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Fuzzy Soft Ideals of Fuzzy Soft Ternary Γ- Semirings

T. Satish[1], D. Madhusudhana Rao[2], T. Srinivas [3], M. Vasantha[4], M. Sajani Lavanya[5]

[1]Assistant Professor, Department of Mathematics, SRKR Engineering College (A), Bhimavaram, A.P, India. e-mail: tsatishmaths@gmail.com

[2]Profesor, Department of Mathematics, Govt. Degree College for women's (A), Guntur, A.P, India.

[3]Assistant Professor, Department of Science and Humanities, Vasireddy Venkatadri Institute of Technology, Namburu, Guntur (Dist), A.P, India.

[4]Assistant Professor, Department of H&S (Mathematics), Malla Reddy Engineering College for Women, Hyderabad, T.S, India

[5]Lecturer in Mathematics, Department of Mathematics, Government College (A), Rajahmundry, A.P, India.

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Abstract:

we introduce the notation of specific classes of fuzzy soft tideals in fuzzy soft ternary Γ-Semirings in this paper. We examine several relations between distinct types of fuzzy soft tideals in fuzzy soft ternary Γ-Semirings.

Keywords: Fuzzy soft ternary \Gamma-semiring (FSTΓSR), fuzzy soft tideals (FSIs), fuzzy soft prime tideal (FSPI), fuzzy soft semiprime tideal (FS-SPI), fuzzy soft strongly prime tideal (FSSPI), fuzzy soft irreducible tideal (FSII), fuzzy soft strongly irreducible tideal (FSSII).

1. Introduction

We deal with certain situations in our daily lives these days for which there is no complete information available. Mathematical models are created to address scenarios including uncertainty. An extension of standard set theory is the basis for the majority of these models. Up until now, the theory of fuzzy sets may have been the most suitable theory to deal with situations involving uncertainty. However, this idea faces significant challenges, most likely as a result of inadequate parameters. People are naturally attempting to get out of this circumstance. D. Molodtsov [1] developed the idea of a soft set for this reason by incorporating sufficient parameters. A generalized mathematical tool for handling uncertain phenomena is the theory of soft sets. Medical fields successfully employ soft sets.

We address Fuzzy soft TΓ-SR in this study. We also aim to use these FSIs to characterize completely regular fuzzy soft TΓ-SRs and to the properties of prime tideal, semiprime tideal, irreducible tideal in the context of fuzzy soft set. Our goal is to present a novel idea of fuzzy soft tideals, such as fuzzy soft prime tideal, fuzzy soft semiprime tideals of fuzzy soft TΓ-SR.

2. Preliminaries

For Preliminaries refer to references.

3. Fuzzy Soft Prime tideal and fuzzy Soft Semi Prime tideal

This section covers fuzzy soft prime tideal (FSPI) and fuzzy soft semi prime tideals (FS-SPI) along with a discussion of some of their characteristics.

Definition 3.1: A FΓSS (f,M,Γ) over a TΓ-SR S is said to be a FSTΓ-SR over S, if $(f,M,\Gamma)\circ(f,M,\Gamma)\circ(f,M,\Gamma)\subseteq(f,M,\Gamma)$.

Def 3.2: A FΓSS $(f,M,\Gamma)(\neq\phi)$ over a TΓ-SR S is said to be fuzzy soft left tideal (FSLI) over S, if $\tilde{M}_S\circ\tilde{M}_S\circ(f,M,\Gamma)\subseteq(f,M,\Gamma)$.

A FΓSS $(f,M,\Gamma)(\neq\phi)$ over a TΓ-SR S is said to be a fuzzy soft right tideal (FSRI) over S, if $(f,M,\Gamma)\circ\tilde{M}_S\circ\tilde{M}_S\subseteq(f,M,\Gamma)$.

A FΓSS $(f,M,\Gamma)(\neq\phi)$ over a TΓ-SR S is said to be a fuzzy soft lateral tideal (FSMI) over S, if $\tilde{M}_S\circ(f,M,\Gamma)\circ\tilde{M}_S\subseteq(f,M,\Gamma)$.

