

# Pseudo Symmetric Ideals in Near Subtraction Semigroups

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**Abstract:**

The current ongoing work, author introduce the theory of pseudo symmetric, PSideal, semiprime ideal, completely semi prime ideal, left pseudo commutative, right pseudo commutative, duo NSSG and PSnear subtraction SG.

**Keywords:** Pseudo symmetric, completely semiprime, duo semigroup, near subtractionsemigroup.

## 1. Introduction

In abstract algebra the theory of near subtraction semi groups is a rapid departing branch. The subtraction algebra was introduced by Abbott in 1967 by using the concept of subtraction algebra, shein developed subtraction SG in the year 1992. In the year of 2007 Dheena.P was first introduced the concept of near subtraction SG. Basically, the NSSG is a fundamental notions of subtraction algebra. Jun et al., investigated about the theory of ideals in near subtraction algebra after that he characterized some elementary axioms. Later on some research scholars developed the basic properties of near subtraction SG. Later they establish a structure in NSSG and also examine different kinds of near subtraction algebra and getting the equivalent conditions for regularity. The fundamental concept of this work is to learn about PS ideals in NSSG and PSNSSG and some of the results were proved regarding NSSG.

## 2. Preliminaries

For fundamental concepts make reference to (1), (2), (3) and (4)

## 3. Main content

**Definition 3.1:** Assume that A be an ideal of a NSSG X which is said to be a PS if  $x, y \in X, xy \in A \Rightarrow xsy \in A$  for each  $s \in X$ .

**EXAMPLE 3.2:** Assume that  $X = \{0, 1, 2, 3, 4, 5\}$  for which ‘-,’ and ‘.’ are defined by

-	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	3	4	3	1

2	2	5	0	2	5	4
3	3	0	3	0	3	3
4	4	0	0	4	0	4
5	5	5	0	5	5	0
.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	4	3	4	0
2	0	4	2	0	4	5
3	0	3	0	3	0	0
4	0	4	4	0	4	0
5	0	0	5	0	0	5

Then  $(A, -, \cdot)$  is a NSSG. Let  $I = \{0, 1, 3, 4\}$  is a PS ideal of  $X$ .

EXAMPLE 3.3: Let  $X = \{p, q, r\}$  for which '-' and ' $\cdot$ ' are defined as follows:

-	$p$	$q$	$r$
$p$	$p$	$p$	$p$
$q$	$q$	$p$	$q$
$r$	$r$	$r$	$p$
$\cdot$	$p$	$q$	$r$
$p$	$p$	$p$	$p$
$q$	$p$	$p$	$p$
$r$	$p$	$q$	$r$

Then  $(X, -, \cdot)$  is a near subtraction normal SG. The ideals of  $X$  are  $\{p\}, \{p, q\}, \{p, q, r\}$  which are PS.

**Theorem 3.4 :** The family of non empty intersection of PS ideals of a NSSG  $X$  a PS is an ideals of  $X$ .

**Proof :** Assume that  $\{A_\alpha\}_{\alpha \in \Delta}$  a family of PSI of  $X$  and let  $A = \bigcap_{\alpha \in \Delta} A_\alpha$ .

By known theorem,  $A$  is an ideal of  $X$ .

Let  $x, y, s \in X, xy \in A, \Gamma y \in A \Rightarrow xy \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow xy \in A_\alpha$  for each  $\alpha \in \Delta$ .

$xy \in A_\alpha, A_\alpha$  is a PSI of  $X, s \in X \Rightarrow xsy \in A_\alpha$

$\Rightarrow xsy \in A_\alpha$  for all  $\alpha \in \Delta \Rightarrow xsy \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow xsy \in A$ .

$\therefore A$  is a PSI of  $X$ .

**Theorem 3.5:** Suppose that  $X$  be a NSSG and  $A$  be ideal in  $X$ . So that  $A$  is a PS iff for all  $a \in X$ , the set  $A_r(a) = \{x \in X : ax \in A\}$  is an ideal of  $X$ .

**Proof:** Assume that  $A$  will be a PSI in  $X$  and  $a \in X$

Let  $x, y \in A_r(a) \Rightarrow ax, ay \in A$ .

