

# Implementing Statistical Distribution for Control Chart Design in Higher Education Institutions

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**Abstract:**

In higher education institutions, continuous improvement of academic and administrative processes is crucial. This paper explores the use of statistical control charts for effective monitoring and management of these processes. By implementing control charts based on suitable statistical distributions, institutions can systematically track performance metrics such as exam scores, student attendance rates, graduation rates, and faculty performance. Choosing the right control chart is essential and depends on the data type. For continuous data, such as exam scores or time-to-graduation, X-bar and R (Mean and Range) charts or X-bar and S (Mean and Standard Deviation) charts are appropriate, as they assume a normal distribution and are useful for subgroup variations. For discrete data, like student absences or pass/fail outcomes, P-charts (Proportion charts) or U-charts (Defect per unit charts) are better suited, handling binomial or Poisson distributions effectively. Implementing control charts involves defining the process to be monitored, collecting and organizing data, selecting the suitable chart based on data type and distribution, and analyzing the chart to identify trends or outliers. This structured approach enables institutions to identify areas needing improvement, assess the impact of interventions, and make data-driven decisions. The use of statistical control charts facilitates proactive management by detecting issues before they escalate. For example, trends in declining exam scores or increasing absenteeism can be identified early, allowing for timely interventions. Monitoring faculty performance through control charts helps maintain high teaching standards and highlights areas for professional development. In conclusion, statistical control charts offer a structured, empirical approach to quality management in higher education. By aligning the choice of control charts with data characteristics, institutions can enhance their monitoring capabilities, leading to improved academic and administrative outcomes. This paper highlights the potential of data-driven management in fostering continuous improvement and excellence in higher education.

**Keywords:** Control Chart, Distribution, Higher Institution, Lower control limit, Upper control limit.

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## 1. INTRODUCTION

Under the socio-investment concept of economic growth, education and universities play a system-forming function. The core of this concept is that social investments, or those made to advance and realize human potential, are what power economic growth and progress (Makhalina et.al., 2020). The development of digital competences among the people, the training of digital personnel for business, lifelong learning, targeted, and corporate training to raise the level of qualification of employees of

firms are all manifestations of the socio-investment model of economic growth. All of this is done in accordance with universities (Liao et.al., 2022). Nonetheless, the higher education services offered by present universities fully adhere to all economic good criteria: they are exclusive, competitive in consumption, clearly divided, and contemplate fee collecting. Effectiveness is defined as the ability of a social investment to pay for itself in terms of economic growth and to be commercially appealing, competitive, and profitable (Johnes et.al, 2022).

Enshun Tian (2006) stated that both internal and external variables may have an impact on the course of higher education reform as globalization of the world economy and internationalization of higher education progress. Higher education has by far demonstrated a clear trend: more partners, including the government, higher education institutions, and society, participate in Quality Assurance and make decisions that can respect their needs and interests, and it is anticipated that a balance between autonomy and accountability will be reached. The establishment of a system of quality assurance in which universities autonomously guarantee their own quality, the government is responsible for macro management, the society participates in oversight, and market adjustment occurs naturally is crucial for China.

According to Vroeijenstijn (1995a), instruction and research should be evaluated independently. Several sorts of competence are needed for the evaluation of research and the evaluation of instruction and learning. While peers and subject-matter experts are required for research, experts for teaching and learning must have a comprehensive understanding of the area. Also, merging the evaluation of research projects with academic programs would require very large committees and take a lot longer on site visits. As higher education does not seek to generate graduates who are devoid of flaws, Watty (2003) contends that the notion of excellence as perfection can be dropped. According to small-scale research with a sample of senior managers at higher education institutions, Lomas (2001) suggests that suitability for purpose and transformation seem to be the two most appropriate criteria of quality.

The core of the described model and its key distinguishing characteristic are, on the one hand, the reliance on highly skilled and innovative human resources with the acceleration of economic growth rate and the humanistic treatment of economic growth, which is to serve society's and each individual's best interests (Bajraktari et.al., 2022). Yet, it is still unclear how much quality assurance and university administration will contribute to the adoption of the socio-investment model of economic development. Every year, new universities that are dynamically growing and forward-thinking are included to the global university rankings. For instance, Times Higher Education (THE) (2022) offers, Emerging Economies University Rankings 2022 and Young University Rankings 2022, in addition to the primary international university rankings. The level of elaboration related with the issue of this study is substantial, according to the assessed literature.

