\}$. Let P = $\{p\}$. Clearly, P is i(rg)-closed set but not a b(rg)-closed set.

Theorem 3.19: Every b(rg)-closed set is a d(rg)-closed set.

Proof: Every balanced set is a decreasing set, as we are aware. Every b(rg)-closed set is a d(rg)-closed set, hence this is true. The following illustration demonstrates that, a d(rg)-closed set does not always have to be a b(rg)-closed set.

Example 3.20: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{p, r\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (q, r), (p, r)\}$. Clearly $(X, \tau, \leq)$ is a topological ordered space. b(rg)-closed sets are $\emptyset$, X. d(rg)-closed sets are $\emptyset$, X, $\{q\}$, $\{p, q\}$. Let P = $\{q\}$. Clearly P is a d(rg)-closed set but not a b(rg)-closed set.

Theorem 3.21: i(rg)-closed and d(rg)-closed are independent notions. This will be demonstrated by the example that follows.

Example 3.22: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{q\}, \{p, q\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}$. Clearly, a topological ordered space is $(X, \tau, \leq)$. i(rg)-closed sets are $\emptyset$, X, $\{r\}$, $\{q, r\}$. d(rg)-closed sets are $\emptyset$, X, $\{p, q\}$, $\{p, r\}$. Let P = $\{p, r\}$. Clearly, P is a d(rg)-closed set but not i(rg)-closed set. Let Q = $\{r\}$. Clearly, Q is i(rg)-closed set but not a d(rg)-closed set.

4. Topological ordered space with irg-closed type sets

Theorem 4.1: A set P ∪ Q is irg-closed if P and Q are irg-closed sets.

Proof: If P ∪ Q ⊆ R and R is regular-open, then P ⊆ R and Q ⊆ R.

But P and Q are irg-closed and therefore icl(P) ⊆ R and icl(Q) ⊆ R.

Therefore, (icl (P) ∪ icl (Q)) ⊆ R ⟹ icl (P ∪ Q) ⊆ R.

Hence, P ∪ Q is irg-closed.

Example 4.2: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{q\}, \{p, q\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q)\}$. Clearly, a topological ordered space is $(X, \tau, \leq)$. Take, P = $\{r\}$, Q = $\{p, q\}$. If (P ∪ Q) = $\{r\} \cup \{p, q\} = \{p, q, r\}$ ⊆ R = X and R is regular-open, then $\{r\}$ ⊆ R and $\{p, q\}$ ⊆ R. But P and Q are irg-closed and therefore icl(P) ⊆ R and icl(Q) ⊆ R. Therefore, (icl(P) ∪ icl (Q)) ⊆ R, and hence icl(P ∪ Q) ⊆ R. Hence P ∪ Q is irg-closed.

Theorem 4.3: Suppose that Q ⊆ P ⊆ X, P is an ig-closed open subset of X and Q is an irg-closed set in relation to P. Then, Q is irg-closed with respect to X.

Proof: Let Q ⊆ R and let R be regular-open. We have Q ⊆ (P ∩ R).

But Q is an irg-closed set relative to P.

Hence $\mathrm{icl}_P(Q) \subseteq (P \cap R)$. →(1)

Note that P ∩ R is regular-open in P.

But $\mathrm{icl}_P(Q) = \mathrm{icl}(Q) \cap P$ →(2)

From (1) and (2),

(P ∩ icl(Q)) ⊆ (P ∩ R)

Consequently P ∩ icl (Q) ⊆ R. Hence, P ∩ (icl(Q) ∪ C(icl(Q))) ⊆ R ∪ C(icl(Q)).

That is P ∩ X ⊆ (R ∪ C(icl(Q))).

So P ⊆ (R ∪ C(icl(Q))) = G, say →(3)

But then G is an open set. Since P is ig-closed in X, from (3) we have

icl(P) ⊆ (R ∪ C(icl(Q))) = G →(4)

But icl(Q) ⊆ icl(P) →(5)

From (4) and (5) we have icl(Q) ⊆ (R ∪ C(icl(Q))).

Hence icl(Q) ⊆ R because icl(Q) ∩ C(icl(Q)) = ∅ ⟹ Q is irg-closed relative to X.

Corollary 4.4: Let P be an ig-closed, open set. Suppose that Q is an i-closed set. Then P ∩ Q is an irg-closed set relative to X.

Proof: We have that P ∩ Q is closed in P. Hence icl(P ∩ Q) = P ∩ Q in P. Let P ∩ Q ⊆ R, Where R is regular-open in P. That is icl(P ∩ Q) ⊆ R. Hence P ∩ Q is an irg-closed set in the ig-closed P. By the theorem [4.3], P ∩ Q is an irg-closed set relative to X.

