Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 7s (2024)

A Comparative Study of Lagrangean Multiplier Method and Integer Programming Approach for Efficiency Design in Parallel-Series Configuration

[1]Bhavani Kapu, [2]Sridhar Akiri, [3]Pavan Kumar Subbara, [4]Srinivasa Rao Velampudi & [5]Ramadevi Surapati

[1, 2 & 5] Department of Mathematics, GITAM School of Science, GITAM (Deemed to be University), Visakhapatnam, Andhra Pradesh, India

[3]Department of Mathematics, GITAM School of Science, GITAM (Deemed to be University), Bangalore Campus, Karnataka, India

[4]Professor, Department of Sciences and Humanities, Raghu Engineering College, Visakhapatnam, Andhra Pradesh, India

e-mail: bkapu@gitam.in, sakiri@gitam.edu, orpavan@gmail.com, vsr.rit@gmail.com & rsurapat@gitam.in

Article History:

Received: *01-06-2024*

Revised: *03-07-2024*

Accepted: *29-07-2024*

Abstract:

The Integrated Redundant Reliability Model (IRRM) represents an innovative approach to reliability engineering, employing a Parallel-Series Configuration to enhance system dependability. The performance of the system hinges on the effectiveness of each component within the parallel-series structure, surpassing the efficiency of a single-system factor with a comparable setup. To address component efficiencies, factors in each phase, and existing constraints, this paper introduces a customized Integrated Reliability Model (IRM) specifically designed for the parallel-series scenario. In this model, redundant components are arranged in parallel within subsystems, providing immediate fallback for a specialized machine that is specifically designed for the assembly of an IC Engine. The interconnected series configuration ensures operational continuity in the event of one subsystem failure, thereby minimizing vulnerabilities associated with both parallel and series configurations. Particularly beneficial in critical systems, the integrated approach aims to enhance reliability levels. The model utilizes Lagrangean methods for computing variable quantities, effectiveness, and phase reliability, taking into account various criteria to improve overall system efficiency. Adjustments to simulation techniques and Integer Programming approaches guarantee integer outputs, contributing to the realism of obtained values. This research provides valuable insights into optimizing system reliability and efficiency through integrated redundancy strategies.

Keywords: Integrated Redundant Reliability, Lagrangean Approach, Component Reliability, Stage Reliability, Integer Programming Approach, System Reliability

Mathematics Subject Classification: Primary 90B25; Secondary: 90C39, 90C59

1. Introduction

Enhancing the reliability of a structure can be achieved through two primary approaches: incorporating redundant units or integrating elements with inherently higher reliability. Simultaneously combining both methods requires additional resources. This study explores the optimization of structural reliability, taking into account resource constraints such as size, value, and load availability. While intrinsic value traditionally forms the basis for reliability assessments, real-world scenarios reveal the pivotal role of often unseen factors like load and size in influencing and elevating structural reliability.

In 1971, Mishra K. B. [01], the redundancy optimization problem was formulated as an integer programming problem using zero-one variables. The solution is derived by employing an algorithm developed by Lawler and Bell. The objective function and limits might include any arbitrary functions without specified constraints. Three distinct forms of the optimization issue are currently being studied. The formulation is unambiguous and the solution is quite effective on a digital computer. The magnitude of the problem that can be resolved is not limited by the quantity of constraints. A mathematical model was presented by Mishra K. B. in 1972 [02]. The model's objective is to maximize the reliability of a system that is composed of numerous stages and has both series and parallel redundancy. Additionally, the model is subject to linear constraints. Many distinct methods for the evaluation of the reliability of general systems have been proposed by Agarwal, K. K. and Gupta, J. S., (1975) [03]. The advantages and disadvantages of each approach are examined. The computational labor and the magnitude of the final derived reliability expression are compared by solving an example using all the methods.

The optimal allocation of components to parallel-series and series-parallel systems to maximize the system's reliability can be solved using majorization and Schur-convex functions, as described by Ei-Neweihi, E., et. al. (1986). The optimal allocation for parallel-series systems [04], is entirely delineated and is contingent upon the ordering of component reliabilities. We present a partial ordering of allocations for series-parallel systems that can result in the optimal allocation. Lastly, we present a method for recasting these problems as integer linear programming problems. The results of this paper demonstrate that the application of Schur functions and a different approach to certain linear integer programming problems can result in the acquisition of complete solutions in certain cases and a more comprehensive understanding in others. Prasad, V. R., et al., (1991), explained the purpose of a parallel-series network [05], is to maximize the system's reliability by selecting and assigning n components from the available options to the n spots in the network. The assignment of a component to any of the n places can impact its reliability. Solving this problem precisely is really intricate. Hence, a heuristic approach is devised that necessitates conventional assignment issues, with k representing the quantity of pathsets within the network. The findings of a comprehensive computer experiment demonstrate that the heuristic approach achieves precise solutions in the majority of instances. In cases where an exact solution is not obtained, the variation from the exact solution is generally minimal.

In their 1991 study, Krishna B. Mishra and Usha Sharma [06], presented a novel mathematical approach for optimizing the reliability and redundancy allocation in a system that utilizes different types of redundancy. Their formulation incorporates numerous objective functions to enhance the reliability design. The paper presents a computationally feasible method for addressing a range of design challenges. The suggested solution strategy combines the direct search technique and a random search procedure to optimize redundancy and reliability simultaneously in each subsystem. It utilizes a multi-criteria optimization procedure based on the min-max approach to generate Pareto optimal solutions. The process is demonstrated using multiple examples. Baxter and Harche (1992), introduced a heuristic method for achieving the optimal assembly of series-parallel systems [07]. This heuristic possesses an asymptotic optimality property, which is obtained by a probabilistic analysis. The calculations determine the limits of the absolute and relative mistakes for each heuristic used to achieve the best assembly of series-parallel systems. Fan, L. T. and Wang, T. (1997), attained the utmost level of redundancy in the parallel system by employing a variation technique [08]. The objective is to optimize the system's profitability. This approach offers a direct and efficient computational way for attaining the most optimal design of parallel systems with multiple stages. Two precise numerical examples are presented in a thorough manner.