## **2. PROCESS CONTROL CHART**

In many scenarios, a single continuous random variable that is typically believed to follow a normal distribution can be used to describe the process quality. Yet, it is becoming more typical for processes to be defined by a number of variables that are typically connected Reynolds and Kim and Reynolds (2005). Statistical Process Control (SPC) permits the detection of a "out of control"

situation as well as the identification of various sources of variance (Besterfield, 1995). Using a function of time or sample size, a control chart "displays the value of the quality criteria of interest" (Montgomery, 2009). As a result, output should be the basis for determining the variability of a quality characteristic, which entails calculating its statistical distribution and parameters (Juran and Gryna, 1988). Murphy (1987) suggested a technique based on discriminant analysis to find the out-of-control variables. We can see this quality control strategy as trying to discern between the process of being in control or out of control. He split the entire set of variables into two subsets before attempting to pinpoint which one was responsible for the out of control signal.

It is crucial to identify the root cause of an out-of-control signal since the goal of multivariate statistical process control is to monitor the process over time and identify any unexpected events that would allow for quality and process improvement. Unlike to univariate control charts, multivariate control charts are more complex, and the cross-correlation between variables makes it challenging to analyze assignable explanations for the out-of-control signal. Because Hotteling (1947) acknowledged that the quality of a product can depend on a number of associated parameters in the 1940s, this has served as the foundation for extensive study in the field of multivariate control charts.

One of the key SPC tools for guaranteeing the excellent quality of the product is the control chart. Random shocks can happen to change the production process even though the product is produced at the target mean and variance. A control chart should a) promptly identify a process change and b) reduce false alarms when the process is genuinely in control state. The efficient control chart recognizes a slight variation in the mean and/or variance.

### 3. PROCESS OF THE CONTROL CHART

The Exponentiated Exponential Distribution (EED) as the quality characteristic of a failure time. Assuming the time between events represented by  $T$  which follows the EED with  $\beta$  the scale parameter and  $\theta$  the location parameter. Using the collected data, we estimate the parameters of the EED, namely the scale parameter  $\beta$  and the location parameter  $\theta$ . This can be done using maximum likelihood estimation or other statistical methods. Gupta and Kundu (2001).

The density function of EED is

$$f(t) = \beta\theta(1 - e^{-\theta x})^{\beta-1}e^{-\theta x}, \text{ for } t > 0 \tag{1}$$

**Practical example:** The plant has been recording the failure times (in months) of a sample of 30 temperature sensors 12, 14, 18, 20, 22, 24, 26, 28, 30, 32, 14, 15, 17, 19, 21, 23, 25, 27, 29, 31,

16, 17, 18, 19, 20, 21, 22, 23, 24, 25. Using statistical software, we estimate the parameters of the Exponentiated Exponential Distribution (EED) from the given failure time data. Let's assume we obtain the following parameter estimates: Scale Parameter ( $\beta$ ) = 2.5 and Location Parameter ( $\Theta$ ) = 0.05. With the estimated parameters, we model the failure times using the EED's probability density function:  $f(t)=2.5 \times 0.05 \times (1 - e^{-0.05t})^{2.5-1} e^{-0.05t}$ .

**Analysis and Prediction:** We can analyze the reliability of the temperature sensors and make predictions about their failure times using the EED model. For example, we can estimate the probability that a sensor will fail within a certain period or calculate the mean time to failure for the population of sensors.

**Quality Control:** Develop control charts based on the EED model to monitor the reliability of temperature sensors over time. Detect deviations from expected failure patterns and implement preventive maintenance strategies to minimize downtime.

**Decision Making:** Based on the analysis of failure times and reliability, make informed decisions about sensor replacement schedules, inventory management, and equipment maintenance to optimize plant operations and reduce costs.

This example demonstrates how the Exponentiated Exponential Distribution can be applied to model the failure times of industrial sensors and make data-driven decisions to ensure the reliability and efficiency of manufacturing processes.

