

Homomorphism of Tripolar Fuzzy Soft Ternary Γ -Semiring

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Abstract:

The notions of a tripolar fuzzy soft ternary Γ -Semirings, a tripolar fuzzy soft ternary Γ -Semiring homomorphism and a tripolar fuzzy soft ideal in ternary Γ -Semirings are discussed, and related properties are investigated. On the other hand, in this paper, we also define the image and pre-image of tripolar fuzzy soft ternary Γ -Semirings. Some properties and results involving these concepts are stated and proved.

Keywords: Soft set, fuzzy Soft set, tripolar fuzzy soft set, tripolar fuzzy soft ternary Γ -Semiring, tripolar fuzzy soft ideal, tripolar fuzzy soft ternary Γ -Semiring homomorphism.

1. Introduction

In the year 2017 Revathi et.al [7, 8] introduced the concept of “fuzzy ideals” in “ternary gamma semirings”. In the year 2018, they [9,10,11] studied “fuzzy regular ternary gamma semirings”, “completely prime fuzzy ideals” and “prime fuzzy ideals in ternary gamma semirings”. In the year 2020 E. Meera Prasad et.al [6] introduced the notion of “fuzzy soft Bi-ideals” over “ternary gamma semirings”. Satish. T et. al [14] studied about “fuzzy soft ideals in ternary gamma semirings”. G. Srinivasa Rao et.al [] studies about ternary gamma semirings extensively. In this paper we introduce the notion “tripolar fuzzy soft ternary Γ -semiring homomorphism”. Throughout this paper we indicate “ternary gamma semiring” as TGSR, “Fuzzy soft ternary gamma Semiring” as FSTGSR, “tripolar fuzzy soft set” as TFSS, “Tripolar Fuzzy soft ternary gamma semiring” as TFSTGSR, “Tripolar fuzzy soft gamma ideal” as TFSI and “Tripolar fuzzy soft ternary Γ -semiring homomorphism” as TFSTGSRH, unless otherwise stated.

2. Preliminaries:

Definition 2.1: A “tripolar fuzzy set” T in a universe set M is an object having the form $T = \{(a, \nu_T(a), \gamma_T(a), \beta_T(a)) \mid a \in M, 0 \leq \nu_T(a) + \gamma_T(a) \leq 1\}$, where $\nu_T: M \rightarrow [0, 1]$, $\gamma_T: M \rightarrow [0, 1]$ and $\beta_T: M \rightarrow [-1, 0]$ such that $0 \leq \nu_T(a) + \gamma_T(a) \leq 1$. Since ν_T characterises the extent that the

element a satisfies the property corresponding the tripolar fuzzy set T , γ_T characterises the extent that the element a satisfies the irrelevant (non property) corresponding the tripolar fuzzy set T and β_T characterises the extent that the element a satisfies to the implicit counter property of the tripolar fuzzy set T . We use the notion $T = \{v_T, \gamma_T, \beta_T\}$ instead of $T = \{(a, v_T(a), \gamma_T(a), \beta_T(a)) \mid a \in M, 0 \leq v_T(a) + \gamma_T(a) \leq 1\}$.

Definition 2.2: A TFSs (δ, V, Γ) over TGSR M is nothing but TFSTGSR over TGSR M if $v(a) = \{(p, v_{\delta(a)}(p), \gamma_{\delta(a)}(p), \beta_{\delta(a)}(p)) \mid p \in M, a \in V\}$, where $v_{\delta(a)}: M \rightarrow [0, 1]$, $\gamma_{\delta(a)}: M \rightarrow [0, 1]$ and $\beta_{\delta(a)}: M \rightarrow [-1, 0]$, $\exists 0 \leq v_{\delta(a)} + \gamma_{\delta(a)} \leq 1$ and $\forall p \in M$ satisfying the following conditions:

- (1) $v_{\delta(a)}(j + m) \geq \min \{v_{\delta(a)}(j), v_{\delta(a)}(m)\}$
- (2) $\gamma_{\delta(a)}(j + m) \leq \max \{\gamma_{\delta(a)}(j), \gamma_{\delta(a)}(m)\}$
- (3) $\beta_{\delta(a)}(j + m) \leq \max \{\beta_{\delta(a)}(j), \beta_{\delta(a)}(m)\}$
- (4) $v_{\delta(a)}(jam\theta k) \geq \min \{v_{\delta(a)}(j), v_{\delta(a)}(m), v_{\delta(a)}(k)\}$
- (5) $\gamma_{\delta(a)}(jam\theta k) \leq \max \{\gamma_{\delta(a)}(j), \gamma_{\delta(a)}(m), \gamma_{\delta(a)}(k)\}$
- (6) $\beta_{\delta(a)}(jam\theta k) \leq \max \{\beta_{\delta(a)}(j), \beta_{\delta(a)}(m), \beta_{\delta(a)}(k)\} \forall j, m, k \in M, a \in V, \alpha, \theta \in \Gamma$.

