

Mean Cordial Labeling in Graph Representations of Human Anatomy and Circulatory Systems

A. Anto Cathrin Aanisha¹, R. Manoharan²

¹-Research Scholar, Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai, India

²-Assistant Professor, Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai, India. Email Id: aanishaanto@gmail.com¹, mano_rl@yahoo.co.in²

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Abstract:

Let h be the map from $W(\Omega)$ to $\{0,1,2\}$. For each edge zw , apply the label $\left\lceil \frac{h(z)+h(w)}{2} \right\rceil$. h is considered a mean cordial labelling if $|z_h(m)-z_h(n)| \leq 1$ and $|f_h(m)-f_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$, $z(y)$ and $f(y)$ denote the count of nodes and arcs, respectively, labeled with y ($y=0,1,2$). In this study, we investigate the notion of mean cordial labeling in the three-dimensional portrayal of the human body, as well as in graphical depictions of blood circulation in the heart and the circulatory systems of the human heart and kidney.

Keywords: Mean cordial labeling, 3D representation of the human body, Blood Circulation Digraphs in Heart and Kidney.

1. Introduction:

Graph theory involves the examination of mathematical constructs referred to as graphs, which serve as tools for illustrating pairwise connections among entities in disciplines such as mathematics and computer science (Prathik et al., 2016). Graph labeling is a field within graph theory, a branch of mathematics, which focuses on assigning labels, usually numbers, to the edges or vertices, or both, of a graph according to certain rules (Gallian, 2022). Graph labeling is crucial due to its wide range of applications across various domains, including circuit design, radar technology, communication network addressing, and more. In various aspects of computer science and communication networks, network representations are playing a vital role (Pir et al., 2023). (Pir & Parthiban, 2022) 's research paper presents prime distance labeling of generalized Petersen graphs and cycles, exploring distinct labeling techniques and the study highlights interesting applications, including potential uses in graph-based cryptography. This innovative approach could enhance cryptographic systems' security measures. Graph labeling has significant applications in web design also. In a web graph, web pages are represented by vertices, and hyperlinks by edges. Labeling these elements helps in efficiently finding and organizing attractive information. Another application is in website communities, where vertices represent classes of objects and edges represent connections between them. In graph theory, this forms a complete graph, denoted as K_n , where each vertex is connected to every other vertex. This complete interconnectivity facilitates comprehensive analysis and navigation within web communities (Dhanalakshmi et al., 2022). The primary goal is to explore the function of graph labeling in the communication sector. Additionally, graph labeling simplifies tasks in various networking-related domains, making it a powerful tool. This summary illustrates the concept, helping researchers

understand its uses and identify potential research directions (Sumathi & Jethruth Emelda Mary, 2022). Furthermore, graph labeling has various application in medical field. Particularly, (Aravindan et al., 2020) applied sum cordial labeling to the study of blood circulation from the human heart to the kidney. Also, cordial labeling is the basement of all other labeling. There are many labeling of cordial labeling. They are sum divisor cordial labeling, prime cordial labeling, edge sum divisor cordial labeling and product cordial labeling etc., Mean cordial labeling is also derived from cordial labeling. introduced a new concept called mean cordial labeling and investigated its behavior in various standard graphs, such as paths, cycles, stars, complete graphs, and combs. The symbol $[x]$ stands for the smallest integer greater than or equal to x (Ponraj et al., 2012). So, here, we explore the concept of prime labeling in the 3D representation of the human body, as well as in digraph representations of blood circulation in the heart and the circulatory systems of the human heart and kidney.

2. Main Results:

Theorem: 2.1

Mean cordial labeling applies in the graphical portrayal of the blood circulation within the heart.

Proof:

Let Ω be the graphical portrayal of the blood circulation within the heart.

Let $W(\Omega) = \{z_1, z_2, \dots, z_{10}\}$ be the node set of Ω and $F(\Omega) = \{f_k = z_k z_{k+1}: 1 \leq k \leq 3; f_4 = z_1 z_4; f_{k+4} = z_{6k-4} z_{10}: 1 \leq k \leq 2; f_{k+6} = z_{k+3} z_{k+4}: 1 \leq k \leq 2; f_{k+8} = z_{k+6} z_{k+7}: 1 \leq k \leq 2\}$ be the arc set of Ω .