The posterior belief of the higher education institute is taken in to consideration from (Nezhad and Niaki, 2010).

$$C(O_k) = C(T_k, O_{k-1}) = \frac{C(O_{k-1})T^* - \mu_{T^*}}{C(O_{k-1})T^* - \mu_{T^*} + (1 - C(O_{k-1}))} \quad (2)$$

From equation (2) the Mean and Standard deviation (SD) is calculated for  $T^*$ , by keeping  $\beta = 3.6$  making the distribution follows normal as stated by Nelson (1994). When the location parameter of EED is equal to 1 then,

$$\begin{aligned} \text{Mean} &= \theta_0^* \Gamma(1 + 0.28) \\ \text{SD} &= \theta_0^* \sqrt{\Gamma(1 + 0.56) - \Gamma(1 + 0.28)^2} \end{aligned}$$

The indicator when  $Z_k = C(O_k)/(1 - C(O_k))$  and for mean=0 and variance k which follows the normal distribution the proposed control chart process of  $\ln(Z_k)$  is given as

$$\ln(Z_k) = \sum_{i=1}^k \frac{T_i^* - \mu_{T_i^*}}{\sigma_{T_i^*}} \quad (3)$$

Therefore, Lower Control Limit (LCL) and Upper Control Limit (UCL) for the equation (3) is derived as

$$LCL = -L\sqrt{k} ; UCL = L\sqrt{k} \text{ and } L = \text{Control Limit}$$

When the process is in control we consider it as Average Run Length (ARL) and when the process is in out of control then we consider equation (4)

$$ARL = 1 - \Phi\left(\frac{UCL}{\sqrt{k}}\right) + \Phi\left(\frac{LCL}{\sqrt{k}}\right) \quad (4)$$

#### 4. RESULTS AND DISCUSSIONS

As a simulation data for other parameter values of ARL, we define ARL=50 as seen in Table 1, ARL=100 are observed in Table 2 and ARL=150 are reported in Table 3. We observe that as the ARL decreases randomly in the control chart process and the shift s increases as observed in Table 1, Table and Table 3. As the constant parameter k increases there is a decreasing in the trend as observed.

The minimum value found in the ARL=50 is 1 when the mean value increased to 1.7 and for  $k=30$ , and for  $k=20$  the minimum mean value observed as 1.8 as seen in Table 1. As the ARL=100 value increases, the same trend is observed as seen in Table 2 for all the  $k=10, 20$  and  $30$ . In Table 3, when the ARL value increased to 150, there also observed the decreasing trend for the increase in  $k$ .

Table 1: The value of ARL=50, when the process is in shift

$s$	$k = 10$	$k = 20$	$k = 30$
1	50	50	50
1.1	43.26	39.09	27.94
1.2	34.67	17.41	9.34
1.3	18.11	12.06	5.17
1.4	10.83	4.22	4.49
1.5	7.24	3.41	2.93
1.6	4.19	2.89	1.08
1.7	3.01	1.37	1.00
1.8	1.87	1.00	1.00
1.9	1.51	1.00	1.00
2.0	1.29	1.00	1.00