Definition 2.3: A TFSs (δ, V, Γ) over TGSR M is nothing but TFSI over TGSR M if

- (1) $v_{\delta(a)}(j + m) \geq \min \{v_{\delta(a)}(j), v_{\delta(a)}(m)\}$
- (2) $\gamma_{\delta(a)}(j + m) \leq \max \{\gamma_{\delta(a)}(j), \gamma_{\delta(a)}(m)\}$
- (3) $\beta_{\delta(a)}(j + m) \leq \max \{\beta_{\delta(a)}(j), \beta_{\delta(a)}(m)\}$
- (4) $v_{\delta(a)}(jam\theta k) \geq \max \{v_{\delta(a)}(j), v_{\delta(a)}(m), v_{\delta(a)}(k)\}$
- (5) $\gamma_{\delta(a)}(jam\theta k) \leq \min \{\gamma_{\delta(a)}(j), \gamma_{\delta(a)}(m), \gamma_{\delta(a)}(k)\}$
- (6) $\beta_{\delta(a)}(jam\theta k) \leq \min \{\beta_{\delta(a)}(j), \beta_{\delta(a)}(m), \beta_{\delta(a)}(k)\} \forall j, m, k \in M, a \in V, \alpha, \theta \in \Gamma$.

Definition 2.4: If M_1 and M_2 are two TGSRs, a function $\pi: M_1 \rightarrow M_2$ is called a homomorphism TGSR if $\pi(j + m) = \pi(j) + \pi(m)$ and $\pi(jam\theta k) = \pi(j)\alpha\pi(m)\theta\pi(k)$, $\forall j, m, k \in M_1, \alpha, \theta \in \Gamma$.

Let M_1 and M_2 two sets and $\psi: M_1 \rightarrow M_2$ is any function. A bipolar fuzzy subset ε of M_1 is nothing but a ε -invariant $\psi(p) = \psi(q) \implies \varepsilon(p) = \varepsilon(q)$.

Let $\psi: M_1 \rightarrow M_2$ is any function, $\varepsilon = (\varepsilon^+, \varepsilon^-)$ and $\eta = (\eta^+, \eta^-)$ are bipolar fuzzy subsets in M_1, M_2 respectively. Then the image $\psi(\varepsilon)$ of ε is the bipolar fuzzy subset $\psi(\varepsilon) = ((\psi(\varepsilon))^+, (\psi(\varepsilon))^-)$ of M_2 defined as

$$(\psi(\varepsilon))^+(p) = \begin{cases} \max\{(\varepsilon)^+(p): p \in \psi^{-1}(p): \text{if } \psi^{-1}(p) \neq \emptyset\} \\ 0 \text{ otherwise} \end{cases}$$

$$(\psi(\varepsilon))^{-}(p) = \begin{cases} \max\{(\varepsilon)^{-}(p): p \in \psi^{-1}(p): \text{if } \psi^{-1}(p) \neq \emptyset\} \\ 0 \text{ otherwise} \end{cases}$$

And the pre-image $\psi^{-1}(\eta)$ of η under ψ is the bipolar fuzzy subset of M_1 defined by for $p \in M_1$ $((\psi^{-1}(\eta))^+(p) = \varepsilon^+(\psi(a))$ and $((\psi^{-1}(\eta))^{-}(p) = \varepsilon^{-}(\psi(a))$. For more preliminaries refer the references and their references.

For more preliminaries consider the references and their references.

3: Homomorphism in tripolar fuzzy soft Ternary Γ -semiring:

In this section the homomorphism concept over TFSTGSR is introduced and studied their properties.

Definition 3.1: Let (δ, V, Γ) and (ε, W, Γ) TFSs over TGSR M_1 and M_2 and $\psi_1: M_1 \rightarrow M_2$, $\psi_2: V \rightarrow W$ were two functions such that V, W were parameter sets for the crisp sets M_1 and M_2 , then (ψ_1, ψ_2) is nothing but tripolar fuzzy soft function from M_1 to M_2 .

Definition 3.2: Let (δ, V, Γ) and (ε, W, Γ) TFSs over TGSR M_1 and M_2 and (ψ_1, ψ_2) is tripolar fuzzy soft function from M_1 to M_2 . Then (ψ_1, ψ_2) is called Tripolar fuzzy soft ternary Γ -semiring homomorphism if satisfying the following laws:

- 1) Ψ_1 is a ternary Γ -semiring homomorphism from M_1 onto M_2 .
- 2) Ψ_2 is a mapping from V onto W .
- 3) $\psi_1(\nu_{\delta(x)}) = \varepsilon_{\psi_2(x)}$, $\psi_1(\gamma_{\delta(x)}) = \varepsilon_{\psi_2(x)}$ and $\psi_1(\beta_{\delta(x)}) = \varepsilon_{\psi_2(x)}$

Note 3.3: If there exist a TFSTGSRH between (δ, V, Γ) and (ε, W, Γ) , then we say that (δ, V, Γ) is soft homomorphic to (ε, W, Γ) .

