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Investigation on Anti-Fuzzy Tssemiring of a Ter.semi-ring

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Abstract:

In this article, we scrutinise the algebraic characteristics of the anti-fuzzy ternary sub-semi-ring of a ternary semi-ring and explore many theorems inside it.

Keywords: Fuzzy set, fuzzy ternary sub-semi-ring, anti-fuzzy ternary sub-semi-ring, anti-fuzzy normal ternary sub-semi-ring, homomorphism, anti-homomorphism, isomorphism, anti-isomorphism.

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1. INTRODUCTION

Numerous ideas exist for universal algebras that extend an associative ring (R, +,..). Several near-rings and semi-ring types, in particular, have shown to be quite beneficial. If both (T, +) and (T,.) are commutative semi-groups with + being a binary operation and. being a ternary multiplication fulfilling $a(b+c)d = abd+acd$, $ab(c + d)= abc + abd$, $(a+b)cd = abd + bcd$, $\forall$ a, b, c, d $\in$ T, then an algebra (T, +,.) is considered a ter.semi-ring. Elaborately ternary semi-ring was researched by G. Srinivasa Rao et al. [10–14]. This paper delves into a few theorems related to the anti-fuzzy ter.sub-semi-ring of a semi-ring.

2. PRELIMINARIES

Def.2.1: Let P $\neq \emptyset$. A fuzzy subset χ of P is a mapping χ : P $\rightarrow$ [0, 1].

Def.2.2: Let T be a ter.semi-ring. A fuzzy subset S of T is said to be a fuzzy ter.sub-semi-ring (FTSSR) of T if (i) $\chi_S(a + b) \geq \min\{\chi_S(a), \chi_S(b)\}$,(ii) $\chi_S(abc) \geq \min\{\chi_S(a), \chi_S(b), \chi_S(c)\}$, $\forall$ a, b, c$\in$T.

Ex.:2.3: Let Z be a ring of integers and S = (Z\N) ⊆ Z , set of all negative integers with zero. Then (Z\N, +, .) forms a ternary semi-ring S with zero with respect to binary addition and ternary multiplication. Define a fuzzy subset A: Z ⟶ [0, 1], we have

$$A(x) = \begin{cases} 1, & if\ x \in (Z\backslash N) \\ 0, & otherwise \end{cases}$$

Then A(x) is a fuzzy ternary sub-semi-ring of S.

Ex.2.4: Consider the set of integers modulo 5, non-positive integers Z_5^- = {0, -1, -2, -3, -4}under the usual addition and ternary multiplication, we have

+	0	-1	-2	-3	-4
0	0	-1	-2	-3	-4
-1	-1	-2	-3	-4	0
-2	-2	-3	-4	0	-4
-3	-3	-4	0	-1	-2
-4	-4	0	-1	-2	-3

.	0	-1	-2	-3	-4
0	0	0	0	0	0
-1	0	1	2	3	4
-2	0	2	4	1	3
-3	0	3	1	4	2
-4	0	4	3	2	1

Clearly (Z_5^-, +, **.**) is a ternary semi ring. Let a fuzzy set $\lambda : Z_5^- \longrightarrow [0, 1]$ be defined as λ (0) = 1, λ (−1) = 0.3, λ (−2) = 1, λ (−3) = 0.3, λ (−4) = 1 and λ (−5) = 0.3. Thus (Z_5^-, +, **.**, λ) is a fuzzy ternary semi-ring.

Ex.2.5: Let R = N, be the set of all-natural numbers with zero and Γ = {0, 1}. Define a mapping R × Γ × R × Γ × R ⟶ R as *aαbβc* by ternary multiplication of *a*, *α*, *b*, *β*, *c* for all *a*, *b*, *c*∈ R and α, β ∈ Γ. Clearly R is a ternary gamma semi-ring. Define : R ⟶ [0, 1] as

$$\lambda(x) = \begin{cases} 0.4, & if\ x\ is\ odd \\ 0.6, & if\ is\ even \\ 0.7, & if\ x = 0 \end{cases}$$

Clearly λ is a fuzzy ternary gamma semi-ring.

Ex.2.6: Let R = [0, 1], Γ = N. Define + and ternary multiplication '**.** ' defined as *a* + *b* = max {*a*, *b*} and *aαbβc* = min {*aαbβc*}

Def.2.7: Let T be a ter.semi-ring. A fuzzy subset S of T is said to be an anti-fuzzy ter.sub-semi-ring (AFSSR) of T when (i) $\chi_S(a+b) \leq \max\{\chi_S(a), \chi_S(b)\}$,(ii) $\chi_S(abc) \leq \max\{\chi_S(a), \chi_S(b), \chi_S(c)\}$, $\forall$ a, b, c$\in$T.