Let h be a map from $W(\Omega)$ to $\{0, 1, 2\}$

Define, $h(z_k) = 0; 1 \leq k \leq 4$

$h(z_5) = 1$

$h(z_{2k+6}) = 1; 1 \leq k \leq 2$

$h(z_{k+5}) = 2; 1 \leq k \leq 2$

$h(z_9) = 2$

So, we can conclude that,

$$|z_h(0) - z_h(1)| = 1, |z_h(0) - z_h(2)| = 1 \text{ and } |z_h(1) - z_h(2)| = 0$$

So, we can say that, $|z_h(m) - z_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Also, we can conclude that,

$$|f_h(0) - f_h(1)| = 1, |f_h(0) - f_h(2)| = 1 \text{ and } |f_h(1) - f_h(2)| = 0$$

So, we can say that, $|f_h(m) - f_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Hence, mean cordial labeling applied in the graphical portrayal of the blood circulation within the heart.

Hence proved.

Example: 2.2

The graphical portrayal of the blood circulation within the heart (Othman et al., 2022) with mean cordial labeling is shown in figure 1.

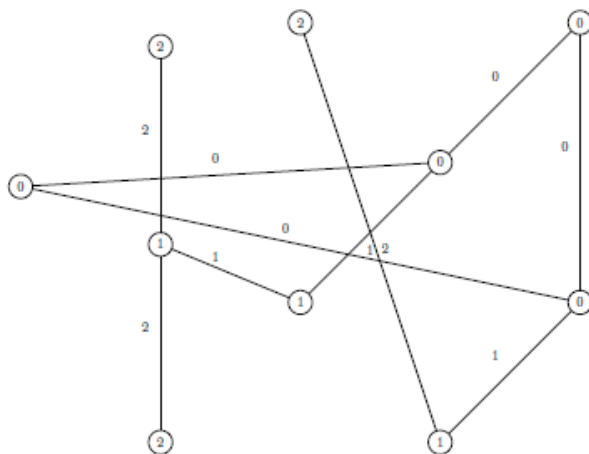


Figure 1

Theorem: 2.3

Mean cordial labeling applies to the graphical portrayal of blood circulation from the human heart to the kidney.

Proof:

Let Ω be the graph representation of blood circulation from the human heart to the kidney.

Let $W(\Omega) = \{z_1, z_2, \dots, z_{12}\}$ be the node set of Ω and $F(\Omega) = \{f_k = z_2 z_{2k-1}: 1 \leq k \leq 2; f_{k+2} = z_5 z_{3k+1}: 1 \leq k \leq 2; f_{k+4} = z_8 z_{7k-5}: 1 \leq k \leq 2; f_7 = z_6 z_{12}; f_8 = z_6 z_7; f_{k+8} = z_{10} z_{3k+6}: 1 \leq k \leq 2; f_{k+10} = z_{11} z_{3k+6}: 1 \leq k \leq 2\}$ be the arc set of Ω .

Let h be a map from $W(\Omega)$ to $\{0, 1, 2\}$

Define, $h(z_1) = 2$

$h(z_{k+2}) = 2; 1 \leq k \leq 3$

$h(z_2) = 1$

$h(z_{k+5}) = 1; 1 \leq k \leq 3$

$h(z_{k+8}) = 0; 1 \leq k \leq 4$

So, we can conclude that,

$$|z_h(0) - z_h(1)| = 0, |z_h(0) - z_h(2)| = 0 \text{ and } |z_h(1) - z_h(2)| = 0$$

So, we can say that, $|z_h(m) - z_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Also, we can conclude that,

$$|f_h(0) - f_h(1)| = 0, |f_h(0) - f_h(2)| = 0 \text{ and } |f_h(1) - f_h(2)| = 0$$

So, we can say that, $|f_h(m) - f_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Hence, mean cordial labeling applied to the graphical portrayal of blood circulation from the human heart to the kidney.

Hence proved.