Prasad V.R. and Ragawachari (1998), expanded upon the research conducted by El-Neweihi and Proschan Sethuraman (1986) to enhance the reliability of a series-parallel system by optimizing the allocation of replaceable components [09]. The paper presented an effective approach for solving the NP-complete reliability optimization problem. An approximation linear programming model is created to minimize the mean deviation of the cut-set risks in a series-parallel system. This model is based on the fact that the dependability of the system improves when the cutset hazards are more homogeneous. A supplementary algorithm is offered to enhance the resulting allocation. The numerical study of this heuristic method and its implementation on a complex problem has produced promising outcomes. Kuo and Prasad (2000), addressed a range of reliability optimization problems and applied their methods to different design problems [10]. In addition, the researchers examined heuristics, metaheuristic algorithms, exact methods, reliability-redundancy allocation, multi-objective optimization, and the assignment of replacement components in dependability systems. In their study, Mettas [2000], introduced a comprehensive model that establishes the minimum level of dependability necessary for various components within a system to achieve the desired overall reliability of the system. The model comprises two components. The initial component entails a formulation of the allocation problem using nonlinear programming. The second component involves formulating a cost function to be utilized in the nonlinear programming technique. It is considered that there is a general relationship between the cost and the reliability of a component in this context.

A comprehensive examination of an over-reliability model, considering multiple constraints, was undertaken to maximize the efficiency of the proposed configuration. The current challenge involves exploring various unknowns - individual elements ($P_{\gamma k}$), element reliability ($r_{\gamma k}$), and stage reliability ($R_{\gamma k}$) - at a specific juncture. These considerations aim to address several constraints, thereby amplifying structural reliability (R_{SA}) and leading to the development of a Unified Reliability Model (URM). Existing literature indicates that enhancing Unified Reliability Models involves incorporating value constraints, establishing a fixed relationship between value and reliability. In a novel approach, this study integrates load and size as supplementary constraints, in addition to value, to formulate an improved redundant reliability system for structures following the parallel-series composition principle.

2. Methods

2.1. Assumptions and Symbols

- $R_{\gamma k}$ is the probability of successful operation of j^{th} component in i^{th}

  parallel path.

- γ is the number of series components at each stage.
- k is number of stages in parallel
- Homogeneity assumption: All components within each stage are considered identical in terms of reliability.
- Statistical independence assumption: Components are treated as statistically independent entities, implying that the failure of one element does not impact the performance of other elements in the system.

R_{SA} = System-Dependability

$R_{\gamma k}$ = Phase-Dependability $0 < R\gamma_k < 1$

$r_{\gamma k}$ = Component-level Reliability within a Phase $'\gamma'$; $0 < r_{\gamma k} < 1$

$P_{\gamma k}$ = Number of Components within each Phase $'k'$

$C_{\gamma k}$ = Price-Component within each Phase $'k'$

$L_{\gamma k}$ = Weight-Component within each Phase $'k'$

$S_{\gamma k}$ = Volume-Component within each Phase $'k'$

$C_{\gamma 0}$ = Maximum Permissible System-Price

$L_{\gamma 0}$ = Maximum Permissible System-Weight

$S_{\gamma 0}$ = Maximum Permissible System-Volume

LMM Lagrangean Multiplier Method

LMMWR Lagrangean Multiplier Method With Rounding-Off

IPM Integer Programming Method

IRRM Integrated Redundant Reliability Model

u_k, v_k, w_k, x_k, y_k, z_k are Constants.

2.2 Computational Evaluation (R_{SA})

The efficiency of the system to the provided worth function

$$\text{Maximize } R_{SA} = 1 - \prod_{\gamma=1}^{m}\left[1 - \prod_{k=1}^{n} R_{\gamma k}\right] \tag{1}$$

The following relationship between worth and efficiency is used to calculate the worth coefficient of each unit in phase $'\gamma k'$.

$$r_{\gamma k} = v_k e^{\frac{c_k}{u_k}} \tag{2}$$

Therefore, $$C_\gamma = u_k \log\frac{r_k}{v_k} \tag{2a}$$

Similarly, $$L_\gamma = w_k \log\frac{r_k}{x_k} \tag{2b}$$

$$S_\gamma = y_k \log \frac{r_J}{z_k} \tag{2c}$$

Since price-components are linear in k,

$$\sum_{j=1}^{n} c_k \,.\, P_{\gamma k} \le C_{\gamma 0} \tag{3a}$$

Similarly weight-components and volume-components are also linear in k,

$$\sum_{j=1}^{n} l_k \,.\, P_{\gamma k} \le L_{\gamma 0} \tag{3b}$$

$$\sum_{j=1}^{n} s_k . P_{\gamma k} \le S_{\gamma 0} \tag{3c}$$

Substituting (2) in (3)

$$\sum_{k=1}^{n} u_k \log \frac{r_k}{v_k} \,.\, P_{\gamma k} - C_{\gamma 0} \le 0 \tag{4a}$$

$$\sum_{k=1}^{n} w_k \log \frac{r_k}{x_k} \,.\, P_{\gamma k} - L_{\gamma 0} \le 0 \tag{4b}$$

$$\sum_{k=1}^{n} y_k \log \frac{r_k}{z_k} \,.\, P_{\gamma k} - S_{\gamma 0} \le 0 \tag{4c}$$

The transformed equation through the relation $X_j = \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]}$ (5)

Where $R_S = 1 - \prod_{k=1}^{n} \left[1 - R_{\gamma k}\right]$ (6)

Subject to the constraints

$$\sum_{k=1}^{n} u_k \log \frac{r_k}{v_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - C_{\gamma 0} \le 0 \tag{7a}$$

$$\sum_{k=1}^{n} w_k \log \frac{r_k}{x_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - L_{\gamma 0} \le 0 \tag{7b}$$

$$\sum_{k=1}^{n} y_k \log \frac{r_k}{z_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - S_{\gamma 0} \le 0 \tag{7c}$$