Def.2.8: Let T be a ter.semi-ring. An anti-fuzzy ter.sub-semi-ring (AFTSSR) S of T is said to be an anti-fuzzy normal ter.sub-semi-ring (AFNSSR) of T if it satisfies the following conditions:

(i) $\chi_S(a+b) = \chi_S(b+a)$, (ii) $\chi_S(abc) = \chi_S(cba)$, $\forall$ a, b, c$\in$T.

Def.2.9: Let (T, +, .) and (T_1, +, .) be any two ter.semi-rings(TSR). Let h: T →T_1 be any function and P be an AFTSSR in T, U be an AFTSSR in h(T) = T_1, defined by χ_U (b) = $\inf_{a \in h^{-1}(b)} \chi_U(a)$, for all *a* in T and *b* in T_1. Then P is called a pre-image of U with respect to h and is denoted by h^{-1}(U).

Def.2.10: Let (T, +, .) and (T_1, +, .) be any two TSRs. A mapping from h: T →$T_1$is said to be a TSR homomorphism if h(*a* + *b*) = h(*a*) + h(*b*), h(*abc*) = h(*a*) h(*b*)h(*c*), $\forall$ *a*, *b*, *c*$\in$T.

Def.2.11: Let (T, +,.) and (T_1, +, .) be any two TSRs. A mapping from h: T →T_1 is said to be a TSR anti-homomorphism if h(*a* + *b*) = h(*a*) + h(*b*), h(*abc*) = h(*c*) h(*b*) h(*a*), $\forall$ *a*, *b*, *c*$\in$ T.

Def.2.12: Let (T, +,.) and (T_1, +, .) be any two TSRs. Function f: T $\to T_1$ is a TSR isomorphism, if f is homomorphism, one-to-one and onto.

Def.2.13: Let (T, +,.) and (T_1, +, .) be any two TSRs. Then the function *f*: T $\to T_1$ is a TSR is said to be a ter.semi-ring anti-isomorphism, if *f* is anti-homomorphism, one-to-one and onto.

Def.2.14: Let P be an AFTSSR of a ter.semi-ring (T, +, ·) and x in T. Then the pseudo anti-fuzzy coset $(x\mathrm{P})^{\mathrm{p}}$ is defined by $((x\chi_P)p)(u) = p(x\chi_P(u))$, for every u in T and for some p in P.

3. PROPERTIES OF ANTI-FUZZY TERNARY TER.SUBSEMIRING OF A TERNARY SEMIRING

Th.3.1: Union of any two AFTSSR of a ter.semi-ring T is an AFTSSR of T.

Pf.: Let P and Q be any two AFTSSRs of a ter.semi-ring T and *p* and *q* in T. Let P={(p, $\chi_P(\alpha)$)/α $\in$T}and Q={(q, $\chi_Q(\alpha)$)/$\alpha\in$T}and also let U = P$\bigcup$Q = {(α, $\chi_U(\alpha)$) /$\alpha\in$T}, where max{ $\chi_P(\alpha)$, $\chi_Q(\alpha)$} = $\chi_U(\alpha)$. Now, $\chi_U(\alpha+\beta)$ = max{ $\chi_P(\alpha+\beta)$, $\chi_Q(\alpha+\beta)$} ≤ max{max { $\chi_P(\alpha)$, $\chi_P(\beta)$ }, max{ $\chi_Q(\alpha)$, $\chi_Q(\beta)$}} = max{max{ $\chi_P(\alpha)$, $\chi_Q(\alpha)$ }, max{ $\chi_P(\beta)$, $\chi_Q(\beta)$ } } = max{ $\chi_U(\alpha)$, $\chi_U(\beta)$}. Therefore, $\chi_U(\alpha+\beta)$ ≤max{ $\chi_U(\alpha)$, $\chi_U(\beta)$}, $\forall\alpha,\beta\in R$. And, $\chi_U(\alpha\beta\gamma)$=max{ $\chi_P(\alpha\beta\gamma)$, $\chi_Q(\alpha\beta\gamma)$)} ≤ max{{max $\chi_P(\alpha)$, $\chi_P(\beta)$, $\chi_P(\gamma)$}, max{max $\chi_Q(\alpha)$, $\chi_Q(\beta)$, $\chi_Q(\gamma)$}}=max{max{ $\chi_P(\alpha)$, $\chi_Q(\alpha)$},max{ $\chi_P(\beta)$, $\chi_Q(\beta)$}, max{ $\chi_P(\gamma)$, $\chi_Q(\gamma)$}}= max{ $\chi_U(\alpha)$, $\chi_U(\beta)$, $\chi_U(\gamma)$}. Therefore, $\chi_U(\alpha\beta\gamma)$ ≤ max{ $\chi_U(\alpha)$, $\chi_U(\beta)$, $\chi_U(\gamma)$}, $\forall$ α, β, $\gamma\in$T. Therefore, U is an AFTSSR of a TSRT. Hence the union of any two AFTSSRs of a TSRT is an AFTSSR of T.