Definition 3.4: If (ψ_1, ψ_2) is a tripolar fuzzy soft function from M_1 to M_2 . The pre image of (ε, W, Γ) under the tripolar fuzzy soft function (ψ_1, ψ_2) devoted by $(\psi_1, \psi_2)^{-1}(\varepsilon, W, \Gamma)$ defined as $(\psi_1, \psi_2)^{-1}(\varepsilon, W, \Gamma) = (\psi_1^{-1}(\varepsilon), \psi_2^{-1}(W))$ is a TFSs.

Theorem 3.5: If (ε, W, Γ) is a TFSTGSR over TGSR M_2 , $\psi: M_1 \rightarrow M_2$ is a monomorphism and for each $w \in W$, define $(\psi\varepsilon)_w(m) = \psi_w(\varepsilon(m))$ for all $m \in M_1$. Then $(\psi\varepsilon, W, \Gamma)$ is a TFSTGSR over TGSR M_2 .

Proof: Suppose $j, m, k \in M$, $w \in W$ and $\alpha, \theta \in \Gamma$. Then

1. $(\psi\varepsilon)_w(j + m) = \psi_w(\varepsilon(j + m)) = \nu_{\psi_w(\varepsilon(j) + \varepsilon(m))} \geq \min \{ \nu_{\psi_w(\varepsilon(j))}, \nu_{\psi_w(\varepsilon(m))} \}$
 $= \min \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m) \}$
2. $(\psi\varepsilon)_w(j + m) = \psi_w(\varepsilon(j + m)) = \gamma_{\psi_w(\varepsilon(j) + \varepsilon(m))} \leq \max \{ \gamma_{\psi_w(\varepsilon(j))}, \gamma_{\psi_w(\varepsilon(m))} \}$
 $= \max \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m) \}$.
3. $(\psi\varepsilon)_w(j + m) = \psi_w(\varepsilon(j + m)) = \beta_{\psi_w(\varepsilon(j) + \varepsilon(m))} \leq \max \{ \beta_{\psi_w(\varepsilon(j))}, \beta_{\psi_w(\varepsilon(m))} \}$
 $= \max \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m) \}$.
4. $(\psi\varepsilon)_w(j \alpha m \theta k) = \psi_w(\varepsilon(j \alpha m \theta k)) = \nu_{\psi_w(\varepsilon(j) \alpha \varepsilon(m) \theta \varepsilon(k))}$
 $\geq \min \{ \nu_{\psi_w(\varepsilon(j))}, \nu_{\psi_w(\varepsilon(m))}, \nu_{\psi_w(\varepsilon(k))} \}$
 $= \min \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m), (\psi\varepsilon)_w(k) \}$.
5. $(\psi\varepsilon)_w(j \alpha m \theta k) = \psi_w(\varepsilon(j \alpha m \theta k)) = \gamma_{\psi_w(\varepsilon(j) \alpha \varepsilon(m) \theta \varepsilon(k))}$
 $\leq \max \{ \gamma_{\psi_w(\varepsilon(j))}, \gamma_{\psi_w(\varepsilon(m))}, \gamma_{\psi_w(\varepsilon(k))} \}$
 $= \max \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m), (\psi\varepsilon)_w(k) \}$.
6. $(\psi\varepsilon)_w(j \alpha m \theta k) = \psi_w(\varepsilon(j \alpha m \theta k)) = \beta_{\psi_w(\varepsilon(j) \alpha \varepsilon(m) \theta \varepsilon(k))}$
 $\leq \max \{ \beta_{\psi_w(\varepsilon(j))}, \beta_{\psi_w(\varepsilon(m))}, \beta_{\psi_w(\varepsilon(k))} \}$
 $= \max \{ (\psi\varepsilon)_w(j), (\psi\varepsilon)_w(m), (\psi\varepsilon)_w(k) \}$.

Therefore $(\psi\varepsilon)_w(m)$ is a tripolar fuzzy soft Γ -subsemiring of M . Thus $(\psi\varepsilon, W, \Gamma)$ is a tripolar fuzzy soft ternary Γ -semiring over M_2 .

Theorem 3.6: If (φ, W, Γ) TFTSR over TGSR M , χ is an endomorphism of M and defined $(\varphi\chi)_w = \varphi_w\chi$ for each $w \in W$. Then $(\varphi\chi, W, \Gamma)$ is a TFSTGSR over M .