A FΓSS $(f,M,\Gamma)(\neq\phi)$ over a TΓ-SR S is said to be a FSI over S if (f,M,Γ) is FSL, FSR and FSMI over S.

Definition 3.3: A proper FSI (f,M,Γ) over a TΓ-SR S is said to be a FSPI over S if for any three proper fuzzy soft ideal (PFSI)s $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ over S satisfying $(h,F_1,\Gamma)\circ(g,F_2,\Gamma)\circ(i,F_3,\Gamma)\subseteq(f,M,\Gamma)\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$, where $F_1,F_2,F_3\subseteq M$.

Definition 3.4: A proper FSI (f,M,Γ)over a TΓ-SR S is said to be fuzzy soft strongly prime tideal (FSSPI) over S if for any PFSIs $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ over S satisfying $\left((h,F_1,\Gamma)\circ(g,F_2,\Gamma)\circ(i,F_3,\Gamma)\right)\cap_R\left((g,F_2,\Gamma)\circ(i,F_3,\Gamma)\circ(h,F_1,\Gamma)\right)\cap_R$ $\left((i,F_3,\Gamma)\circ(h,F_1,\Gamma)\circ(g,F_2,\Gamma)\right)\subseteq(f,M,\Gamma)\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)\ or\ (g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$, where $F_1,F_2,F_3\subseteq M$.

We now go over some findings for FSSPIs and FSPIs over TΓ-SR.

Theorem 3.5: Let (f,M,Γ) be a FSPI over TΓ-SR S. Then $f(\chi)$is a FSPI of S for all $\forall\chi\in M$, where $f(\chi)\neq\phi$.

Proof: Let (f,M,Γ) be a FSPI over a TΓ-SR S. Let $\chi\in M$ be such that $f(\chi)\neq\phi$. Let U,V,W be tideals of S $\ni UVW\subseteq f(\chi)$. Define $h(\chi)=U,\ g(\chi)=V,\ i(\chi)=W$ and $h(\psi)=g(\psi)=i(\psi)=f(\psi),\forall\,\psi\in F-(\chi)$. Then $(h,F_1,\Gamma)\circ(g,F_2,\Gamma)\circ(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$.

$\Rightarrow (h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Hence $U\subseteq f(\chi)$ or $V\subseteq f(\chi)$ or $W\subseteq f(\chi)$. Since χ is arbitrarily element of M, $f(\chi)$ is a FPI of S, $\forall\,\chi\in M$.

Note 3.6: The above result's converse is untrue. i.e. if $f(\chi)$ is a FPI of TΓ-SR S, $\forall\,\chi\in M$, where $M\subseteq S$, it may possible that (f,M,Γ) is not a FSPI over S.

Theorem 3.7: Let (f,M,Γ) be a FSSPI over a TΓ-SR S. Then $f(\chi)$ is a fuzzy strongly prime ideal (FSPI) of S for all $\chi\in M$, if $f(\chi)\neq\phi$.

Proof: Let (f,M,Γ) be a FSSPI over TΓ-SR S and $\chi\in M$ be $\ni$ $f(\chi)\neq\phi$. Let U,V,W be tideals of S $\ni UVW\cap_R VWU\cap_R WUV\subseteq f(\chi)$. Let us define $h(\chi)=U, g(\chi)=V, i(\chi)=W$ and $h(\psi)=g(\psi)=i(\psi)=f(\psi),\forall\,\psi\in M-\{\chi\}$. Then we find that $\left((h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\right)\cap_R\left((g,F_1,\Gamma)\square\,(i,F_2,\Gamma)\square\,(h,F_3,\Gamma)\right)\cap_R$ $\left((i,F_3,\Gamma)\square\,(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\right)\subseteq(f,M,\Gamma).\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Therefore, $U\subseteq f(\chi)$ or $V\subseteq f(\chi)$ or $W\subseteq f(\chi)$. Hence each $f(\chi)$ is a FSPI over S for $f(\chi)\neq\phi$.

Proposition 3.8: Every FSSPI over a TΓ-SR S is a FSPI over S.