Th.3.2: The arbitrary union of a family of AFTSSRs of TSRT is an AFTSSR of T.

Pf.: Let {S_i :i$\in\Delta$ } be an arbitrary family of AFTSSRs of a TSRT and let P = $\bigcup_{i\in\Delta} S_i$. Let α, β and γ in T. Then, $\chi_P(\alpha+\beta) = \sup_{i\in\Delta}\chi_{S_i}(\alpha+\beta) \le \sup_{i\in\Delta}\max\{\chi_{S_i}(\alpha), \chi_{S_i}(\beta)\}$ =max{ $\sup_{i\in\Delta}\{\chi_{S_i}(\alpha)$, $\sup_{i\in\Delta}\{\chi_{S_i}(\beta)$ }= max{ $\chi_P(\alpha)$, $\chi_P(\beta)$}. Therefore, $\chi_P(\alpha+\beta)$ ≤ max{ $\chi_P(\alpha)$, $\chi_P(\beta)$}, $\forall$ α, β, $\gamma\in$ T. And, $\chi_P(\alpha\beta\gamma) = \sup_{i\in\Delta}\chi_{S_i}(\alpha\beta\gamma) \le \sup_{i\in\Delta}\max\{\chi_{S_i}(\alpha), \chi_{S_i}(\beta), \chi_{S_i}(\gamma)\}$}= max{ $\sup_{i\in\Delta}\{\chi_{S_i}(\alpha)$, $\sup_{i\in\Delta}\{\chi_{S_i}(\beta)\}$, $\sup_{i\in\Delta}\{\chi_{S_i}(\gamma)\}$}= max{ $\chi_P(\alpha)$, $\chi_P(\beta)$, $\chi_P(\gamma)$}, Therefore, $\chi_P(\alpha\beta\gamma)$≤ max{ $\chi_P(\alpha)$, $\chi_P(\beta)$, $\chi_P(\gamma)$ }, $\forall$ α, β, $\gamma\in$ T. That is, P is an AFTSSR of a TSRT. Thus, the union of a family of AFTSSRs of R is an AFTSSR of T.

Th.3.3: If P, Q and R, any AFTSSRs of the TSRs T_1, T_2 and T_3 respectively, then anti-product P × Q × R is an AFTSSR of T_1 x T_2 x T_3.

Proof: Let P, Q and R are any AFTSSRs of the TSRs T_1, T_2 and T_3 respectively. Let a_1, a_2 and a_3 be in T_1, b_1, b_2 and b_3 be in T_2, c_1, c_2 and c_3 be in T_3. Then (a_1, b_1, c_1), (a_2, b_2, c_2) and (a_3, b_3, c_3) are in