Proof: Suppose $j, m, k \in M, w \in W$ and $\alpha, \theta \in \Gamma$. Then

1. $(v\chi)_w(j + m) = v_{\varphi(w)}(\chi(j + m)) = v_{\varphi(w)}(\chi(j) + \chi(m))$
 $\geq \min \{ v_{\varphi(w)}(\chi(j)), v_{\varphi(w)}(\chi(m)) \}$
 $= \min \{ ((v\chi)_w(j)), ((v\chi)_w(m)) \}$
2. $(\gamma\chi)_w(j + m) = \gamma_{\varphi(w)}(\chi(j + m)) = \gamma_{\varphi(w)}(\chi(j) + \chi(m))$
 $\leq \max \{ \gamma_{\varphi(w)}(\chi(j)), \gamma_{\varphi(w)}(\chi(m)) \}$
 $= \max \{ ((\gamma\chi)_w(j)), ((\gamma\chi)_w(m)) \}$
3. $(\beta\chi)_w(j + m) = \beta_{\varphi(w)}(\chi(j + m)) = \beta_{\varphi(w)}(\chi(j) + \chi(m))$
 $\leq \max \{ \beta_{\varphi(w)}(\chi(j)), \beta_{\varphi(w)}(\chi(m)) \}$
 $= \max \{ ((\beta\chi)_w(j)), ((\beta\chi)_w(m)) \}$
4. $(v\chi)_w(j\alpha m\theta k) = v_{\varphi(w)}(\chi(j\alpha m\theta k)) = v_{\varphi(w)}(\chi(j)\alpha\chi(m)\theta\chi(k))$
 $\geq \min \{ v_{\varphi(w)}(\chi(j)), v_{\varphi(w)}(\chi(m)), v_{\varphi(w)}(\chi(k)) \}$
 $= \min \{ (v\chi)_w(j), (v\chi)_w(m), (v\chi)_w(k) \}$
5. $(\gamma\chi)_w(j\alpha m\theta k) = \gamma_{\varphi(w)}(\chi(j\alpha m\theta k)) = \gamma_{\varphi(w)}(\chi(j)\alpha\chi(m)\theta\chi(k))$
 $\leq \max \{ \gamma_{\varphi(w)}(\chi(j)), \gamma_{\varphi(w)}(\chi(m)), \gamma_{\varphi(w)}(\chi(k)) \}$
 $= \max \{ (\gamma\chi)_w(j), (\gamma\chi)_w(m), (\gamma\chi)_w(k) \}$
6. $(\beta\chi)_w(j\alpha m\theta k) = \beta_{\varphi(w)}(\chi(j\alpha m\theta k)) = \beta_{\varphi(w)}(\chi(j)\alpha\chi(m)\theta\chi(k))$
 $\leq \max \{ \beta_{\varphi(w)}(\chi(j)), \beta_{\varphi(w)}(\chi(m)), \beta_{\varphi(w)}(\chi(k)) \}$
 $= \max \{ (\beta\chi)_w(j), (\beta\chi)_w(m), (\beta\chi)_w(k) \}$

Thus $(\varphi\chi)_w$ is a TFGSR of M . Then $(\varphi\chi, W, \Gamma)$ is a TFSTGSR over M .

Theorem 3.7: If $\varphi : M_1 \rightarrow M_2$ is an epimorphism of TGSR and (ρ, W, Γ) is a “tripolar fuzzy soft right ideal” over M_2 . If for each $w \in W, \zeta_w = \varphi^{-1}(\rho_w)$ then (ζ, W, Γ) is a “tripolar fuzzy soft right ideal” over M_1 .

Proof: If $w \in W$ and $\alpha, \theta \in \Gamma$. Then ρ_x is a “tripolar fuzzy soft right ideal” over M_2 . If $j, m, k \in M_1$ and $\alpha, \theta \in \Gamma$, then:

1. $\varphi^{-1}(v_w(j + m)) = v_{\rho(w)}(\varphi(j + m)) = v_{\rho(w)}(\varphi(j) + \varphi(m))$
 $\geq \min \{ v_{\rho(w)}(\varphi(j)), v_{\rho(w)}(\varphi(m)) \}$
 $= \min \{ \varphi^{-1}(v_w(j)), \varphi^{-1}(v_w(m)) \}$
2. $\varphi^{-1}(\gamma_w)(j + m) = \gamma_{\rho(w)}(\varphi(j + m)) = \gamma_{\rho(w)}(\varphi(j) + \varphi(m))$
 $\leq \max \{ \gamma_{\rho(w)}(\varphi(j)), \gamma_{\rho(w)}(\varphi(m)) \}$
 $= \max \{ \varphi^{-1}(\gamma_w)(j), \varphi^{-1}(\gamma_w)(m) \}$
3. $\varphi^{-1}(\beta_w)(j + m) = \beta_{\rho(w)}(\varphi(j + m)) = \beta_{\rho(w)}(\varphi(j) + \varphi(m))$
 $\leq \max \{ \beta_{\rho(w)}(\varphi(j)), \beta_{\rho(w)}(\varphi(m)) \}$

$$= \max\{\varphi^{-1}(\beta_w)(j), \varphi^{-1}(\beta_w)(m)\}.$$

$$4. \varphi^{-1}(v_w(j\alpha m\theta k)) = v_{\rho(w)}(\varphi(j\alpha m\theta k)) = v_{\rho(w)}(\varphi(j)\alpha\varphi(m)\theta\varphi(k)) \\ \geq v_{\rho(w)}(\varphi(j)) = \varphi^{-1}(v_w(j)).$$

$$5. \varphi^{-1}(\gamma_w(j\alpha m\theta k)) = \gamma_{\rho(w)}(\varphi(j\alpha m\theta k)) = \gamma_{\rho(w)}(\varphi(j)\alpha\varphi(m)\theta\varphi(k)) \\ \leq \gamma_{\rho(w)}(\varphi(j)) = \varphi^{-1}(\gamma_w(j)).$$

$$6. \varphi^{-1}(\beta_w(j\alpha m\theta k)) = \beta_{\rho(w)}(\varphi(j\alpha m\theta k)) = \beta_{\rho(w)}(\varphi(j)\alpha\varphi(m)\theta\varphi(k)) \\ \leq \beta_{\rho(w)}(\varphi(j)) = \varphi^{-1}(\beta_w(j)).$$

Therefore $\zeta_w = \varphi^{-1}(\rho_w)$ is a “tripolar fuzzy right ideal” of M_1 . Thus (ζ, W, Γ) is a “tripolar fuzzy soft right ideal” over M_1 .