Positivity restrictions $k \ge 0$

A Lagrangean function is defined as

$$L_F = R_k + \varphi_1 \left[\sum_{j=1}^{n} u_k \log \frac{r_k}{v_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - u_k C_{\gamma 0}\right] + \varphi_2 \left[\sum_{j=1}^{n} w_k \log \frac{r_k}{x_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - L_{\gamma 0}\right] + \varphi_3 \left[\sum_{j=1}^{n} y_k \log \frac{r_k}{z_k} \,.\, \frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - S_{\gamma 0}\right] \tag{8}$$

The Lagrangean function can be used to find the ideal point and separating it by R_k, r_k, φ_1, φ_2 and φ_3.

$$\frac{\partial L_F}{\partial R_k} = 1 + \varphi_1 [\sum_{k=1}^{n} u_k \log \frac{r_k}{v_k} \,.\, \frac{1}{R_k \mathrm{Log}[r_k]}] + \varphi_2 \left[\sum_{k=1}^{n} w_k \log \frac{r_k}{x_k} \,.\, \frac{1}{R_k \mathrm{Log}[r_k]}\right] + \varphi_3 \left[\sum_{k=1}^{n} y_k \log \frac{r_k}{z_k} \,.\, \frac{1}{R_k \mathrm{Log}[r_k]}\right] \tag{9}$$

$$\frac{\partial L_F}{\partial r_k} = \varphi_1 [\sum_{k=1}^{n} u_k \log R_k] \left[\frac{1}{r_k \mathrm{Log}[r_k]} + r_k \log \frac{r_k}{v_k}\right] + \varphi_2 [\sum_{k=1}^{n} w_k \log R_k] \left[\frac{1}{r_k \mathrm{Log}[r_k]} + r_k \log \frac{r_k}{x_k}\right] +$$

$$\varphi_3\ [\sum_{k=1}^{n}\ y_k \log R_k\ ]\left[\frac{1}{r_k \mathrm{Log}[r_k]} + r_k \log\frac{r_k}{z_k}\right] \tag{10}$$

$$\frac{\partial L_F}{\partial \varphi_1} = \sum_{k=1}^{n}\ u_k \log\frac{r_k}{v_k}\frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - C_{\gamma 0} \tag{11}$$

$$\frac{\partial L_F}{\partial\ \varphi_2} = \sum_{k=1}^{n}\ w_k \log\frac{r_k}{x_k}\ .\frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - L_{\gamma 0} \tag{12}$$

$$\frac{\partial L_F}{\partial \varphi_3} = \sum_{k=1}^{n}\ y_k \log\frac{r_k}{z_k}\ .\frac{\mathrm{Log}[R_k]}{\mathrm{Log}[r_k]} - S_{\gamma 0} \tag{13}$$

Where φ_1, φ_2 and φ_3 are Lagrangean multipliers.

The determination of quantities such as the number of elements in each phase ($P_{\gamma k}$), optimal component reliability ($r_{\gamma k}$), stage reliability ($R_{\gamma k}$), and structural reliability (R_{SA}) is achieved using the Lagrangean method. Reliability engineering places significant emphasis on the optimal allocation of components in parallel-series systems, a critical consideration in this field. Researchers have extensively investigated this domain, focusing on exact algorithms, redundancy allocations, optimal assignment of interchangeable components, and heuristic approaches. Sankaraiah G., et. al., (2011), conducted a study to examine the impact of various restrictions on system reliability [12]. An integrated redundant reliability system is analyzed by modeling and solving it using a Lagrangian multiplier. This approach provides a solution for the number of components in the system and the dependability of each component at each stage, represented as real-valued values. A heuristic algorithm and an integer programming technique are used to thoroughly study the problem and obtain a solution in the form of an integer. The validity of these methods is confirmed by sensitivity analysis. Sasikala, Palle., et. al., (2013), developed an Integrated Reliability Model for a parallel-series redundant system by utilizing the Lagrangean Multiplier Method [13]. This method was used to determine the optimal values for the number of Components, Component Reliabilities, and Stage Reliabilities in order to maximize System Reliability. To obtain an integer solution, the Dynamic Programming Method was applied. This method produces a specific (numeric) solution concerning price-component, weight-component, and volume-component considerations.
Through the utilization of the Lagrangean approach, the authors carried out an experiment involving a newly developed mathematical function. We found out the number of components, the reliabilities of the components, the reliabilities of the phases, and the reliability of the system by utilizing the Integrated Redundant Reliability (IRR) model in MATLAB. This model was used to determine the reliability of the system. The number of components was given actual values that were assigned to them. It is important to note that the values should be integers. so that the writers can test with integer programming by leveraging Python programming in order to achieve integer values while concurrently boosting the number of components, component reliabilities, stage reliabilities, and system reliability in order for the authors to test with integer programming. These are the values that were found, and they are discussed in part 6, in addition to being referenced in sections 3, 4, and 5.

2.3 Investigative Challenge

In the exploration of optimization techniques for determining various parameters in a specific mechanical system, this study assumes a proportional relationship between factors such as price-component, weight-component, and volume-component, and the dependability of the system. It is crucial to acknowledge that this assumption may not be applicable to electronic systems. Consequently, the evaluation of maximum component-level reliability ($r_{\gamma k}$), stage-reliability ($R_{\gamma k}$), the quantity of elements per stage ($P_{\gamma k}$), and structural accuracy (R_{SA}) is relevant to any given mechanical system. In their study [14], Sridhar, Akiri et. al., (2013) proposed a new approach to optimize a Redundant IRM with multiple constraints. They aimed to address the hidden effects of extra constraints, beyond the cost constraint, when optimizing the system. Their focus was on examining the k-out-of-n configuration system. Mostafa Abouei Ardakan and Ali Zeinal Hamadani (2014), proposed that each subsystem can adopt either an active or standby redundancy strategy [15]. They aimed to identify the optimal method for these subsystems by employing a suitable mathematical model. Building upon this premise, a new approach is presented that combines conventional active and standby techniques. The new approach is referred to as a mixed strategy, which involves the simultaneous utilization of both active and cold-standby techniques inside a single subsystem. Hence, the task at hand is to ascertain the specific type of components, the level of redundancy, and the quantity of active and cold-standby units for each subsystem, with the objective of optimizing the overall reliability of the system.