Proof: Let (f,M,Γ) be a FSSPI over a TΓ-SR S and $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ be FSIs over S $\ni (h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Then $\left((h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\right.$ $\left.(i,F_3,\Gamma)\right)\cap_R\left((g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\square\,(h,F_1,\Gamma)\right)\cap_R\left((i,F_3,\Gamma)\square\,(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\right)\subseteq$ $(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(f,M,\Gamma).\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq (f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Hence (f,M,Γ) is a FSPI over S.

Definition 3.9: Let $\left\{(f_j,M_j,\Gamma)\right\}_{j\in I}$ be a collection of FSIs over a TΓ-SR S. This collection is said to be a chain of tideals if $(f_1,M_1,\Gamma)\subseteq(f_2,M_2,\Gamma)\subseteq(f_3,M_3,\Gamma)\subseteq$..........

Proposition 3.10: The collection of FSIs $\left\{(f_j,M_j,\Gamma)\right\}_{j\in I}$ is a chain of FSIs over TΓ-SR S iff $\left\{f_j(x)\right\}_{j\in I}$ is a chain of tideals of S, $\forall\,x\in M$.

Proof: Let S be a TΓ-SR. Then (f_j,M_j,Γ) is FSI over S, $\forall\,j\in I$ iff $f_j(x)$ are ideals of S, $\forall\,j\in I$ and $x\in M$. Again $(f_1,F_1,\Gamma)\subseteq(f_2,F_2,\Gamma)\subseteq(f_3,F_3,\Gamma)\subseteq$.......... iff $f_1(x)\subseteq f_2(x)\subseteq$ $f_3(x)\subseteq$.......... $\forall\,x\in M$. Therefore $\left\{f_j(x)\right\}_{j\in I}$ is a chain of tideals $\forall\,x\in M$.

Proposition 3.11: Let $\left\{(f_j,M_j,\Gamma)\right\}_{j\in I}$ be a chain of FSPIs over TΓ-SR S. Then $\bigcap_{j\in I}(f_j,M_j,\Gamma)$ is a FSPI over S.

Proof: Let $(f,M,\Gamma)=\bigcap_{j\in I}(f_j,M_j,\Gamma)$. Suppose $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ are any proper FSIs over S $\ni (h,F_1,\Gamma)\square(g,F_2,\Gamma)\square(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. $\Rightarrow (h,F_1,\Gamma)\square(g,F_2,\Gamma)\square(i,F_3,\Gamma)\subseteq(f_j,M,\Gamma),\forall j\in I$. Therefore $(h,F_1,\Gamma)\square(g,F_2,\Gamma)\square(i,F_3,\Gamma)\subseteq(f_1,M,\Gamma)$. Then by the definition of FSPI $(h,F_1,\Gamma)\subseteq(f_1,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f_1,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f_1,M,\Gamma)$. And by the definition of chain, we have $(f_1,F,\Gamma)\subseteq(f_j,M,\Gamma),\forall j\in I$. So $(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Hence (f,M,Γ) is a FSPI over S.

Proposition 3.12: Let (f,M,Γ) be a FSPI over TΓ-SR S and (g,N,Γ) be any FSI over S. Then $(f,M,\Gamma)\cap_R(g,N,\Gamma)$ is a FSPI of (g,N,Γ).