Theorem 3.7 is also true for tripolar fuzzy left ideal.

Proposition 3.8. If M_1 and M_2 are TGSRs, $\chi : M_1 \rightarrow M_2$ is a TGSRH and φ is a χ -invariant bipolar fuzzy subset of M_1 , if $w = \chi(u)$ then $\chi(\varphi)(w) = \varphi(u); u \in M_1$.

Proof: Straight forward.

Theorem 3.9: If (φ, W, Γ) is a tripolar fuzzy soft right ideal over TGSR M_1 and χ is a homomorphism from M_1 onto M_2 . For each $w \in W$, φ_w is a χ -invariant bipolar fuzzy right ideal of M_1 , if $\zeta_w = \chi(\varphi_w)$ then (ζ, W, Γ) is a tripolar fuzzy soft right ideal over M_2 .

Proof: Let $m_1, m_2, m_3 \in M_2, w \in W, \alpha, \beta \in \Gamma$. Then $\exists m_4, m_5, m_6 \in M_1 \ni \chi(m_4) = m_1, \chi(m_5) = m_2, \chi(m_6) = m_3, m_1 + m_2 = \chi(m_4 + m_5)$ & $m_1\alpha m_2\beta m_3 = \chi(m_4\alpha m_5\beta m_6)$. φ_w is a χ -invariant. Thus, by proposition 3.8, we have:

1. $v_{\zeta(w)}(m_1 + m_2) = \chi(v_{\varphi_w})(m_1 + m_2) = v_{\varphi_w}(m_4 + m_5) \\ \geq \min\{v_{\varphi_w}(m_4), v_{\varphi_w}(m_5)\} \\ = \min\{\chi(v_{\varphi_w})(m_1), \chi(v_{\varphi_w})(m_2)\} \\ = \min\{v_{\zeta(w)}(m_1), v_{\zeta(w)}(m_2)\}$
2. $\gamma_{\zeta(w)}(m_1 + m_2) = \chi(\gamma_{\varphi_w})(m_1 + m_2) = \gamma_{\varphi_w}(m_4 + m_5) \\ \leq \max\{\gamma_{\varphi_w}(m_4), \gamma_{\varphi_w}(m_5)\} \\ = \max\{\chi(\gamma_{\varphi_w})(m_1), \chi(\gamma_{\varphi_w})(m_2)\} \\ = \max\{\gamma_{\zeta(w)}(m_1), \gamma_{\zeta(w)}(m_2)\}$
3. $\beta_{\zeta(w)}(m_1 + m_2) = \chi(\beta_{\varphi_w})(m_1 + m_2) = \beta_{\varphi_w}(m_4 + m_5) \\ \leq \max\{\beta_{\varphi_w}(m_4), \beta_{\varphi_w}(m_5)\} \\ = \max\{\chi(\beta_{\varphi_w})(m_1), \chi(\beta_{\varphi_w})(m_2)\} \\ = \max\{\beta_{\zeta(w)}(m_1), \beta_{\zeta(w)}(m_2)\}$
4. $v_{\zeta(w)}(m_1\alpha m_2\beta m_3) = \chi(v_{\varphi_w})(m_1\alpha m_2\beta m_3) = v_{\varphi_w}(\chi(m_4\alpha m_5\beta m_6)) \\ = v_{\varphi_w}(\chi(m_4)\alpha\chi(m_5)\beta\chi(m_6)) \geq v_{\varphi_w}(\chi(m_4)) \\ = \chi(v_{\varphi_w})(m_1) = v_{\zeta(w)}(m_1).$
5. $\gamma_{\zeta(w)}(m_1\alpha m_2\beta m_3) = \chi(\gamma_{\varphi_w})(m_1\alpha m_2\beta m_3) = \gamma_{\varphi_w}(\chi(m_4\alpha m_5\beta m_6))$

$$= \gamma_{\varphi_w}(\chi(m_4)\alpha\chi(m_5)\beta\chi(m_6)) \leq \gamma_{\varphi_w}(\chi(m_4))$$

$$= \chi(\gamma_{\varphi_w})(m_1) = \gamma_{\zeta(w)}(m_1).$$

$$6. \quad \beta_{\zeta(w)}(m_1\alpha m_2\beta m_3) = \chi(\beta_{\varphi_w})(m_1\alpha m_2\beta m_3) = \beta_{\varphi_w}(\chi(m_4)\alpha m_5\beta m_6))$$

$$= \beta_{\varphi_w}(\chi(m_4)\alpha\chi(m_5)\beta\chi(m_6)) \leq \beta_{\varphi_w}(\chi(m_4))$$

$$= \chi(\beta_{\varphi_w})(m_1) = \beta_{\zeta(w)}(m_1).$$

then ζ_w is a tripolar fuzzy ideal of M_2 . Hence (ζ, W, Γ) is a tripolar fuzzy soft right ideal over M_2 .