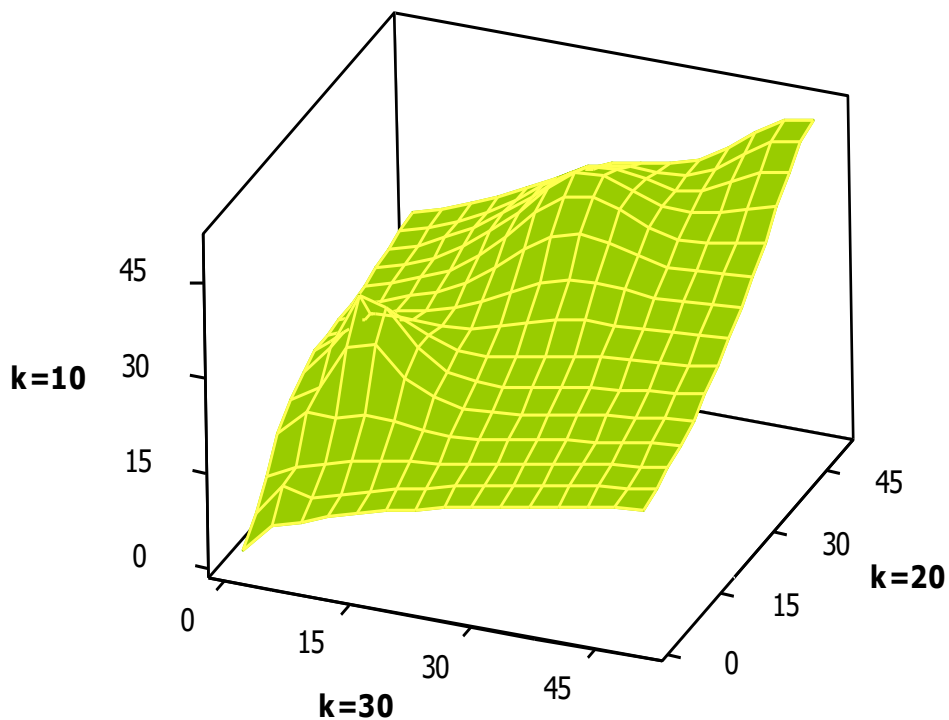


Figure 1: Trend values of  $k=10, 20$  and  $30$  when ARL=50

Table 2: The value of ARL=100, when the process is in shift

<b>s</b>	<b>k = 10</b>	<b>k = 20</b>	<b>k = 30</b>
1	100	100	100
1.1	71.40	65.93	59.39
1.2	43.02	37.24	25.67
1.3	25.23	19.59	12.11
1.4	11.58	9.13	4.24
1.5	8.97	5.51	3.07
1.6	6.05	3.36	1.93
1.7	4.36	1.91	1.00
1.8	3.07	1.00	1.00
1.9	1.92	1.00	1.00
2.0	1.60	1.00	1.00

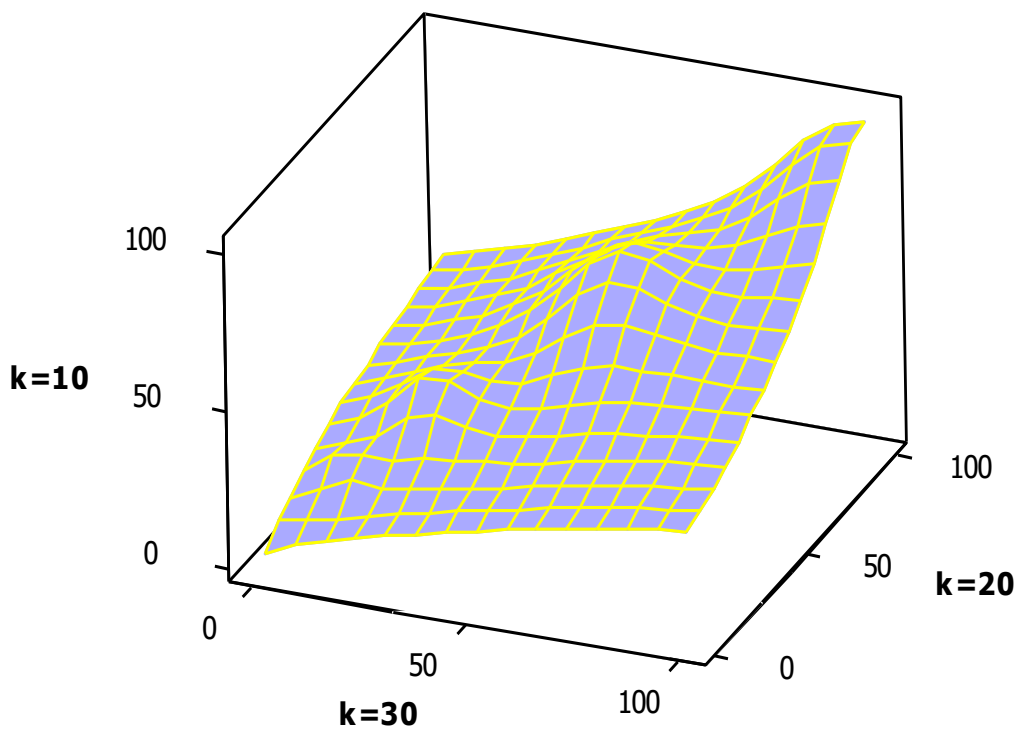


Figure 2: Trend values of k=10, 20 and 30 when ARL=100

Table 3: The value of ARL=150, when the process is in shift

<b>s</b>	<b>k = 10</b>	<b>k = 20</b>	<b>k = 30</b>
1	150	150	150
1.1	109.07	83.40	67.22
1.2	53.05	38.89	31.75
1.3	27.33	23.45	19.06
1.4	18.61	13.07	9.44

1.5	13.33	7.28	5.39
1.6	8.09	4.96	2.04
1.7	4.96	2.12	1.00
1.8	3.11	1.00	1.00
1.9	2.07	1.00	1.00
2.0	1.71	1.00	1.00