Proof: Let $(h,P,\Gamma)=(f,M,\Gamma)\cap_R(g,N,\Gamma)$. Then $P=M\cap N$. Let $(h_1,P_1,\Gamma),(h_2,P_2,\Gamma),(h_3,P_3,\Gamma)$ be FSIs of (g,N,Γ) such that $(h_1,P_1,\Gamma)\square(h_2,P_2,\Gamma)\square(h_3,P_3,\Gamma)\subseteq(f,M,\Gamma)\cap_R(g,N,\Gamma)$. Therefore $(h_1,P_1,\Gamma)\square(h_2,P_2,\Gamma)\square(h_3,P_3,\Gamma)\subseteq(h,P,\Gamma)$. This implies that $(h_1,P_1,\Gamma)\subseteq(h,P,\Gamma)$ or $(h_2,P_2,\Gamma)\square\subseteq(h,P,\Gamma)$ or $(h_3,P_3,\Gamma)\subseteq(h,P,\Gamma)$. Since $(h_1,P_1,\Gamma),(h_2,P_2,\Gamma),(h_3,P_3,\Gamma)$ all are FSIs of (g,N,Γ), we have $(h_1,P_1,\Gamma)\subseteq(f,M,\Gamma)\cap_R(g,N,\Gamma)$ or $(h_2,P_2,\Gamma)\subseteq(f,M,\Gamma)\cap_R(g,N,\Gamma)$ or $(h_3,P_3,\Gamma)\subseteq(f,M,\Gamma)\cap_R(g,N,\Gamma)$. Thus $(f,M_1,\Gamma)\cap_R(g,N,\Gamma)=(h,P,\Gamma)$ is FSPI of (g,N,Γ).

Definition 3.13: A PFS (L, R, M) tideal (f,M,Γ) over TΓ-SR S is said to be a FS (L,R,M) SPI over S, if for any PFS (L,R,M) tideal (g,N,Γ) over S, $(g,N,\Gamma)\square(g,N,\Gamma)\square(g,N,\Gamma)\subseteq(f,M,\Gamma)\Rightarrow(g,N,\Gamma)\subseteq(f,M,\Gamma)$.

Lemma 3.14: A PFSI (f,M,Γ) over a TΓ-SR S is fuzzy soft semiprime tideal (FS-SPI) over S iff $f(\chi)\neq\phi$ is SPI of S for each $\chi\in M$.

Proof: Let (f,M,Γ) be a FS-SPI over TΓ-SR S. Therefore, (f,M,Γ) is a FSI over S. Let $\chi\in M$. Then $f(\chi)$ is an tideal of S. Let U be an ideal of S such that $U^3\subseteq f(\chi)$. Let us define a FSI (g,N,Γ) over S, $\ni N=\{\chi\}, g(\chi)=U$. Therefore $(g,N_2,\Gamma)\square(g,N_2,\Gamma)\square(g,N,\Gamma)\subseteq(f,M,\Gamma).\Rightarrow(g,N,\Gamma)\subseteq(f,M,\Gamma)\Rightarrow g(\chi)\subseteq f(\chi)\Rightarrow U\subseteq f(\chi)$. Thus $f(\chi)$ is a SPI of S, $\forall\chi\in M$.

Conversely, suppose that $f(\chi)$ is a SPI of S, $\forall\chi\in M$. Let (g,N,Γ) be FSI over S, $\ni(g,N,\Gamma)\square(g,N,\Gamma)\square(g,N,\Gamma)\subseteq(f,M,\Gamma)$. So, $g(\chi)g(\chi)g(\chi)\subseteq f(\chi),\forall\chi\in N\subseteq M$. Since $f(\chi)$ is a SPI of S, $g(\chi)\subseteq f(\chi),\forall\chi\in N\subseteq M.\Rightarrow(g,N,\Gamma)\subseteq(f,M,\Gamma)$. Hence (f,M,Γ) is a FS-SPI over TΓ-SR S.

Proposition 3.15: Every FSPI over TΓ-SR S is a FS-SPI tideal over S.

Proof: Let (f,M,Γ) be a FSPI over S and (g,N,Γ) be any PFSI over S, $\ni (g,N,\Gamma)\square\,(g,N,\Gamma)\square\,(g,N,\Gamma)\subseteq(f,M,\Gamma)$. Since (f,M,Γ) is FSPI over S we have, $(g,N,\Gamma)\subseteq(f,M,\Gamma)$. Hence (f,M,Γ) is a FS-SPI over S.

Corollary 3.16: Every FSSPI over a TΓ-SR S is a FS-SPI over S.

Definition 3.17: Let (f,M,Γ) be a FSI over TΓ-SR S. Then (f,M,Γ) is said to be fuzzy soft irreducible tideal (FSII) over S if for FSIs $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ over S satisfying $(h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)=(f,M,\Gamma)$. $\Rightarrow (h,F_1,\Gamma)=(f,M,\Gamma)$ or $(g,F_2,\Gamma)=(f,M,\Gamma)$ or $(i,F_3,\Gamma)=(f,M,\Gamma)$.