Theorem 3.10: If (φ_1, W_1, Γ) and (φ_2, W_2, Γ) are two bipolar FSTGSR over M_1 and M_2 respectively, and (χ, ψ) is a TFSTGSRH from (φ_1, W_1, Γ) onto (φ_2, W_2, Γ) . Then $(\chi(\varphi_1), M_2, \Gamma)$ is a TFSTGSR over M_2 .

Proof: By definition 3.2., χ is a TGSRH from M_1 into M_2 and ψ is a mapping from W_1 into W_2 . For each $w_2 \in W_2 \exists w_1 \in W_1 \ni \psi(w_1) = w_2$.

Define $(\chi(\varphi))_{w_2} = \chi(\varphi_{w_1})$. If $m_4, m_5, m_6 \in M_2, \alpha, \theta \in \Gamma$.

Then $\exists m_1, m_2, m_3 \in M_1 \ni \chi(m_1) = m_4, \chi(m_2) = m_5, \chi(m_3) = m_6, \chi(m_1 + m_2) = m_4 + m_5$ & $\chi(m_1\alpha m_2\theta m_3) = m_4\alpha m_5\theta m_6$. Thus, we get

$$1. \quad (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_1 + m_2) = (\chi(v_{(\varphi_1)}(w_1)))(m_1 + m_2) = v_{(\varphi_1)}(w_1)(m_4 + m_5)$$

$$\geq \min \{ v_{(\varphi_1)}(w_1)(m_4), v_{(\varphi_1)}(w_1)(m_5) \}$$

$$= \min \{ (\chi(v_{(\varphi_1)}(w_1)))(m_1), (\chi(v_{(\varphi_1)}(w_1)))(m_2) \}$$

$$= \min \{ (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_2) \}$$

$$2. \quad (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_1 + m_2) = (\chi(\gamma_{(\varphi_1)}(w_1)))(m_1 + m_2) = \gamma_{(\varphi_1)}(w_1)(m_4 + m_5)$$

$$\leq \max \{ \gamma_{(\varphi_1)}(w_1)(m_4), \gamma_{(\varphi_1)}(w_1)(m_5) \}$$

$$= \max \{ (\chi(\gamma_{(\varphi_1)}(w_1)))(m_1), (\chi(\gamma_{(\varphi_1)}(w_1)))(m_2) \}$$

$$= \max \{ (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_2) \}$$

$$3. \quad (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_1 + m_2) = (\chi(\beta_{(\varphi_1)}(w_1)))(m_1 + m_2) = \beta_{(\varphi_1)}(w_1)(m_4 + m_5)$$

$$\leq \max \{ \beta_{(\varphi_1)}(w_1)(m_4), \beta_{(\varphi_1)}(w_1)(m_5) \}$$

$$= \max \{ (\chi(\beta_{(\varphi_1)}(w_1)))(m_1), (\chi(\beta_{(\varphi_1)}(w_1)))(m_2) \}$$

$$= \max \{ (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_2) \}$$

$$4. \quad (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_1\alpha m_2\theta m_3) = (\chi(v_{(\varphi_1)}(w_1)))(m_1\alpha m_2\theta m_3) = v_{(\varphi_1)}(w_1)(m_4\alpha m_5\theta m_6)$$

$$\geq \min \{ v_{(\varphi_1)}(w_1)(m_4), v_{(\varphi_1)}(w_1)(m_5), v_{(\varphi_1)}(w_1)(m_6) \}$$

$$= \min \{ (\chi(v_{(\varphi_1)}(w_1)))(m_1), (\chi(v_{(\varphi_1)}(w_1)))(m_2), (\chi(v_{(\varphi_1)}(w_1)))(m_3) \}$$

$$= \min \{ (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_2), (\chi(v_{(\varphi_1)}))_{\psi(w_1)}(m_3) \}$$

$$5. \quad (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_1\alpha m_2\theta m_3) = (\chi(\gamma_{(\varphi_1)}(w_1)))(m_1\alpha m_2\theta m_3) = \gamma_{(\varphi_1)}(w_1)(m_4\alpha m_5\theta m_6)$$

$$\leq \max \{ \gamma_{(\varphi_1)}(w_1)(m_4), \gamma_{(\varphi_1)}(w_1)(m_5), \gamma_{(\varphi_1)}(w_1)(m_6) \}$$

$$= \max \{ (\chi(\gamma_{(\varphi_1)}(w_1)))(m_1), (\chi(\gamma_{(\varphi_1)}(w_1)))(m_2), (\chi(\gamma_{(\varphi_1)}(w_1)))(m_3) \}$$