Theorem 4.5: If a set P is irg-closed then icl(P)\P contains no nonempty regular-closed set.

Proof: Suppose that P is irg-closed. Let S be a regular-closed subset of icl(P)\P.

Then S ⊆ (icl(P) ∩ C(P)) and so P ⊆ C(S). But P is irg-closed.

Therefore icl(P) ⊆ C(S). → (1)

Consequently S ⊆ C(icl(P)) → (2)

We have already S ⊆ icl(P) → (3)

From (2) and (3), S ⊆ (icl(P) ∩ C(icl(P))) = ∅

Thus S = ∅. Therefore icl(P)\P contains no nonempty regular-closed set.

Corollary 4.6: Let P be an irg-closed set. If P is regular-closed then cl(int(P))\P is regular-closed.

Proof: Let P be an irg-closed. If P is regular-closed i.e., cl(int(P)) = P. Then cl(int(P))\P = P\P = ∅. But, ∅ is always regular-closed. As a result, cl(int(P))\P is regular-closed.

On the other hand, imagine that cl(int(P))\P is regular-closed. However, P is irg-closed. Additionally, the regular-closed set cl(int(P))\P is contained in icl(P)\P. By above theorem [4.5], cl(int(P))\P = ∅. Hence cl(int(P)) = P. Therefore P is regular-closed.

Theorem 4.7: If P is ig-closed then P is irg-closed.

Proof: Suppose that P ⊆ R, Where R is regular-open. Now R regular-open implies that R is open. Thus P ⊆ R and R is open. But P is ig-cosed. Hence icl(P) ⊆ R. Therefore, P is irg-closed.

The following illustration demonstrates that an irg-closed set need not always be an ig-closed set.

Example 4.8: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{q\}, \{p, q\}\}$ and ≤ = {(*p*, *p*), (*q*, *q*), (*r*, *r*), (*p*, *q*), (*r*, *q*)}. Clearly $(X, \tau, \leq)$ is a topological ordered space. irg-closed sets are ∅, *X*, {*p*, *q*}, {*q*, *r*}. ig-closed sets are ∅, *X*, {*q*, *r*}. Let P = {*p*, *q*}. Clearly P is an irg-closed set but not an ig-closed set.

Theorem 4.9: If P is irg-closed and P ⊆ Q ⊆ icl(P) then icl(Q)\Q contains no nonempty regular-closed set.

Proof: Suppose P is irg-closed and P ⊆ Q ⊆ icl(P).

Since P ⊆ Q ⟹ C(Q) ⊆ C(P) → (1)

Since Q ⊆ icl(P) ⟹ icl(Q) ⊆ icl(icl(P)) ⊆ icl(P) → (2)

That is icl(Q) ⊆ icl(P).

From (1) and (2), (icl(Q) ∩ C(Q)) ⊆ (icl(P) ∩ C(P)).

Which implies (icl(Q)\Q) ⊆ (icl(P)\P). Now P is irg-closed. Hence, icl(P)\P has no nonempty regular-closed subsets neither does icl(Q)\Q.

Theorem 4.10: Assume that P is irg-closed in X and P ⊆ Y ⊆ X. If Y is open in X, Then P is irg-closed relative to Y.

Proof: Assume that R is regular-open in X and that P ⊆ Y ∩ R. Therefore, P ⊆ R and hence icl(P) ⊆ R. It follows from this that (Y ∩ icl(P)) ⊆ Y ∩ R. Therefore P is irg-closed with respect to Y.

Theorem 4.11: Let X be a regular space. Prove that every compact subset of X is an irg-closed set.

Proof: Assume P R, where R is regular-open. R is open since it is regular-open right now. But in the typical space X, P is compact. Consequently, there exists an open set O in which P ⊆ O ⊆ cl(O) ⊆ R.

Since P ⊆ cl(O) ⟹ icl(P) ⊆ icl(cl(O)) = cl(cl(O)) = cl(O) ⊆ R. That is icl(P) ⊆ R. Hence P is irg-closed in X.

5. ir*g*- closed type sets in topological ordered spaces:

Theorem 5.1: A set P ∪ Q is ir*g*-closed if P and Q are ir*g*-closed sets.

Proof: If P ∪ Q ⊆ R and R is g-open, then P ⊆ R and Q ⊆ R. But P and Q are ir*g*-closed and therefore ircl (P) ⊆ R and ircl (Q) ⊆ R. Therefore, (ircl(P) ∪ ircl(Q)) ⊆ R, and hence ircl(P ∪ Q) ⊆ R. Hence P ∪ Q is ir*g*-closed.