Debasis Bhattacharya and Soma Roychowdhury, (2017), this study reduces redundancy allocation costs while meeting reliability goals [16]. The number of redundancies is the choice variable. Any cohesive system, simple or complicated, can use this study's methodology. System architecture complexity directly affects computational complexity. The redundancy allocation problem solved in this paper yields a deterministic optimal solution in polynomial time for a well-known NP-hard problem. This strategy is illustrated with numerical examples. A study examined the ideal solution's sensitivity, the augmented system's reliability, and the cost of redundancy in relation to reliability targets. No component life distribution assumptions are made in the analysis. A thorough examination of existing research on the optimal assignment of components in parallel-series systems unveils valuable insights into precise algorithms, redundancy allocations, the utilization of interchangeable components, and heuristic approaches. These investigations contribute significantly to advancing our comprehension of reliability engineering in intricate systems. This study specifically concentrates on assessing the structural accuracy of a specialized machine designed for assembling an IC Engine (Show in the following figure1).

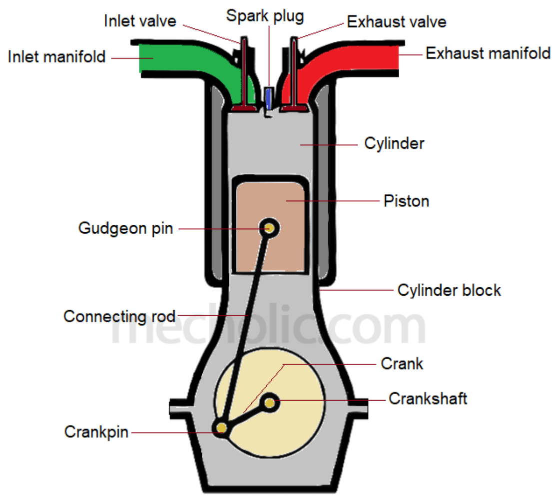

Figure 1: The Structure of IC Engine

The price, weight, and volume of an IC engine exhibit variations influenced by components like factors like gudgeon pin, cylinder, piston, connecting rod and crankshaft. Generally IC engine tailored for residential or small-scale applications tend to possess a comparatively lower cost, smaller size, and lighter weight in comparison to their three-phase counterparts. The IC engine is priced at approximately $1600, encapsulating the core machinery cost within the structure. Notably, the IC engine's weight is registered at 650 pounds, underscoring its substantial structural presence. Beyond the monetary and weight aspects, the IC engine occupies a volume of 750cm^3, accentuating its compact size within the overall structure. It's crucial to emphasize that these figures are illustrative and designed to allow for potential adjustments to suit various contextual and environmental requirements, fostering inclusivity and adaptability across diverse scenarios.

2.4 Fixed Parameters for Components in the Case Study: Preset Constants

The constants needed for the case problem can be found in Table 1, presenting the essential data.

Table 1. Component's-price, Component's-weight, and Component's-volume Pre-fixed Constant Values

Phase	Price & Its Reliability		Weight & Its Reliability		Volume & Its Reliability	
	u_k	v_k	w_k	x_k	y_k	z_k
1	110	0.85	45	0.92	40	0.94
2	120	0.88	55	0.88	50	0.89
3	130	0.91	65	0.91	60	0.86

The tables below present a visual representation of the efficacy of individual factors, phases, and the number of factors in each stage, along with the structural effectiveness.

2.4.1 Precision in Addressing Price-Component Constraints through the Lagrangean Multiplier Technique.

The efficiency design based on value-related metrics is detailed in Table 2.

Table 2. Price- Component Constraint Analysis by using Lagrangean Multiplier Method

Phase	u_k	v_k	$r_{\gamma k}$	Log $r_{\gamma k}$	$R_{\gamma k}$	Log $R_{\gamma k}$	$P_{\gamma k}$	$C_{\gamma k}$	$C_{\gamma k} . P_{\gamma k}$
01	110	0.85	0.9141	0.0390	0.7177	0.1441	3.6932	93.5	345.3119
02	120	0.88	0.9245	0.0341	0.7098	0.1489	4.3664	105.6	461.0917
03	130	0.91	0.9247	0.0340	0.6461	0.1897	5.5796	118.3	660.0620
Final Price-Component									1466.4655

2.4.2 Precision in Addressing Load-Component Constraints through the Lagrangean Multiplier Technique.

The efficiency design based on load-related metrics is detailed in Table 3.

Table 3. Weight- Component Constraint Analysis by using Lagrangean Multiplier Method

Phase	w_k	x_k	$r_{\gamma k}$	Log $r_{\gamma k}$	$R_{\gamma k}$	Log $R_{\gamma k}$	$P_{\gamma k}$	$L_{\gamma k}$	$L_{\gamma k} . P_{\gamma k}$
01	45	0.92	0.9141	0.0390	0.7177	0.1441	3.6932	41.4	152.8974
02	55	0.88	0.9245	0.0341	0.7098	0.1489	4.3664	48.4	211.3337
03	65	0.91	0.9247	0.0340	0.6461	0.1897	5.5796	59.15	330.0310
Final Weight-Component									694.2621

2.4.3 Precision in Addressing Size-Component Constraints through the Lagrangean Multiplier Technique.

The efficiency design based on size-related metrics is detailed in Table 4.

Table 4. Volume-Component Constraint Analysis by using Lagrangean Multiplier Method

Phase	y_k	z_k	$r_{\gamma k}$	Log $r_{\gamma k}$	$R_{\gamma k}$	Log $R_{\gamma k}$	$P_{\gamma k}$	$S_{\gamma k}$	$S_{\gamma k}.P_{\gamma k}$
01	40	0.94	0.9141	0.0390	0.7177	0.1441	3.6932	37.6	138.8634
02	50	0.89	0.9245	0.0341	0.7098	0.1489	4.3664	44.5	194.3047
03	60	0.86	0.9247	0.0340	0.6461	0.1897	5.5796	51.6	287.9053
Final Volume-Component									621.0734

The reliability of the structure (R_{SA}), without any rounding-off, is directly linked to the specific components of the price-component, weight-component, and volume-component, maintaining a constant proportionality factor of 0.8967.