$$\begin{aligned}
 &= \max \{ (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_2), (\chi(\gamma_{(\varphi_1)}))_{\psi(w_1)}(m_3) \} \\
 6. \quad &(\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_1 \alpha m_2 \theta m_3) = (\chi(\beta_{(\varphi_1)}(w_1)))(m_1 \alpha m_2 \theta m_3) = \beta_{(\varphi_1)}(w_1)(m_4 \alpha m_5 \theta m_6) \\
 &\leq \max \{ \beta_{(\varphi_1)}(w_1)(m_4), \beta_{(\varphi_1)}(w_1)(m_5), \beta_{(\varphi_1)}(w_1)(m_6) \} \\
 &= \max \{ (\chi(\beta_{(\varphi_1)}(w_1)))(m_1), (\chi(\beta_{(\varphi_1)}(w_1)))(m_2), (\chi(\beta_{(\varphi_1)}(w_1)))(m_6) \} \\
 &= \max \{ (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_1), (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_2), (\chi(\beta_{(\varphi_1)}))_{\psi(w_1)}(m_3) \}
 \end{aligned}$$

Then $(\chi(\varphi))_{w_2}$ is a “tripolar fuzzy ternary Γ -sub semiring” of M_2 . Hence $(\chi(\varphi_1), M_2, \Gamma)$ is a “tripolar fuzzy soft ternary Γ -semiring” over M_2 .

Theorem 3.11: If M_1 and M_2 are TGSRs, $\chi : M_1 \rightarrow M_2$ is a TGSRH, (φ_1, W_1, Γ) and (φ_2, W_2, Γ) are TFSTGSR over M_1 and (φ_1, W_1, Γ) is a “tripolar fuzzy soft ternary Γ -sub semiring” of (φ_2, W_2, Γ) . Then $(\chi(\varphi_1), W_1, \Gamma)$ and $(\chi(\varphi_2), W_2, \Gamma)$ are “tripolar fuzzy soft ternary Γ -sub semirings” over M_2 and $(\chi(\varphi_1), W_1, \Gamma)$ is a “tripolar fuzzy soft ternary Γ -sub semiring” of $(\chi(\varphi_2), W_2, \Gamma)$.

Proof: Since $(\chi(\varphi_1))_{w_1} = (\chi(\varphi_1(w_1)))$ is a tripolar fuzzy ternary Γ -sub semiring of $M_2 \forall w_1 \in W_1$ and $(\chi(\varphi_2))_{w_2} = (\chi(\varphi_2(w_2)))$ is a tripolar fuzzy ternary Γ -sub semiring of $M_2 \forall w_2 \in W_2$. Hence $(\chi(\varphi_1), W_1, \Gamma)$ and $(\chi(\varphi_2), W_2, \Gamma)$ are tripolar fuzzy soft ternary Γ -semiring over M_2 . Since (φ_1, W_1, Γ) is a tripolar fuzzy soft ternary Γ -sub semiring of (φ_2, W_2, Γ) . $\varphi_1(w_1)$ is a tripolar fuzzy ternary sub semiring of $\varphi_2(w_2)$. Hence $(\chi(\varphi_1(w_1)))$ is a tripolar fuzzy ternary Γ -sub semiring of $(\chi(\varphi_2(w_2))) \forall w_1 \in W_1$. Therefore $(\chi(\varphi_1), W_1, \Gamma)$ is a tripolar fuzzy soft ternary Γ -sub semiring of $(\chi(\varphi_2), W_2, \Gamma)$.

Theorem 3.12: If (φ_1, W_1, Γ) and (φ_2, W_2, Γ) are TFSTGSRs over M_1 and M_2 respectively and (χ, ψ) is a TFSH from (φ_1, W_1, Γ) onto (φ_2, W_2, Γ) . then the pre-image of (φ_2, W_2, Γ) under TFSTGSRH is a “tripolar fuzzy soft ternary Γ -sub semiring” of (φ_1, W_1, Γ) over M_1 .

Proof: By definition 3.4., $(\chi, \psi)^{-1}(\varphi_2, W_2, \Gamma) = (\chi^{-1}(\varphi_2), \psi^{-1}(W_2))$.

Define $(\chi^{-1}(\varphi_2))_{w_1}(m_1) = \varphi_{2, \psi(w_1)}(\chi(m_1)) \forall m_1 \in M_1 \& w_1 \in \psi^{-1}(W_2)$.