Example 5.2: Let $X = \{p, q, r\}$, $\tau = \{\emptyset, X, \{p\}, \{q\}, \{p, q\}\}$ and $\leq = \{(p, p), (q, q), (r, r), (p, q), (q, r)\}$. Clearly $(X, \tau, \leq)$ is a topological ordered space. Take P =$\{r\}$, Q = $\{q, r\}$. If (P ∪ Q) =$\{r\}\cup\{q, r\}$ = $\{q, r\}$ ⊆ R = X and R is g-open, then $\{r\}$ ⊆ R and $\{p, q\}$ ⊆ R. But P and Q are ir*g*-closed and therefore ircl (P) ⊆ R and ircl (Q) ⊆ R. Therefore, (ircl(P) ∪ ircl(Q)) ⊆ R, and hence ircl(P ∪ Q) ⊆ R. Hence P ∪ Q is ir*g*-closed.

Theorem 5.3: If a set P is ir*g*-closed then ircl(P)\P contains no nonempty regular-closed set.

Proof: Suppose that P is ir*g*-closed. Let S be a regular-closed subset of ircl(P)\P.
Then S ⊆ (ircl(P) ∩ C(P)) and so P ⊆ C(S). But P is ir*g*-closed.
Therefore ircl(P) ⊆ C(S). →(1) Consequently S ⊆ C(ircl(P)) →(2)
We have already S ⊆ ircl(P) →(3). From (2) and (3) S ⊆ (ircl(P) ∩ C(ircl(P))) = ∅.
Thus S = ∅. Therefore, ircl (P)\P contains no nonempty regular-closed set.

Corollary 5.4: If P is an ir*g*-closed set, then P is regular-closed if and only if cl(int (P))\P is regular closed.

Proof: Make P an irg-closed. If cl(int(P)) = P, then P is regular-closed. If so, cl(int(P))\P = P\P = ∅. However, ∅ is always regular closed. cl(int(P))\P is hence regular-closed.

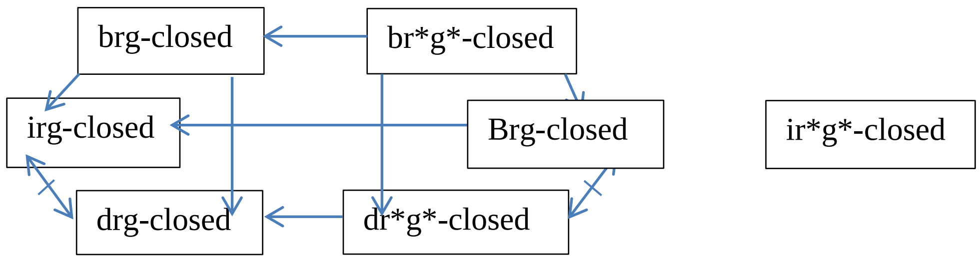
Assume, on the other hand, that cl(int(P))\P is regular-closed. P is, however, irg-closed. The regular-closed set cl(int(P))\P is also contained in ircl(P)\P. The statement "cl(int(P))\P = ∅." is based on the aforementioned theorem. As a result, cl(int(P)) = P. As a result, P is regular closed.

Theorem 5.5: In the event that P is ir*g*-closed and P ⊆ Q ⊆ ircl(P), then ircl(Q)\Q does not contain any nonempty regular-closed sets.

Proof: If P is ir*g*-closed and P ⊆ Q ⊆ ircl(P). Since C(P)⊆C(Q) follows from P⊆Q → (1)
Q ⊆ ircl(P) implies that ircl(Q) ⊆ ircl(ircl(P)) = ircl(P). In this case ircl(Q) ⊆ ircl(P) → (2)
From (1) & (2) (ircl(Q) ∩ C(Q)) ⊆ (ircl(P) ∩ C(P)) Implies (ircl(Q)\Q) ⊆ (ircl(P)\P).
P is now ir*g*-closed. As a result, neither ircl(P)\P nor ircl(Q)\Q have any nonempty regular-closed subsets.

Theorem 5.6: Let P ⊆ Y ⊆ X and suppose that P is ir*g*-closed in X. Then P is ir*g*-closed relative to Y, provided Y is open in X.

Proof: Let P ⊆ Y ∩ R and suppose that R is g-open in X. Then P ⊆ R and hence ircl(P) ⊆ R. This implies that (Y ∩ ircl(P)) ⊆ Y ∩ R. Thus P is ir*g*-closed relative to Y.