T_1 x T_2 x T_3. Now, $\chi_{P\times Q\times R}[(a_1,b_1,c_1)+(a_2,b_2,c_2)+(a_3,b_3,c_3)]$ = $\chi_{P\times Q\times R}[(a_1+a_2+a_3,b_1+b_2+b_3,c_1+c_2+c_3)]$ = max{ $\chi_P(a_1+a_2+a_3)$, $\chi_Q(b_1+b_2+b_3)$, $\chi_R(c_1+c_2+c_3)$ } $\leq$ max{max{ $\chi_P(a_1)$, $\chi_P(a_2)$, $\chi_P(a_3)$ },max{ $\chi_Q(b_1)$, $\chi_Q(b_2)$, $\chi_Q(b_3)$)}, max{ $\chi_R(c_1)$, $\chi_R(c_2)$, $\chi_R(c_3)$ } = max{max{ $\chi_P(a_1)$, $\chi_Q(b_1)$, $\chi_R(c_1)$ },max{ $\chi_P(a_2)$, $\chi_Q(b_2)$, $\chi_R(c_2)$ }, max{ $\chi_P(a_3)$, $\chi_Q(b_3)$, $\chi_R(c_3)$ } = max{ $\chi_{P\times Q\times R}(a_1,b_1,c_1)$, $\chi_{P\times Q\times R}(a_2,b_2,c_2)$, $\chi_{P\times Q\times R}(a_3,b_3,c_3)$ }. Therefore, $\chi_{P\times Q\times R}[(a_1+a_2+a_3,b_1+b_2+b_3,c_1+c_2+c_3)]$ $\leq$ max{ $\chi_{P\times Q\times R}(a_1,b_1,c_1)$, $\chi_{P\times Q\times R}(a_2,b_2,c_2)$, $\chi_{P\times Q\times R}(a_3,b_3,c_3)$ }. Also, $\chi_{P\times Q\times R}[(a_1,b_1,c_1)(a_2,b_2,c_2)(a_3,b_3,c_3)]$ = $\chi_{P\times Q\times R}(a_1a_2a_3,b_1b_2b_3,c_1c_2c_3)$ = max{ $\chi_P(a_1a_2a_3)$, $\chi_Q(b_1b_2b_3)$, $\chi_R(c_1c_2c_3)$ } $\leq$ max{max{ $\chi_P(a_1)$, $\chi_P(a_2)$, $\chi_P(a_3)$ },max{ $\chi_Q(b_1)$, $\chi_Q(b_2)$, $\chi_Q(b_3)$)}, max { $\chi_R(c_1)$, $\chi_R(c_2)$, $\chi_R(c_3)$ }} = max{max{ $\chi_P(a_1)$, $\chi_Q(b_1)$, $\chi_R(c_1)$ }, max{ max{ $\chi_P(a_2)$, $\chi_Q(b_2)$, $\chi_R(c_2)$ }, max{ $\chi_P(a_3)$, $\chi_Q(b_3)$, $\chi_R(c_3)$ }}= max{ $\chi_{P\times Q\times R}(a_1,b_1,c_1)$, $\chi_{P\times Q\times R}(a_2,b_2,c_2)$, $\chi_{P\times Q\times R}(a_3,b_3,c_3)$ }. Therefore, $\chi_{P\times Q\times R}[(a_1,b_1,c_1)(a_2,b_2,c_2)(a_3,b_3,c_3)]$ $\leq$ max{ $\chi_{P\times Q\times R}(a_1,b_1,c_1)$, $\chi_{P\times Q\times R}(a_2,b_2,c_2)$, $\chi_{P\times Q\times R}(a_3,b_3,c_3)$ }. Hence P × Q × R is an AFTSSR of TSR of T_1 x T_2 x T_3.

Th.3.4: P is an AFTSSR of T if and only if U is an AFTSSR of T×T×T, when P is a fuzzy subset of a TSRT and U is a strongest anti-fuzzy relation of T.