If $m_1, m_2, m_3 \in M_1 \& \alpha, \theta \in \Gamma$. Then

1. $(\chi^{-1}(\lambda_{\varphi_2}))_{w_1}(m_1 + m_2) = \lambda_{\varphi_2, \psi(w_1)}(\chi(m_1 + m_2)) = \lambda_{\varphi_2, \psi(w_1)}(\chi(m_1) + \chi(m_2))$
 $\geq \min \{ \lambda_{\varphi_2, \psi(w_1)}(\chi(m_1)), \lambda_{\varphi_2, \psi(w_1)}(\chi(m_2)) \}$
 $= \min \{ \chi^{-1}(\lambda_{\varphi_2})(m_1), \chi^{-1}(\lambda_{\varphi_2})(m_2) \}$
2. $(\chi^{-1}(\gamma_{\varphi_2}))_{w_1}(m_1 + m_2) = \gamma_{\varphi_2, \psi(w_1)}(\chi(m_1 + m_2)) = \gamma_{\varphi_2, \psi(w_1)}(\chi(m_1) + \chi(m_2))$
 $\leq \max \{ \gamma_{\varphi_2, \psi(w_1)}(\chi(m_1)), \gamma_{\varphi_2, \psi(w_1)}(\chi(m_2)) \}$
 $= \max \{ \chi^{-1}(\gamma_{\varphi_2})(m_1), \chi^{-1}(\gamma_{\varphi_2})(m_2) \}$
3. $(\chi^{-1}(\beta_{\varphi_2}))_{w_1}(m_1 + m_2) = \beta_{\varphi_2, \psi(w_1)}(\chi(m_1 + m_2)) = \beta_{\varphi_2, \psi(w_1)}(\chi(m_1) + \chi(m_2))$
 $\leq \max \{ \beta_{\varphi_2, \psi(w_1)}(\chi(m_1)), \beta_{\varphi_2, \psi(w_1)}(\chi(m_2)) \}$
 $= \max \{ \chi^{-1}(\beta_{\varphi_2})(m_1), \chi^{-1}(\beta_{\varphi_2})(m_2) \}$

$$4. \quad (\chi^{-1}(\lambda_{\varphi_2}))_{w_1}(m_1 \alpha m_2 \theta m_3) = \lambda_{\varphi_2 \psi(w_1)}(\chi(m_1 \alpha m_2 \theta m_3)) = \lambda_{\varphi_2 \psi(w_1)}(\chi(m_1) \alpha \chi(m_2) \theta \chi(m_3))$$

$$\geq \min \{ \lambda_{\varphi_2 \psi(w_1)}(\chi(m_1)), \lambda_{\varphi_2 \psi(w_1)}(\chi(m_2)), \lambda_{\varphi_2 \psi(w_1)}(\chi(m_3)) \}$$

$$= \min \{ \chi^{-1}(\lambda_{\varphi_2})(m_1), \chi^{-1}(\lambda_{\varphi_2})(m_2), \chi^{-1}(\lambda_{\varphi_2})(m_3) \}$$

$$5. \quad (\chi^{-1}(\gamma_{\varphi_2}))_{w_1}(m_1 \alpha m_2 \theta m_3) = \gamma_{\varphi_2 \psi(w_1)}(\chi(m_1 \alpha m_2 \theta m_3)) = \gamma_{\varphi_2 \psi(w_1)}(\chi(m_1) \alpha \chi(m_2) \theta \chi(m_3))$$

$$\leq \max \{ \gamma_{\varphi_2 \psi(w_1)}(\chi(m_1)), \gamma_{\varphi_2 \psi(w_1)}(\chi(m_2)), \gamma_{\varphi_2 \psi(w_1)}(\chi(m_3)) \}$$

$$= \max \{ \chi^{-1}(\gamma_{\varphi_2})(m_1), \chi^{-1}(\gamma_{\varphi_2})(m_2), \chi^{-1}(\gamma_{\varphi_2})(m_3) \}$$

$$6. \quad (\chi^{-1}(\beta_{\varphi_2}))_{w_1}(m_1 \alpha m_2 \theta m_3) = \beta_{\varphi_2 \psi(w_1)}(\chi(m_1 \alpha m_2 \theta m_3)) = \beta_{\varphi_2 \psi(w_1)}(\chi(m_1) \alpha \chi(m_2) \theta \chi(m_3))$$

$$\leq \max \{ \beta_{\varphi_2 \psi(w_1)}(\chi(m_1)), \beta_{\varphi_2 \psi(w_1)}(\chi(m_2)), \beta_{\varphi_2 \psi(w_1)}(\chi(m_3)) \}$$

$$= \max \{ \chi^{-1}(\beta_{\varphi_2})(m_1), \chi^{-1}(\beta_{\varphi_2})(m_2), \chi^{-1}(\beta_{\varphi_2})(m_3) \}$$

Thus $(\chi^{-1}(\varphi_2))_{w_1}$ is a “tripolar fuzzy ternary Γ -sub semiring” of $M_1 \forall w_1 \in \psi^{-1}(W_2)$.

$\therefore (\chi^{-1}(\varphi_2), \psi^{-1}(W_2))$ is a “tripolar fuzzy soft ternary Γ -sub semiring” of (φ_1, W_1, Γ) over M_1 .

4. Conclusion:

Here, we investigate the notion of TFSTGSRH and studied some properties of homomorphic image and pre image of TFSTGSR. These notions are basic supporting structures for development the structure of soft set. This work can be extended to the properties of different concepts of kernel of TFSTGSRH, “tripolar fuzzy soft filters over ternary Γ -semirings”.

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