Example: 2.4

The graphical portrayal of blood circulation from the human heart to the kidney (Aravindan et al., 2020) with mean cordial labeling is shown in figure 2.

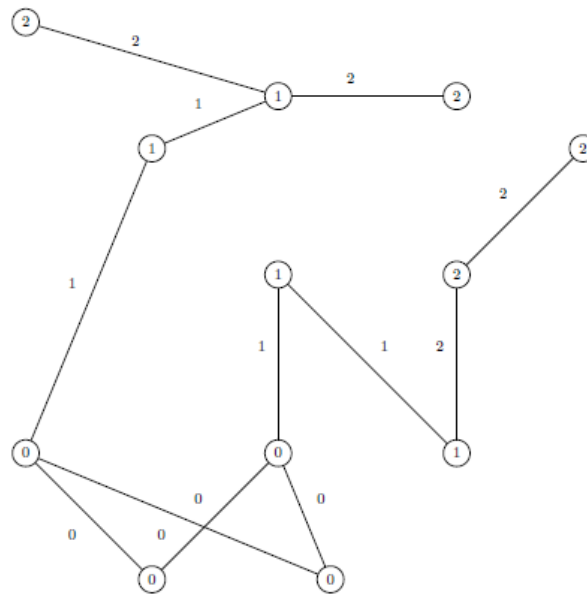


Figure 2

Theorem: 2.5

Mean cordial labeling applies to the 3D graphical portrayal of the human body.

Proof:

Let Ω be the 3D graphical portrayal of the human body.

Let $W(\Omega) = \{z_1, z_2, \dots, z_{11}\}$ be the node set of Ω and $F(\Omega) = \{f_k = z_k z_{k+1}: 1 \leq k \leq 3; f_{k+3} = z_5 z_{4k-2}: 1 \leq k \leq 2; f_{k+5} = z_7 z_{8k-6}: 1 \leq k \leq 2; f_8 = z_{10} z_{11}; f_{k+8} = z_8 z_{2k+5}: 1 \leq k \leq 2\}$ be the arc set of Ω .

Let h be a map from $W(\Omega)$ to $\{0, 1, 2\}$

Define, $h(z_k) = 1; 1 \leq k \leq 3$

$h(z_9) = 1$

$h(z_{k+3}) = 2; 1 \leq k \leq 3$

$h(z_{k+6}) = 0; 1 \leq k \leq 2$

$h(z_{k+9}) = 0; 1 \leq k \leq 2$

So, we can conclude that,

$$|z_h(0) - z_h(1)| = 0, |z_h(0) - z_h(2)| = 1 \text{ and } |z_h(1) - z_h(2)| = 1$$

So, we can say that, $|z_h(m) - z_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Also, we can conclude that,

$$|f_h(0) - f_h(1)| = 1, |f_h(0) - f_h(2)| = 0 \text{ and } |f_h(1) - f_h(2)| = 1$$

So, we can say that, $|f_h(m) - f_h(n)| \leq 1$, where $m, n \in \{0, 1, 2\}$

Hence, mean cordial labeling applied to the 3D graphical portrayal of the human body.

Example: 2.6

The 3D graphical portrayal of the human body (Thome et al., 2006) with mean cordial labeling is shown in figure 3.

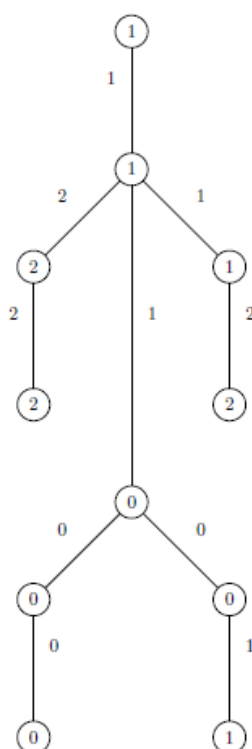


Figure 3

3. Conclusion:

In this paper, we have shown that 3D representation of the human body, the graph representations of blood circulation in the heart and the circulatory systems of the human heart and kidney have the property of mean cordial labeling. This type of labeling techniques may provide valuable tools for further research in computational biology and bioinformatics, facilitating more accurate and insightful modeling of human physiological processes.

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