Pf.: Given that P is an AFTSSR of a TSR *T*. Then for any $a = (a_1, b_1, c_1)$, $b = (a_2, b_2, c_2)$ and $c=(a_3, b_3, c_3)$, are in T x T x T. We have, $\chi_U(a+b) = \chi_U[(a_1, b_1, c_1)+(a_2, b_2, c_2)]$ $= \chi_U(a_1+a_2,b_1+b_2,c_1+c_2)$= max $\chi_U(a_1+a_2,b_1+b_2,c_1+c_2)$)} $\leq$ max{max{ $\chi_U(a_1)$, $\chi_U(a_2)$ },max{ $\chi_U(b_1)$, $\chi_U(b_2)$)} max{ $\chi_U(c_1)$, $\chi_U(c_2)$ }}= max{max{ $\chi_U(a_1)$, $\chi_U(a_2)$ },max{ $\chi_U(b_1)$, $\chi_U(b_2)$)}, max{ $\chi_U(c_1)$, $\chi_U(c_2)$ }} = max{ $\chi_U(a_1,b_1,c_1)$, $\chi_U(a_2,b_2,c_2)$)} = max{ $\chi_U(a)$, $\chi_U(b)$ }. Therefore, $\chi_U(a+b)$ $\leq$ max{ $\chi_U(a)$, $\chi_U(b)$ }, $\forall$ *a*, *b*, *c* $\in$ T × T × T. And, $\chi_U(abc)$ = $\chi_U[(a_1,b_1,c_1)(a_2,b_2,c_2)(a_3,b_3,c_3)]$ = $\chi_U(a_1a_2a_3,b_1b_2b_3,c_1c_2c_3)$ = max { $\chi_U(a_1a_2a_3)$, $\chi_U(b_1b_2b_3)$, $\chi_U(c_1c_2c_3)$ } $\leq$ max {max{ $\chi_U(a_1)$, $\chi_U(a_2)$, $\chi_U(a_3)$ }, max{ $\chi_U(b_1)$, $\chi_U(b_2)$, $\chi_U(b_3)$ }, max{ $\chi_U(c_1)$, $\chi_U(c_2)$, $\chi_U(c_3)$ }} = max{{max{ $\chi_U(a_1)$, $\chi_U(a_2)$, $\chi_U(a_3)$ }, max{ $\chi_U(b_1)$, $\chi_U(b_2)$, $\chi_U(b_3)$ }, max{ $\chi_U(c_1)$, $\chi_U(c_2)$, $\chi_U(c_3)$ }} = max{ $\chi_U(a_1,b_1,c_1)$, $\chi_U(a_2,b_2,c_2)$, $\chi_U(a_3,b_3,c_3)$ }= max{ $\chi_U(a)$, $\chi_U(b)$, $\chi_U(c)$ }. Therefore, $\chi_U(abc) \leq$ max{ $\chi_U(a)$, $\chi_U(b)$, $\chi_U(c)$ }, $\forall$ *a*, *b*, *c* $\in$ T×T×T. This proves that P is an AFTSSR of T×T×T. Conversely assume that U is an AFTSSR of T×T×T, then for any $a = (a_1, b_1, c_1)$, $b=(a_2, b_2, c_2)$ and $c=(a_3, b_3, c_3)$,are in T x T x T, we have max{ $\chi_U(a_1+a_2)$, $\chi_U(b_1+b_2)$, $\chi_U(c_1+c_2)$ } = $\chi_U[(a_1+b_1+c_1,a_2+b_2+c_2)]$= $\chi_U[(a_1, b_1, c_1)+(a_2, b_2, c_2)]$ $= \chi_U(a+b) \leq$ { $\chi_U(a)$, $\chi_U(b)$ } = max{ $\chi_U(a_1,b_1,c_1)$, $\chi_U(a_2,b_2,c_2)$) }= max{max{ $\chi_U(a_1)$, $\chi_U(b_1)$, $\chi_U(c_1)$ }, max{ $\chi_U(a_2)$, $\chi_U(b_2)$, $\chi_U(c_2)$ }. If $\chi_U(a_1+a_2) \geq \chi_U(b_1+b_2)$, $\Rightarrow$ $\chi_U(a_1) \geq \chi_U(b_1)$, $\chi_U(a_2) \geq \chi_U(b_2)$, we get, $\chi_U(a_1+a_2) \leq$ max{ $\chi_U(a_1)$, $\chi_U(a_2)$ }, $\forall$ a_1 and a_2 in T. And, max { $\chi_U(a_1a_2a_3)$, $\chi_U(b_1b_2b_3)$, $\chi_U(c_1c_2c_3)$ }= $\chi_U(a_1b_1c_1,a_2b_2c_2,a_3b_3c_3)$ =

$\chi_U[(a_1,b_1,c_1)(a_2,b_2,c_2)(a_3,b_3,c_3)]$ = $\chi_U(abc) \leq$ max{ $\chi_U(a)$, $\chi_U(b)$, $\chi_U(c)$ }= max{ $\chi_U(a_1,b_1,c_1)$, $\chi_U(a_2,b_2,c_2)$, $\chi_U(a_3,b_3,c_3)$ } =max{ max{ $\chi_U(a_1)$, $\chi_U(a_2)$, $\chi_U(a_3)$ }, max{ $\chi_U(b_1)$, $\chi_U(b_2)$, $\chi_U(b_3)$ }, max{ $\chi_U(c_1)$, $\chi_U(c_2)$, $\chi_U(c_3)$ }}.If $\chi_U(a_1a_2a_3) \geq \chi_U(b_1b_2b_3)$, $\chi_U(a_1) \geq \chi_U(b_1)$, $\chi_U(a_2) \geq \chi_U(b_2)$, $\chi_U(a_3) \geq \chi_U(b_3)$, we get $\chi_U(a_1a_2a_3) \leq$ max{ $\chi_U(a_1)$, $\chi_U(a_2)$, $\chi_U(a_3)$}, $\forall$ a_1, a_2, a_3 in T. Therefore, P is an AFTSSR of T.