3. Enhancing Efficiency via the Utilization of the Lagrangean Multiplier Technique for Optimization

The efficiency of the design involves aggregating the $'\gamma k'$ integers, rounding each $'k'$ value to the nearest whole number. Detailed tables outline the acceptable outcomes for the price-component, weight-component, and volume-component. The current undertaking requires the calculation of variances attributed to the price-component, weight-component, and volume-component, as well as construction capacity, both prior to and following the rounding off of $'\gamma k'$ to the nearest integer, in order to extract comprehensive insights.

3.1 Enhancing Efficiency through Lagrangean Multiplier Method: Precision in Handling Price, Weight, and Volume Components

Table 5. Efficiency design relating to price-component, weight-component, and volume-component Constraint Analysis by using Lagrangean Multiplier Method with Rounding Off is shown in the following table.

Phase	$r_{\gamma k}$	$R_{\gamma k}$	$P_{\gamma k}$	$C_{\gamma k}$	$P_{\gamma k}.C_{\gamma k}$	$L_{\gamma k}$	$P_{\gamma k}.L_{\gamma k}$	$S_{\gamma k}$	$P_{\gamma k}.S_{\gamma k}$
01	0.9141	0.7177	4	94	376	41	164	38	152
02	0.9245	0.7098	4	106	424	48	192	45	180
03	0.9247	0.6461	6	118	708	59	354	52	312
$Total - Price, Weight \& Volume$				1508		710		644	
$System\ Reliability\ (R_{SA})$									0.9392

3.2 Analyzing LMM Approach: Impact of Rounding-Off on Price, Weight, and Volume Components

In the context of the Lagrangean multiplier method framework, the identification of the quantity of components, component reliability, stage reliability, and overall system reliability is accomplished through a MATLAB program that is directly proportional to Price, Weight, and Volume. During this computational process, real-valued solutions were acquired, proving incompatible with the objectives of the ongoing IRRM study. Consequently, the widely employed rounding-off method was implemented to derive integer-valued solutions. The ensuing outcomes delineate the distinctions between the LMM approach without rounding-off and the LMM approach with rounding-off.

3.2.1 Fluctuation in Price-Component under LMM & LMMWR Frame Work = 02.83%

3.2.2 Fluctuation in Weight-Component under LMM & LMMWR Frame Work = 02.27%

3.2.3 Fluctuation in Volume-Component under LMM & LMMWR Frame Work = 03.69%

3.2.4 Fluctuation in System Reliability under LMM & LMMWR Frame Work = 04.74%

4 Integer Programming Approach

Utilizing integer programming necessitates the input of component reliabilities to ascertain the number of components in each stage, stage reliabilities, and system reliability. However, a notable drawback of integer programming is its inability to be directly employed, i.e., without the component reliabilities input, in constructing the integrated reliability model.

In the work [17] of Karim Guilani et. al., (2017), it is stated that components might exist in three different states. Furthermore, a Bi-Objective Reliability Allocation Problem (BORAP) is formulated for a system consisting of sequential subsystems. Each subsystem has non-repairable tri-state components that are connected in parallel, and the subsystem operates according to a k-out-of-n policy. Sridhar, Akiri, et al. (2021), investigated, developed, analyzed, and improved an integrated coherent redundant reliability design, which has not been previously published [18]. The Lagrangean multiplier approach can be used to design and evaluate the system, yielding accurate unit counts and design reliability. Practical implementation is achieved with an integer solution, and design reliability is optimized using integer and dynamic programming. Redundancy costs relative to reliability. The analysis makes no component life distribution assumptions.

Sridhar et al. (2022), investigated the rise of systems and software modeling, including intelligent characters, HMIs, menu generators, UXA, picture archiving, and software systems [19]. This book helps students, researchers, academics, scientists, and industry practitioners comprehend worldwide trends, problems, and practices. This software models, simulates, and optimizes software reliability. Provides dependability modeling and resource allocation methods, software, and applications. Cost modeling and optimization for complicated systems. Srinivasa Rao, Velampudi, et al., (2022), did a literature review on the optimization of system reliability using redundancy and the integration of reliability models with redundancy [20]. In addition, they provided suggestions for further improvements. Srinivasa Rao, Velampudi, et al., (2022), conducted a study to analyze the effects of incorporating more hidden restrictions on enhancing the reliability of a structure [21]. The analytical purpose considers the integrated redundancy of a well-organized system. The Lagrangean multiplier approach provides a solution for the reliability of elements, phases, and structures. An advanced Heuristic algorithm is used to provide an integer solution that is close to optimal, but not a completely bounded answer. This approach then leads to the implementation of the Dynamic programming method. An illustrative numerical example demonstrates the acquired results.

In their study, Srinivasa Rao, Velampudi, et al., (2023), proposed considering the efficiencies of the factors and the quantity of factors in each phase, as well as the various limitations, in order to optimize the system's efficiency [22]. In order to improve the efficiency of the system, the authors utilized various techniques from the Lagrangean approach to calculate the quantities and effectiveness of the variables, as well as the reliabilities of the phase, under diverse parameters such as load, size, and cost. The dynamic programming strategy and simulation method have been modified to achieve an integer outcome and to observe the actual values. Srinivasa Rao, Velampudi, et al., (2024), utilized the integrated model mentioned above to assess the reliability and efficiency of different components in a Muffle Box Furnace machine [23]. They employed Lagrangean methods to determine the price-component, weight-component, and volume-component of various system configurations. The objective was to optimize the overall system performance. In order to get a realistic outcome inside a set of whole numbers, we utilized the integer programming approach along with the dynamic programming strategy.