$a(x - y) = ax - ay \in A$ , since  $A$  is an ideal of  $X$  and  $ax, ay \in A$ .

In this manner  $x - y \in A_r(a)$  and hence  $A_r(a)$  is a sub algebra of  $X$ .

Presently  $x \in A_r(a)$  implies that  $ax \in A$ . Presently  $ax \in A$ ,  $A$  is PS implies  $asx \in A$  implies that  $sx \in A_r(a)$

Presently  $ax \in A$ ,  $s \in S$ ,  $A$  is an ideal implies that  $asx \in A$  implies that  $sx \in A_r(a)$

Consequently  $A_r(a)$  is an ideal in  $X$  for each  $a \in X$ .

On opposite way guess that  $A_r(a)$  is an ideal in  $X$  for every  $a \in X$ . Let  $x, y \in X$  and  $xy \in A$ .

$xy \in A$  implies that  $y \in A_r(a)$

$y \in A_r(a)$ ,  $A_r(a)$  is an ideal implies  $sy \in A_r(a)$  for every  $s \in X$  implies that  $xsy \in A$  for each  $s \in X$

$\therefore A$  is a PSI of  $X$ .

**Theorem 3.6:** Let  $X$  be a NSSG and  $A$  is an ideal in  $X$ . Then  $A$  is PS iff for all  $a \in X$  the set  $A_l(a) = \{x \in X : xa \in A\}$  is an ideal of  $X$ .

**Proof:** Assume that  $A$  will be a PSI in  $X$  and  $a \in X$ .

Let  $x, y \in A_l(a)$  implies  $xa, ya \in A$ .

$(x-y)a = xa - ya \in A$ , since  $A$  is an ideal of  $X$  and  $xa, ya \in A$

In this way  $x-y \in A_l(a)$  and consequently  $A_l(a)$  is a sub algebra of  $X$ .

Presently  $x \in A_l(a)$  implies  $xa \in A$ . Presently  $xa \in A$ ,  $A$  is PS implies  $xsa \in A$  implies  $sx \in A_l(a)$

Presently  $xa \in A$ ,  $s \in X$ ,  $A$  is an ideal  $sxa \in A$  implies that  $sx \in A_l(a)$ .

In this way  $A_l(a)$  is an optimal in  $X$  for each of the  $a \in X$ .

On the other hand guess that  $A_l(a)$  is an ideal in  $S$  for each  $a \in X$ . Let  $x, y \in X$  and  $xy \in A$ .

$xy \in A$  implies  $x \in A_l(y)$ ,  $A_l(y)$  is an ideal implies that  $xs \in A_l(y)$  for each  $s \in X$ ,  $xsy \in A$  for each  $s \in X$ .

Thus  $A$  will be a PSI of  $X$ .

**Corollary 3.7:** Suppose  $X$  be a NSSG and  $A$  is an ideal in  $X$ . Then  $A$  is PS iff for each  $a \in X$  the set  $A(a) = \{x \in X : xa, ax \in A\}$  is an ideal in  $X$ .

**Theorem 3.8:** Let  $A$  be any PSI in a NSSG  $X$  and  $x_1, x_2, \dots, x_n \in X$ . Then

- 1)  $x_1 x_2 \in A$  implies  $x_2 x_1 \in A$ .
- 2)  $x_1 x_2 \dots x_n \in A$  if and only if  $\langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \subseteq A$ .

**Proof:** 1) Suppose  $x_1 x_2 \in A$ . Then  $(x_2 x_1)^2 = x_2 (x_1 x_2) x_1 \in A$  and hence  $x_2 x_1 \in A$ .

2) Let  $x_1 x_2 \dots x_n \in A$ . Since  $A$  is PSI of  $X$ .

Then by corollary 3.6,  $x_1 \in A_1(x_2 x_3 \dots x_n) \Rightarrow \langle x_1 \rangle \subseteq A_1(x_2 x_3 \dots x_n)$

$\Rightarrow \langle x_1 \rangle x_2 x_3 \dots x_n \subseteq A$ . By property (1), we have  $x_2 x_3 \dots x_n \langle x_1 \rangle \subseteq A$ .