Th.3.5: P is an AFTSSR of a TSR (T, +, ·) if and only if $\chi_U(\alpha+\beta) \leq$ max{ $\chi_U(\alpha)$, $\chi_U(\beta)$ }, $\chi_U(\alpha\beta\gamma) \leq$ max{ $\chi_U(\alpha)$, $\chi_U(\beta)$, $\chi_U(\gamma)$}, $\forall$ α, β and γ in T.

Proof: It is trivial.

Th.3.6: If P is an AFTSSR of a TSR (T, +, ·), then H = {α/ α∈T: $\chi_P(\alpha)$ =0} is either H=∅ or a ter.sub-semi-ring(TSSR) of T.

Pf.: If each element doesn't satisfies this condition, then H =∅. If α, β∈ H, then $\chi_U(\alpha+\beta) \leq$ max { $\chi_U(\alpha)$, $\chi_U(\beta)$ }= maximum{0, 0}=0. ∴ $\chi_U(\alpha+\beta)$ = 0. And, $\chi_U(\alpha\beta\gamma) \leq$ maximum{ $\chi_U(\alpha)$, $\chi_U(\beta)$, $\chi_U(\gamma)$ } = maximum{0,0,0} = 0. ∴ $\chi_U(\alpha\beta\gamma)$=0 ⇒ α + β, *αβγ* ∈ H. Therefore, H is a TSSR of T. Hence H=∅ or a TSSR of T.

Th.3.7:If P be an AFTSSR of a TSR (T, +, ·), then if $\chi_U(\alpha+\beta)$ = 1, then either $\chi_U(\alpha)$ =1 or $\chi_U(\beta)$ = 1, $\forall$ α, β in T.

Pf.: Let *α* and β in T. By the definition $\chi_U(\alpha+\beta) \leq$ max { $\chi_U(\alpha)$, $\chi_U(\beta)$ }, ⇒ 1 ≤ max { $\chi_U(\alpha)$, $\chi_U(\beta)$ }. Therefore, either $\chi_U(\alpha)$ = 1 or $\chi_U(\beta)$ = 1.

Th.3.8: Let P be an AFTSSR of a TSR T and f, an isomorphism from a TSRT onto S. Then P∘f is an AFTSSR of T.

Pf.: Let *α*, *β* and γ in T and P be an AFTSSR of a TSRT. Then we have, ($\chi_P \circ f$)(α + β) = χ_P (*f*(α + *β*)) = χ_P (*f*(*α*)+ *f*(*β*)) ≤ max { χ_P (*f*(*α*)), χ_P (*f*(β))} ≤ max{($\chi_P \circ f$) (α),($\chi_P \circ f$)(β)}, ⇒ ($\chi_P \circ f$) (α + *β*) ≤ max {($\chi_P \circ f$)(*α*),($\chi_P \circ f$)(*β*)}. And ($\chi_P \circ f$)(*αβγ*) = χ_P (*f*(*αβγ*)) = χ_P (*f*(*α*)*f*(β)*f*(γ)) ≤ max{ χ_P (*f*(α)), χ_P (*f*(β)), χ_P (*f*(γ))}≤ max {($\chi_P \circ f$)(α), ($\chi_P \circ f$)(*β*), ($\chi_P \circ f$)(γ)}⇒ ($\chi_P \circ f$)(*αβγ*) ≤ max{($\chi_P \circ f$)(α),($\chi_P \circ f$)(β), ($\chi_P \circ f$)(γ) }. Thus ($\chi_P \circ f$) is an AFTSSR of a TSRT.

Th.3.9: Let P be an AFTSSR of a TSRT and *h* be an anti-isomorphism from a TSRT onto S. Then $\chi_P \circ h$ is an AFTSSR of T.

Pf.: Let *a*, *b* and *c* in T and P be an AFTSSR of a TSRT. Then we have, ($\chi_P \circ h$)(*α* + *β*)= χ_P (h(*α* + *β*)) = χ_P (h(α*)* + h(β)) ≤ max{ χ_P (h(α)), χ_P (h(β)) } ≤ max{($\chi_P \circ h$) (α),($\chi_P \circ h$)(*β*)}, ⇒($\chi_P \circ h$)(*α* + *β*) ≤ max{($\chi_P \circ h$)(*α*),($\chi_P \circ h$)(*β*)}. And ($\chi_P \circ h$)(*αβγ*)= χ_P (h(*αβγ*))= χ_P (h(γ)h(β)h(α))

$\leq \max\{\chi_P(h(\alpha)), \chi_P(h(\beta), \chi_P(h(\gamma))\} \leq \max\{(\chi_P \circ h)(\alpha), (\chi_P \circ h)(\beta), (\chi_P \circ h)(\gamma)\}, \Rightarrow (\chi_P \circ h)(\alpha\beta\gamma) \leq \max\{(\chi_P \circ h)(\alpha), (\chi_P \circ h)(\beta), (\chi_P \circ h)(\gamma)\}$. Therefore, $\chi_P \circ h$ is AFTSSR of a TSRT.