Definition 3.18: Let (f,M,Γ) be a FSI over TΓ-SR S. Then (f,M,Γ) is said to be fuzzy soft strongly irreducible tideal (FSSII) over S if for any FSIs $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ over S satisfying $(h,F_1,\Gamma)\cap_R\ (g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)=(f,M,\Gamma).\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$.

The following are some properties of FSIIs and FSSIIs over TΓ-SR S.

Lemma 3.19: Let (f,M,Γ) be a FSII over TΓ-SR S. Then $f(\chi)$ is irreducible tideal of S, $\forall\ \chi\in M$, where $f(\chi)\neq\phi$.

Proof: Let (f,M,Γ) be a FSII over TΓ-SR S. Let $\chi\in M$. Then $f(\chi)$ is a tideal of S. Let U,V,W be tideals of S, $\ni U\cap V\cap W=f(\chi)$. Now define $h(\chi)=U, g(\chi)=V, i(\chi)=W$ and $h(\psi)=g(\psi)=i(\psi)=f(\psi), \psi\in M-\{\chi\}$. Then $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ are FSIs over S, $\ni (h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)=(f,M,\Gamma)$. Since (f,M,Γ) is FSII over S, $(h,F_1,\Gamma)=(f,M,\Gamma)$ or $(g,F_2,\Gamma)=(f,M,\Gamma)$ or $(i,F_3,\Gamma)=(f,M,\Gamma)$. Therefore, $h(\chi)=f(\chi)$ or $g(\chi)=f(\chi)$ or $i(\chi)=f(\chi),\forall\chi\in M.\Rightarrow U=f(\chi)$ or $V=f(\chi)$ or $W=f(\chi)$. So $f(\chi)$ is irreducible tideal of S. Since χ is an arbitrary element of M, $f(\chi)$ is irreducible tideal of S, $\forall\ \chi\in M$.

Note 3.20: In general, lemma 3.19's converse is untrue.

Lemma 3.21: Let (f,M,Γ) be a FSSII over TΓ-SR S. Then $f(\chi)$ is a strongly irreducible tideal (SII) of S, $\forall\ \chi\in M$ where $f(\chi)\neq\phi$.

Lemma 3.22: Every FSSII over TΓ-SR S is a FSII over S.

Proposition 3.23: Every FSPI over TΓ-SR S is a FSSII over S.

Proof: Let (f,M,Γ) be a FSPI over S. Let $(h,F_1,\Gamma),(g,F_2,\Gamma),(i,F_3,\Gamma)$ be FSIs over S $\ni (h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Now $(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq$

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$(h,F_1,\Gamma),(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(g,F_2,\Gamma)$, and $(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(i,F_3,\Gamma)$. Therefore, $(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. $\Rightarrow(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. So (f,M,Γ) is a SII tideal over S.

Note 3.24: Since every FSPI over TΓ-SR S is FSSII over S and every FSSII over TΓ-SR S is FSII over S, every FSPI over TΓ-SR S is a SII over S.

Theorem 3.25: A FS-SPI over TΓ-SR S is FS-SPI if it is FSII over S.

Proof: Let (f,M,Γ) be a FS-SPI over S. Let $(h,F_1,\Gamma),(h,F_2,\Gamma),(i,F_3,\Gamma)$ be PFSIs over S, $\ni(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Suppose (f,M,Γ) is FSII over S.

Now

$((h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma))\square\,((h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma))$

$\square\,((h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma))\subseteq(h,F_1,\Gamma)\square\,(g,F_2,\Gamma)\square\,(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$.