Conclusion: In this study, we provided (r*g*)*closed sets and (r*g*)*open sets and examined some of their properties. This class of sets can be used to study the ideas of continuity, compactness, and connectedness in different topological spaces, such as fuzzy and bi-topological ones.

References:

[1] A.Narmadha & Nagaveni, On regular b-open sets in Topological spaces, Int.Journal of math. Analysis, Vol. 7, 2013, No.19, 937-948.

[2] C. Mugundan, N. Nagaveni, A Weaker form of closed sets, 2011, 949-961.

[3] Y. Gnanambal, "On generalized pre-regular closed sets in Topological Spaces", Indian J. Pure App. Maths, 28(1997), 351-360.

[4] G. SRINIVASA RAO, et al. "g-CLOSED TYPE SETS AND g*-CLOSED TYPE SETS IN TOPOLOGICAL ORDERED SPACES.", 5(6)2014, 1276-1285.

[5] G. Srinivasarao., D. Madhusudanrao, and N. Srinivasarao. "Applications of ig, dg, bg-Closed type sets in topological ordered spaces".8(1) (2015), 12-22

[6] G. Srinivasarao, D. Madhusudanarao, and N. Srinivasarao. "Separation Axioms using ig*, dg*, bg*-Closed Type Sets in Topological Ordered Spaces". *International Journal of Advances in Engineering & Technology* 7.6 (2015): 1840-1850.

[7] K. Mariappa and S. Sekar, On Regular Generalised b-closed Set, Int. Journal of Math. Analysis, vol, 2013, No.13, 613-624.

[8] M. E. Abd El-Monsef, S. N. El. Deeb and R. A. Mohamoud, β open sets and β continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-80.

[9] M. K. R. S. Veerakumar, Between closed sets and g - closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 21 (2000) 1-19.

[10] M. K. R. S. Veera Kumar, g#-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ Ser. A., Math., 24(2003),1-13.

[11] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2)(1970),89-96.

[12] N. Meenakumari and T. Indira, r*g* closed sets in topological spaces, Annals of Pure and Applied Mathematics vol.6, No. 2, 2014, 125-132.

[13] N. Palaniappan & K. C. Rao, Regular generalized closed sets, Kyungpook Math. 3 (2) (1993), 211.

[14] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g** closed sets in Topological Spaces, IJMA 3 (5), (2012), 1-15.

[15] R. Devi, H. Maki, and K. Balachandran, Generalized α- closed maps and α generalized closed maps, Indian J. Pure. Appl. Math, 29(1)(1998), 37-49.

[16] Savithri. D & Janaki. C, On Regular Generalized closed sets in Topological spaces, IJMA- 4(4)2013, 162-169.

[17] S. P. Arya and T. M. Nour, characterizations of S normal spaces, Indian J. Pure app. Math, 21(1990)

[18] S. S. Benchelli and R. S. Wali, On rw-closed sets in topological spaces, Bull.malayas. math.soc.(2007),99-110.

[19] G. Srinivasa Rao, D. Madhusudhanarao and P. Siva Prasad, **Simple Ternary Semi-rings**, The Global Journal of Mathematics & Mathematical Sciences, 9(2) (2016), 185-196.

[20] D. Madhusudhana Rao, G. Srinivasa Rao, **Special Elements in ternary semi rings**, International Journal of Engineering Research and Applications, 4(11) (2014), 123-130.

[21] G. Srinivasa Rao, D. Madhusudhana Rao, **Structure of certain ideals in ternary semi rings**, Int. J. of Innovative Science and Modern Engg., 3(3) (2015), 49-56.

[22] G. Srinivasa Rao, D. Madhusudhana Rao, **A Study on Ternary Semi rings**, Int. J. of Math. Archive, 5(12) (2014), 24-30.

[23] G. Srinivasa Rao, D. Madhusudhana Rao, **Characteristics of Ternary Semi rings**, Int.J. of Engg. Res. and Mgt., 2(1) (2015), 3-6.

[24] G. Srinivasa Rao, A. Nagamalleswara Rao, P.L.N. Varma, D.Madhusudhana Rao, Ch. Ramprasad, **Prime Bi-interior ideals in TGSR**, Malaya Journal of Mathematika, Vol.9, No.1, pp:542-546, 2021.

[25] G. Srinivasa Rao, A. Nagamalleswara Rao, P.L.N. Varma, D. Madhusudhana Rao, Ch. Ramprasad, **Bi-interior ideals in TGSR**, Advances in Mathematics Scientific Journal, 10 (2021), No.3, pp: 1183-1195.