Therefore, the component reliabilities obtained from the preceding method, i.e., the Lagrangean approach, can be applied as the input for integer programming. This allows for the determination of stage reliabilities, system reliability, stage cost, and system cost. Integer programming offers flexibility in establishing the number of components in each stage, along with stage reliabilities and system reliability, within the given constraints.

4.2 Procedure for Integer Programming Method

Integer Programming (IP) is a mathematical optimization technique used to solve optimization problems where some or all of the decision variables are required to take integer values. The general procedure for solving an Integer Programming problem involves the following steps:

1. Formulate the Problem: Clearly define the objective function you want to optimize and the constraints that need to be satisfied. Identify the decision variables and their potential integer values.
2. Define Decision Variables: Specify the decision variables of the problem and their integer restrictions. For example, if you're optimizing the number of items to be produced, the decision variable might be the quantity of each item, which needs to be an integer.
3. Formulate the Objective Function: Write down the mathematical expression that represents the objective you want to maximize or minimize. This could involve costs, profits, or any other measurable quantity.
4. Formulate Constraints: Write down the mathematical expressions that represent the constraints of the problem, such as resource limitations, capacity constraints, demand requirements, etc.
5. Choose Optimization Direction: Decide whether you want to maximize or minimize the objective function.
6. Solve the Linear Relaxation: Start by solving the relaxed version of the problem, where integer constraints are ignored, and the variables can take fractional values. This is called the Linear Programming (LP) relaxation.
7. Obtain Initial Solution: The LP relaxation might provide fractional values for the decision variables. Round these fractional values to the nearest integer or apply a rounding strategy to obtain an initial feasible solution for the Integer Programming problem.
8. Check Feasibility: Check if the initial rounded solution satisfies all the integer and constraint requirements. If it does, you've found an integer solution, and you can proceed to step 10. If not, proceed to step 9.

9. Implement Branch and Bound: Integer Programming problems can be solved using the Branch and Bound method. This technique involves systematically branching on a variable, solving sub problems, and bounding the feasible solutions to prune unproductive branches.
   i. Branching: Choose a fractional variable from the current solution and create two sub problems by fixing the chosen variable to its integer values, i.e., floor and ceiling.
   ii. Bounding: Solve the relaxed LP problems for the sub problems. If the optimal objective value of a sub problem is worse than the best integer solution found so far, prune that branch.
   iii. Recursive Process: Recursively apply the branching and bounding steps on the sub problems until either a feasible integer solution is found or the sub problems are shown to be infeasible or non-promising.
10. Obtain Integer Solution: When the Branch and Bound process terminates, you will have an integer solution that either satisfies the constraints or proves that no feasible integer solution exists.

Our plan for solving the current study problem is to use the Integer Programming method to come up with solutions in integer values. This is different from the usual use of the Lagrangean Multiplier Method, which usually leads to solutions in real values. We are changing the way we do things because we need to get better at guessing the right number of parts a system needs to keep its resilience. We want to use the Integer Redundancy and Reliability (IRR) Model to not only check the dependability of individual parts, but also to do detailed checks on the dependability of each stage and the dependability of the whole system.

Over the past forty years, there have been considerable breakthroughs in the field of integer programming, with a primary focus on linear integer programming. Nevertheless, there has been significant advancement in the theoretical and methodological elements of nonlinear integer programming in recent years. The advancements mentioned have resulted in the utilization of nonlinear integer programming in diverse fields including scientific computing, engineering, management science, and operations research. The applications of this concept are wide-ranging and include portfolio selection, capital budgeting, industrial planning, resource allocation, computer networks, dependability networks, and chemical engineering. Nonlinear programming is a mathematical technique used to find the best solutions for different situations. In a nonlinear programming issue, the objective function is nonlinear, and there may be nonlinear interactions in one or more constraints, or both.

An integer programming problem, in which all or some of the decision variables are constrained to nonnegative integer values, is referred to as an integer programming problem. This challenge is especially important in commerce and industry, as fractional solutions are not practicable because the units cannot be divided. It is not reasonable to discuss fractions of individuals, such as 2.3, working on a project, or fractions of machines, such as 8.7, in a workshop. One possible method is to obtain an integer solution by rounding off the optimal variable values. However, this strategy may be imprecise since there is no assurance that the divergence from the exact integer answer will not be significant enough to preserve feasibility.

Python, a computer language created by Guido van Rossum, was initially released on February 20, 1991. Python is a computer language named after a BBC sketch show called Monty Python's

Flying Circus. The python is a sizable serpent. Python is a user-friendly and high-level programming language. The design theory emphasizes the importance of code readability through the strategic use of ample spacing between lines. Python is a versatile tool specifically created to streamline the creation and solving of linear, nonlinear, and integer optimization models. The package provides a complete and integrated solution, including a powerful language for describing optimization models, a comprehensive environment for creating and updating problems, and a set of efficient built-in solvers.

5 Results

The application of the Lagrangean multiplier approach has offered an authentic numerical solution for the analyzed mathematical models of Integrated Redundant Reliability Systems, fulfilling the need for a non-decimal answer. Using an Integer Programming approach, the researcher calculated the revised phase reliability ($R_{\gamma k}$), resulting in stage reliability values of 0.8424, 0.8243, and 0.8353. The research utilizes the Integer Programming technique to provide the results for the specified mathematical function in tables 6, 7, and 8 consecutively, making it easier to draw important conclusions.

5.2 In-Depth Analysis of Constraints in Price, Weight, and Volume Components: A Comprehensive Examination through Integer Programming Method

Exploring Efficiency Design: Table 6 Provides In-Depth Information on the Interplay Between Price, Weight, and Volume Components

Table 6. Constraint Analysis of Efficiency Design for Price, Weight, and Volume Components: Integer Programming Method Illustrated in the Table.