$\Rightarrow(h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Therefore, $((h,F_1,\Gamma)\cap_R(g,F_2,\Gamma)\cap_R(i,F_3,\Gamma))\cup(f,M,\Gamma)=(f,M,\Gamma)$. $\Rightarrow(h,F_1,\Gamma)\cup(f,M,\Gamma)=(f,M,\Gamma)$ or $(g,F_2,\Gamma)\cup(f,M,\Gamma)=(f,M,\Gamma)$ or $(i,F_3,\Gamma)\cup(f,M,\Gamma)=(f,M,\Gamma)$. Therefore, $(h,F_1,\Gamma)\subseteq(f,M,\Gamma)$ or $(g,F_2,\Gamma)\subseteq(f,M,\Gamma)$ or $(i,F_3,\Gamma)\subseteq(f,M,\Gamma)$. Hence (f,M,Γ) is a FSPI over S.

Theorem 3.26: If (f,M,Γ) is a FSI over TΓ-SR S with identity and $\chi\in S$. Then the equivalent conditions are as follows.

(i) (f,M,Γ) is FS-SPI over S.

(ii) $f(\chi)\neq\phi$ is a SPI of S.

(iii) $\varpi S\,S\varpi\,S\,S\,\varpi\subseteq f(\chi)\Rightarrow\varpi\in f(\chi)$.

(iv) $(S\,S\,\varpi)(S\,S\,\varpi)(S\,S\,\varpi)\subseteq f(\chi)\Rightarrow\varpi\in f(\chi)$, $(\varpi\,S\,S)(\varpi\,S\,S)(\varpi\,S\,S)\subseteq f$
$\varpi\in f(\chi)$, $(S\,\varpi S)(S\,\varpi S)(S\,\varpi S)\subseteq f(\chi)\Rightarrow\varpi\in f(\chi)$.

Proof: $(i)\Leftrightarrow(ii)$: based on Lemma 3.14.

$(ii)\Leftrightarrow(iii)$

Suppose that $f(\chi)$ is a SPI of S, $\forall\ \chi\in M\ \ and\ \ f(\chi)\neq\phi$. Let $(\chi\,S\,S\,\chi\,S\,S\,\chi)\subseteq f(\chi)$. Therefore $S\,S(\varpi S\,S\,\varpi S\,S\varpi)SS\subseteq SS\ f(\chi)S\,S\subseteq(SSf(\chi))SS\subseteq f(\chi)SS\subseteq f(\chi)$. Now $(SS\,\varpi\,SS)(SS\,\varpi\,SS)(SS\varpi\,SS)\subseteq SS\,\varpi\,SS\,\varpi\,SS\,\varpi SS\subseteq f(\chi)$. Therefore, $SS\,\varpi\,SS\subseteq f(\chi)$. Again $SS\,\varpi\,SS$ is an ideal containing ϖ Thus $\varpi\in f(\chi)$.

$(iii)\Leftrightarrow(iv)$

Let $(SS\varpi)(SS\varpi)(SS\varpi)\subseteq f(\chi)$. Now $\varpi SS\varpi SS\varpi \subseteq SS(\varpi SS\varpi SS\varpi)\subseteq f(\chi)$. $\Rightarrow \varpi \in f(\chi)$. Again $(\varpi SS)(\varpi SS)(\varpi SS)\subseteq f(\chi)$. Therefore, $(\varpi SS\varpi SS)SS(\varpi SS\varpi SS\varpi)SS\subseteq f(\chi).\Rightarrow \varpi \in f(\chi)$. Similarly, $(S\varpi S)(S\varpi S)(S\varpi S)\subseteq f(\chi)\Rightarrow \varpi \in f(\chi)$.

$(iv)\Leftrightarrow(v)$

Since (f,M,Γ) is a FSI over S, $f(\chi)\neq\phi$ is ideal of S, $\forall\,\chi\in M$. Let U be an tideal of S, $\ni U^3\subseteq f(\chi)$. Let $\varpi\in U$. Now $(SSU)(SSU)(SSU)\subseteq U^3\subseteq f(\chi)$. Hence $(SS\varpi)(SS\varpi)(SS\varpi)\subseteq(SSU)(SSU)(SSU)\subseteq f(\chi).\Rightarrow \varpi\in f(\chi)$. Thus $U\subseteq f(\chi)$. Therefore, $f(\chi)$ is a SPI of S, $\forall\ \chi\in M,\ni f(\chi)\neq\phi$.

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