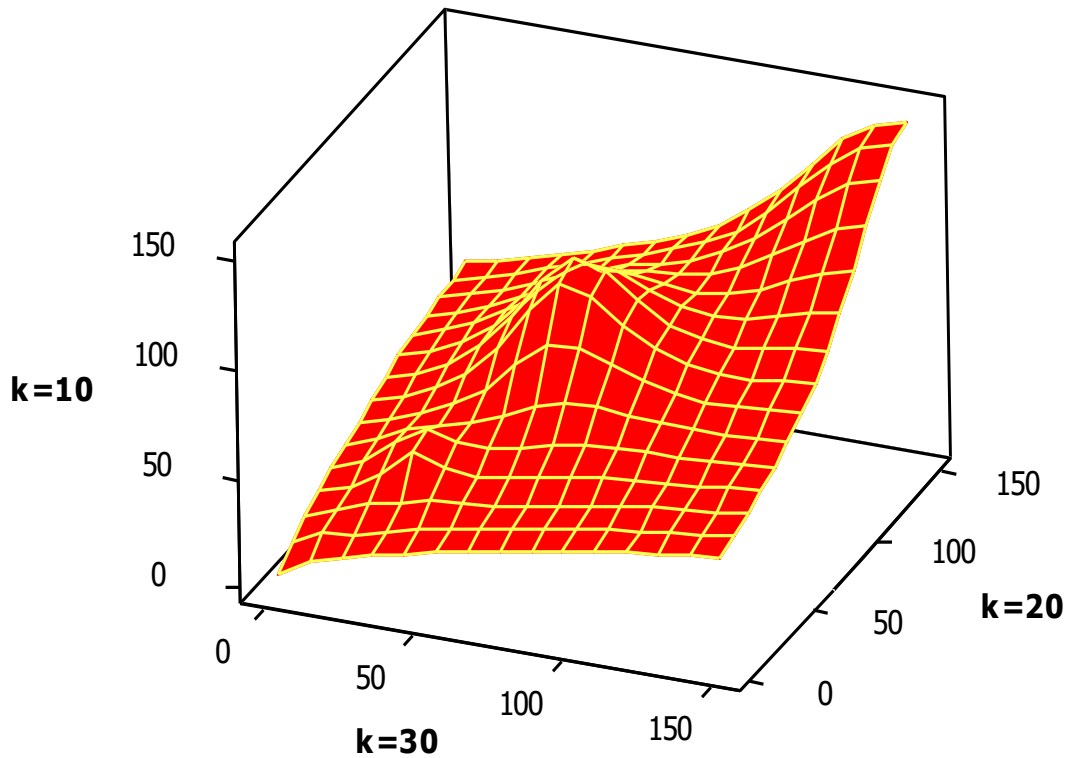


Figure 3: Trend values of k=10, 20 and 30 when ARL=150

**Using Python Programming code:**

```
import numpy as np
import matplotlib.pyplot as plt
def calculate_arl(shift, k_values):
    arl_values = []
    for k in k_values:
        arl = 1 / (1 - shift * (1 - np.exp(-k)))
        arl_values.append(arl)
    return arl_values
```

# Define shifts and parameter values (k)

```
shifts = np.linspace(1, 2, 11)
```

```
k_values = [10, 20, 30]
```

```
# Calculate ARL values for different shifts and k values
```

```
arl_values = {}
```

```
for k in k_values:
```

```
    arl_values[k] = [calculate_arl(shift, [k])[0] for shift in shifts]
```

```
# Plot trends for each k value
```

```
plt.figure(figsize=(10, 6))
```

```
for k, arls in arl_values.items():
```

```
    plt.plot(shifts, arls, label=f'k={k}')
```

```
plt.title('Trend values of k=10, 20, and 30 for different shifts')
```

```
plt.xlabel('Shift (s)')
```

```
plt.ylabel('Average Run Length (ARL)')
```

```
plt.legend()
```

```
plt.grid(True)
```

```
plt.show()
```

## 5. CONCLUSION

Though quality assurance practices in tertiary education vary among countries, there are some common elements of quality assurance systems that apply in most or, at least, several major types of mechanisms can be identified approaches to quality, level and scope of quality review, key stakeholders involved in the process, methods and instruments, and the consequences of quality monitoring. Growing internationalization of higher education serves to reinforce tendencies in establishing commonalities between national quality assurance frameworks.

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