Phase	$r_{\gamma k}$	$R_{\gamma k}$	$P_{\gamma k}$	$C_{\gamma k}$	$C_{\gamma k} \cdot P_{\gamma k}$	$L_{\gamma k}$	$L_{\gamma k} \cdot P_{\gamma k}$	$S_{\gamma k}$	$S_{\gamma k} \cdot P_{\gamma k}$
01	0.9345	0.7626	4	*37*	*148*	*39*	*156*	*37*	*148*
02	0.9482	0.8083	4	*44*	*176*	*46*	*184*	*44*	*176*
03	0.9555	0.7610	6	*50*	*300*	*56*	*336*	*50*	*300*
$Total - Price, Weight \,\&\, Volum$				1426		676		624	
$System\ Reliability\ (R_{SA})$									0.9621

5.3 Evaluating Optimization Strategies in Integrated Redundant Reliability Parallel-Series Systems: A Comparative Analysis of Lagrangean Multiplier Method (LMM) with Rounding-Off and Integer Programming Approach for Price Components

Analytical Comparison: Examining Price-Related Efficiency Design Using Lagrangean Multiplier Method with Rounding-Off and Integer Programming Approach – Findings from Table 7.

Table 7. Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Integer Programming Method Approaches for Price-Component

		LMM With Rounding Off				Integer Programming Method			
Phase	$P_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$C_{\gamma k}$	$P_{\gamma k} \cdot C_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$C_{\gamma k}$	$P_{\gamma k} \cdot C_{\gamma k}$
01	4	0.9141	0.7177	38	152	0.9345	0.7626	*37*	*148*
02	4	0.9245	0.7098	45	180	0.9482	0.8083	*44*	*176*
03	6	0.9247	0.6461	52	312	0.9555	0.7610	*50*	*300*
Final Price-Component				1508				1426	

System Reliability (R_{SA})	LMMWR	0.9392	IPM	0.9621

5.4 Evaluating Optimization Strategies in Integrated Redundant Reliability Parallel-Series Systems: A Comparative Analysis of Lagrangean Multiplier Method (LMM) with Rounding-Off and Integer Programming Approach for Load Components

Analytical Comparison: Examining Load-Related Efficiency Design Using Lagrangean Multiplier Method with Rounding-Off and Integer Programming Approach – Findings from Table 8.

Table 8. Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Integer Programming Method Approaches for Weight-Component

		LMM With Rounding Off				Integer Programming Method			
Phase	$P_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$L_{\gamma k}$	$P_{\gamma k}.L_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$L_{\gamma k}$	$P_{\gamma k}.L_{\gamma k}$
01	4	0.9141	0.7177	41	164	0.9345	0.7626	*39*	*156*
02	4	0.9245	0.7098	48	192	0.9482	0.8083	*46*	*184*
03	6	0.9247	0.6461	59	354	0.9555	0.7610	*56*	*336*
Final Weight-Component				710				676	
System Reliability (R_{SA})			LMMWR		0.9392	IPM			0.9621

5.5 Evaluating Optimization Strategies in Integrated Redundant Reliability Parallel-Series Systems: A Comparative Analysis of Lagrangean Multiplier Method (LMM) with Rounding-Off and Integer Programming Approach for Size Components

Analytical Comparison: Examining Volume-Related Efficiency Design Using Lagrangean Multiplier Method with Rounding-Off and Integer Programming Approach – Findings from Table 9.

Table 9. Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Integer Programming Method Approaches for Volume-Component

		LMM With Rounding Off				Integer Programming Method			
Phase	$P_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$S_{\gamma k}$	$P_{\gamma k}.S_{\gamma k}$	$r_{\gamma k}$	$R_{\gamma k}$	$S_{\gamma k}$	$P_{\gamma k}.S_{\gamma k}$
01	4	0.9141	0.7177	38	152	0.9345	0.7626	*37*	*148*
02	4	0.9245	0.7098	45	180	0.9482	0.8083	*44*	*176*
03	6	0.9247	0.6461	52	312	0.9555	0.7610	*50*	*300*
Final Volume-Component				644				624	
System Reliability (R_{SA})			LMMWR		0.9392	IPM			0.9621

5.5.1 Fluctuation in Price-Component under IPM Frame Work = 05.44%

5.5.2 Fluctuation in Weight-Component under IPM Frame Work = 04.71%

5.5.3 Fluctuation in Volume-Component under IPM Frame Work = 3.11%

5.5.4 Fluctuation in System Reliability under IPM Frame Work = 2.44%

6 Discussion

This paper presents a novel reliability architecture specifically designed for a parallel-series configuration system with different efficiency objectives. The Lagrangean multiplier approach is used to determine the values of components ($P_{\gamma k}$), component reliability ($r_{\gamma k}$), stage reliability ($R_{\gamma k}$), and system reliability (R_{SA}) when working with data represented in real numbers. The component efficiencies ($r_{\gamma k}$) are 0.9141, 0.9245 and 0.9247, the stage reliabilities ($R_{\gamma k}$) are 0.7177, 0.7098, and

0.6461, and the structural dependability (R_{SA}) is 0.9392.

The Integer Programming approach is used to obtain integer solutions for practical purposes. This method yields component reliabilities ($r_{\gamma k}$) of 0.9345, 0.9482 and 0.9555, stage reliabilities ($R_{\gamma k}$) of 0.7626, 0.8083 and 0.7610, and system reliability (R_{SA}) of 0.9621. The integer solutions are produced using inputs drawn from the Lagrangean approach, which guarantees the practical applicability of the model. The Integrated Redundant Reliability model is developed by applying a heuristic approach to examine several constraints on the mathematical function. The input for the IPM approach is obtained from the Lagrangean method for the given case problem.

The model's results underscore a significant benefit of the Lagrangean approach, namely that the quantities of components in each stage (P_k) are represented as whole numbers, making it very applicable in real-world scenarios. Although classical techniques like integer programming can produce integer values, the time required to compute solutions may exponentially rise as the issue size (number of variables) increases. A heuristic strategy is used in this case to effectively locate integer solutions. In the Lagrangian approach, the system's reliability generally increases as the cost of the system increases (a similar pattern is observed when P_k is rounded off). On the other hand, the LMM technique shows that values associated with cost, weight, and volume are roughly in line with the values of the case issue. This leads to a decrease in the number of components in each stage (P_k) and a slight drop in system reliability.