Th.3.10: Suppose P be an AFTSSR of a TSR (T, +, .). The pseudo anti-fuzzy coset $(\chi_P)^p$ is an AFTSSR of a TSRT, for a in T.

Pf.: Let P be an AFTSSR of a TSR T. For every α, β and γ in T, we have, $((x\chi_P)^p)(\alpha+\beta) = p(x)\chi_P(\alpha+\beta) \leq p(x)\max\{\chi_P(\alpha), \chi_P(\beta)\} = \max\{p(x)\chi_P(\alpha), p(x)\chi_P(\beta)\} = \max\{((x\chi_P)^p)(\alpha), ((x\chi_P)^p)(\beta)\}$. $\therefore ((x\chi_P)^p)(\alpha+\beta) \leq \max\{((x\chi_P)^p)(\alpha), ((x\chi_P)^p)(\beta)\}$. Now, $((x\chi_P)^p)(\alpha\beta\gamma) = p(\alpha)\chi_P(\alpha\beta\gamma) \leq p(x)\max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\} = \max\{p(x)\chi_P(\alpha), p(x)\chi_P(\beta)\} = \max\{((x\chi_P)^p)(\alpha), ((x\chi_P)^p)(\beta), ((x\chi_P)^p)(\gamma)\}$.
$\therefore ((x\chi_P)^p)(\alpha\beta\gamma) \leq \max\{((x\chi_P)^p))(\alpha), ((x\chi_P)^p)(\beta), (((x\chi_P)^p)(\gamma)\}$. Hence $((x\chi_P)^p)$ is an AFTSSR of a TSRT.

Th.3.11: Let (T, +, .) and (T_1, +, .) be any two TSRs. The homomorphic image of an AFTSSR of T is an AFTSSR of T_1.

Pf.: Given (T, +, **.**) and (T_1, +, **.**) are two TSRs. Let $f: T \to T_1$ be a homomorphism. Then, $f(\alpha+\beta) = f(\alpha)+f(\beta)$ and $f(\alpha\beta\gamma) = f(\alpha)f(\beta)f(\gamma)$, $\forall$ α, β and γ in T. Let Q = f(P), where P is an AFTSSR of T. To prove, Q is an AFTSSR of T_1. Now, for $f(\alpha), f(\beta), f(\gamma)$ in T_1, $\chi_Q(f(\alpha)+f(\beta)) = \chi_Q(f(\alpha+\beta)) \leq \chi_P(\alpha+\beta) \leq \max\{\chi_P(\alpha), \chi_P(\beta)\} \Rightarrow \chi_Q(f(\alpha)+f(\beta)) \leq \max\{\chi_Q(f(\alpha)), \chi_Q(f(\beta))\}$. Again, $\chi_Q(f(\alpha)f(\beta)f(\gamma)) = \chi_Q(f(\alpha\beta\gamma)) \leq \chi_Q(\alpha\beta\gamma) \leq \max\{\chi_Q(\alpha), \chi_Q(\beta), \chi_Q(\gamma)\} \Rightarrow \chi_Q(f(\alpha)f(\beta)f(\gamma)) \leq \max\{\chi_Q(\alpha), \chi_Q(\beta), \chi_Q(\gamma)\}$. Hence Q is an AFTSSR of T_1.

Th.3.12: Let (T, +, **.**) and (T_1, +, **.**) be any two TSRs. The homomorphic pre-image of an AFTSSR of T_1 is an AFTSSR of T.

Pf.: Given (T, +, **.**) and (T_1, +, **.**) are two TSRs. Let $f: T \to T_1$ be a homomorphism. Then, $f(\alpha+\beta) = f(\alpha)+f(\beta)$ and $f(\alpha\beta\gamma) = f(\alpha)f(\beta)f(\gamma)$, $\forall \alpha, \beta, \gamma \in T$. Let Q = f(P), where P is an AFTSSR of T. We have to prove that P is AFTSSR of T. Let α, β and γ in T. Then, $\chi_P(\alpha+\beta) = \chi_Q(f(\alpha+\beta)) = \chi_Q(f(\alpha)+f(\beta)) \leq \max\{\chi_Q(f(\alpha)), \chi_Q(f(\beta))\} = \max\{\chi_P(\alpha), \chi_P(\beta)\} \Rightarrow \chi_P(\alpha+\beta) \leq \max\{\chi_P(\alpha), \chi_P(\beta)\}$. Again, $\chi_P(f(\alpha\beta\gamma)) = \chi_Q(f(\alpha\beta\gamma)) = \chi_Q(f(\alpha)f(\beta)f(\gamma)) \leq \max\{\chi_Q(\alpha), \chi_Q(\beta), \chi_Q(\gamma)\}\} = \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\} \Rightarrow \chi_P(f(\alpha\beta\gamma)) \leq \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\}$. Hence P is an AFTSSR of T.

Th.3.13: If (T, +, .) and (T_1, +, .) are two TSRs, then anti-homomorphic image of an AFTSSR of T is an AFTSSR of T_1.

Pf.: Given (T, +, .) and (T_1, +, .) are two TSRs. Let $f: T \to T_1$ a homomorphism. Then, $f(\alpha+\beta) = f(\beta)+f(\alpha)$ and $f(\alpha\beta\gamma) = f(\gamma)f(\beta)f(\alpha)$, $\forall \alpha, \beta, \gamma \in T$. Let Q = f(P), where P is an AFTSSR of T. We prove that Q is an AFTSSR of T_1. Now, for $f(\alpha), f(\beta), f(\gamma)$ in T_1, $\chi_Q(f(\alpha)+f(\beta)) = \chi_Q(f(\alpha+\beta)) \leq \chi_P(\beta+\alpha)$

$\le \max\{\chi_P(\beta), \chi_P(\alpha)\} = \max\{\chi_P(\alpha), \chi_P(\beta)\} \Rightarrow \chi_Q(f(\alpha) + f(\beta)) \le \max\{\chi_P(\alpha), \chi_P(\beta)\}$. Again, $\chi_Q(f(\alpha)f(\beta)f(\gamma)) = \chi_Q(f(\gamma\beta\alpha))) \le \chi_P(\gamma\beta\alpha) \le \max\{\chi_P(\gamma), \chi_P(\beta), \chi_P(\alpha)\} = \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\}$, $\Rightarrow \chi_Q(f(\alpha)f(\beta)f(\gamma)) \le \max\{\chi_Q(f(\alpha)), \chi_Q(f(\beta)), \chi_Q(f(\gamma))\}$. Hence Q is an AFTSSR of T_1.

Th.3.14: Let (T, +, .) and (T_1, +, .) be any two TSRs. The anti-homo-morphic pre-image of an AFTSSR of T_1 is an AFTSSR of T.

Pf.: Given (T, +, .) and (T_1, +, .) are two TSRs. Let $f: T \to T_1$ be a homomorphism. Then, $f(\alpha + \beta) = f(\beta) + f(\alpha)$ and $f(\alpha\beta\gamma) = f(\gamma)f(\beta)f(\alpha)$, $\forall \alpha, \beta, \gamma \in T$. Let Q = f(P), where P is an AFTSSR of T. We have to prove that P is an AFTSSR of T. Let α, β and γ in T. Then $\chi_P(\alpha + \beta) = \chi_Q(f(\alpha + \beta)) = \chi_Q(f(\alpha) + f(\beta)) \le \max\{\chi_Q(f(\gamma)), \chi_Q(f(\beta)), \chi_Q(f(\alpha))\} = \max\{\chi_Q(f(\alpha)), \chi_Q(f(\beta)), \chi_Q(f(\gamma))\} = \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\}$, $\Rightarrow \chi_P(\alpha + \beta) \le \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma)\}$. Again, $\chi_P((\alpha\beta\gamma)) = \chi_Q(f(\alpha\beta\gamma)) = \chi_Q(f(\gamma)f(\beta)f(\alpha)) \le \max\{\{\chi_Q(f(\gamma)), \chi_Q(f(\beta)), \chi_Q(f(\alpha))\} = \max\{\chi_Q(f(\alpha)), \chi_Q(f(\beta)), \chi_Q(f(\gamma))\} = \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma))\} \Rightarrow \chi_P((\alpha\beta\gamma)) \le \max\{\chi_P(\alpha), \chi_P(\beta), \chi_P(\gamma))\}$. Hence P is an AFTSSR of T.

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