In future research, the authors propose exploring a novel technique that establishes constraints on the minimum and maximum levels of reliability for components, with the goal of improving the overall dependability of the system. The aim is to enhance the breadth and flexibility of Integrated Reliability Models (IRMs) by integrating redundancy, using existing heuristic methodologies.

Acknowledgement

This research was not funded by any grant.

Compliance with Ethical Standards

There is no conflict of interest between any parties in this paper. Also, ethical clearance is not required as there are no humans or animals involved in this project. The consent of all the authors are taken in confidence.

References

[1] Mishra, K. B. (1971). A method of solving redundancy optimization problems. IEEE Transactions on Reliability, 20(3):117-20.

[2] Mishra, K. B. (1972). Reliability optimization of a series-parallel system. IEEE Transactions on Reliability, 21(4):230-238.

[3] Agarwal, K. K. and Gupta, J. S. (1975). On minimizing the expense of reliable systems. IEEE Transactions on Reliability, 24(3):205-215.

[4] Ei-Neweihi, E., F. Proschan and J. Sethuraman (1986). Optimal allocation of components in parallel – series and series – parallel system", Journal of Applied Probability, 23(01): 770-777.

[5] Prasad, V.R., Y.P. Aneja, and K.P.K. Nair, (1991). A heuristic approach to optimal assignment of components to a parallel-series network", IEEE Transaction of Reliability, 40(02): 555-558.

[6] Krishna B. Misra, Usha Sharma, **(1991),** Multicriteria optimization for combined reliability and redundancy allocation in systems employing mixed redundancies, Microelectronics Reliability.31,323-335. https://doi.org/10.1016/0026-2714(91)90217-U.

[7] Baxter, L.A. and Harche, F. (1992). On the optimal assembly of series parallel systems. Operations Research Letters, 11(1): 153-157.

[8] Fan, L. T. and Wang, T. (1997). Optimization of system reliability. IEEE Transactions on Reliability, 16(2):81-86.

[9] Prasad. V.R., and Ragawachari, (1998),"optimal allocation of interchangeable components to series–parallel reliability system", IEEE Transaction of Reliability, 47(03): 255-260.

[10] Kuo, W. and Prasad, V. R. (2000). An annotated overview of system-reliability optimization. IEEE Transactions on Reliability, 49(2):176-187.

[11] Mettas, A. (2000). Reliability allocation and optimization for complex systems - In Annual reliability and maintainability symposium. Proceedings of International symposium on product quality and integrity (Cat. No. 00CH37055). IEEE Transactions on Reliability, 216-221.

[12] Sankaraiah, G., Sarma, B. D., Umashankar, C. and Sarma, Y. V. S. (2011). Designing, modelling and optimizing of an integrated reliability redundant system. South African Journal of Industrial Engineering, 22(2):100-106.

[13] Sasikala, P., Sridhar, A., Kumar, S. P. and Umashankar, C. (2013). Optimization of IRM Parallel-Series Redundant System, International Journal of Engineering Research & Technology, 2(2):1-6.

[14] Sridhar, A., Kumar, S. P., Reddy, Y. R., Sankaraiah, G. and Umashankar, C. (2013). The k-out-of-n Redundant IRM Optimization with multiple constraints. International Journal for Research in Science & Advanced Technologies, 3(4):436-438.

[15] Mostafa Abouei Ardakan, Ali Zeinal Hamadani, (2014), Reliability optimization of series-parallel systems with mixed redundancy strategy in subsystems, Reliability Engineering & System Safety.130 (1), 132-139. https://doi.org/10.1016/j.ress.2014.06.001.

[16] Debasis Bhattacharya, Soma Roychowdhury, (2017), A redundancy strategy for minimising cost in systems with non-disjoint subsystems under reliability constraint, International Journal of System Assurance Engineering and Management. 8 (1), 645-655. https://doi.org/10.1007/s13198-016-0499-y.

[17] Pe. Pour Karim Guilani ' A. Zaretalab 'S.T. A. Niaki , Pa. Pourkarim Guilani , **(2017),** A bi-objective model to optimize reliability and cost of k-out-of-n series-parallel systems with tri-state components, International Journal of Science and Technology. 24(3), 1585-1602. DOI: 10.24200/SCI.2017.4137.

[18] Sridhar, A., Sasikala, P., Kumar, S. P. and Sarma, Y. V. S. (2021). Design and evaluation of coherent redundant system reliability. Systems Reliability Engineering, edited by Amit Kumar and Mangey Ram, De Gruyter, 137-152.

[19] Sridhar, A., Sasikala, P., Kumar, S. P. and Sarma, Y. V. S. (2022). Design and evaluation of parallel-series IRM system. System Assurances Modeling and Management, edited by Prashant Johri, Adarsh Anand, Juri Vain, Jagvinder Singh, and Mohammad Quasim, Academic Press (Elsevier), 189-208.

[20] Srinivasa Rao V., Sridhar Akiri, Pavan Kumar S., (2022), k-out-of-n Systems Growth Study Focusing on Redundant Reliability Systems by using Heuristic Programming Approach. Mathematical Statistician and Engineering Applications. 71(4), 5602-5613.

[21] Srinivasa Rao V., Sridhar Akiri, Kesava Rao Y., (2022), Design and Implementation of An Integrated Redundant Reliability System for k-out-of-n Configuration, Journal of Neuroquantology. 20(19), 998-1009.

[22] Srinivasa Rao Velampudi, Sridhar Akiri, Pavan Kumar Subbara & Yadavalli V S S., (2023), A Comprehensive Case Study on Integrated Redundant Reliability Model using k out of n Configuration, Journal of Reliability Theory & Applications. 18(1[72]), 110-119.

[23] Srinivasa Rao Velampudi, Sridhar Akiri & Pavan Kumar Subbara, (2024), Integrated Redundant Reliability Model using k out of n Configuration with Integer Programming Approach, Statistics and Applications. 22(1), 137-147.