Now  $x_2 \in A_1(x_3 x_4 \dots x_n \langle x_1 \rangle) \Rightarrow \langle x_2 \rangle \subseteq A_1(x_3 x_4 \dots x_n \langle x_1 \rangle)$ .

Therefore  $\langle x_2 \rangle x_3 x_4 \dots x_n \langle x_1 \rangle \subseteq A \Rightarrow x_3 x_4 \dots x_n \langle x_1 \rangle \langle x_2 \rangle \subseteq A$ .

Continuing this process we have  $x_1 x_2 \dots x_n \in A \Rightarrow \langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \subseteq A$ .

Conversely suppose that  $\langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \subseteq A$

Then  $x_1 x_2 \dots x_n \in \langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \subseteq A$ . Therefore  $x_1 x_2 \dots x_n \in A$

**Corollary 3.9:** If  $A$  is a PSI in a NSSG  $X$ , then for each  $n \in \mathbb{N}$   $a^n \in A \Rightarrow \langle a \rangle^n \subseteq A$ .

**Proof:** By the above corollary 3.8 substitute  $a_1 = a_2 = a_3 = \dots = a_n = a$

**Corollary 3.10 :** Let  $A$  be a PSI in a NSSG  $X$ . If  $a^n \in A$ ,  $n \in \mathbb{N}$  then  $\langle a s \rangle^n, \langle s a \rangle^n \in X$ .

**Theorem 3.11:** Each CSPI  $A$  in a NSSG  $X$  is a PSI of  $X$ .

**Proof:** Assume that  $A$  be a CSPI of the NSSG  $X$ .

Let  $x, y \in X$  and  $xy \in A \Rightarrow (yx)^2 = (yx)(yx) = y(xy)x \in A$ .  $(yx)^2 \in A$ ,  $A$  is CSPI implies  $yx \in A$ . If  $s \in X$ ,  $(xsy)^2 = (xsy)(xsy) = xs(yx)sy \in A$ .

$(xsy)^2 \in A$ ,  $A$  is CSP implies that  $xsy \in A$ .

Thus  $A$  will be a PSI of  $X$ .

**Note 3.12 :** The reverse proof of the theorem 3.11 which is absurdity i.e., a PSI of a NSSG which is not to be CSP.

**Example 3.13:** :: Let  $X = \{p, q, r\}$ . Define a binary operations ‘-’ and ‘.’ on  $X$  as follows

-	$p$	$q$	$r$
$r$	$p$	$p$	$p$
$q$	$q$	$p$	$q$
$r$	$r$	$r$	$p$
.	$p$	$q$	$r$
$p$	$p$	$p$	$p$

$q$	$p$	$p$	$p$
$r$	$p$	$q$	$r$

Clearly  $(X, -, \cdot)$  is a NSSG and  $\{p\}, \{p, q\}, X$  are the ideal of  $X$ .

Here  $pp \in \{p\} \Rightarrow ppp, pqp, prp \in \{p\}$

$pq \in \{p\} \Rightarrow ppq, pqq, prq \in \{p\}$

$pr \in \{p\} \Rightarrow ppr, pqr, prr \in \{p\}$

$qp \in \{p\} \Rightarrow qpp, qqp, qrp \in \{p\}$

$qr \in \{p\} \Rightarrow qpr, qqr, qrr \in \{p\}$

$rp \in \{p\} \Rightarrow rpp, rqp, rrp \in \{p\}$ . Therefore  $\{p\}$  is a PSI in  $X$ .

Here  $p^2 = p \in \{p\}$  but  $b \notin \{p\}$ . Therefore  $\{p\}$  is not a CSPI.

**Corollary 3.14 :** Each CPI of a NSSG  $X$  is a PSI of a NSSGX.

**Proof :** Let  $A$  is a CPI of a NSSGX.

By known theorem,  $A$  is a CSPI of  $X$ .

By known theorem,  $A$  is PSI of  $X$ .

**Theorem 3.15:** If  $A$  is a PSI of a NSSGX then  $A_2 = \{x/x^n \in A \text{ for every } n \in \mathbb{N}\}$  is a CSPI of  $X$ .

**Proof :** Clearly  $A \subseteq A_2$  and subsequently  $A_2$  is a non-empty subset of  $X$ . Assume that  $x, y \in A_2$  and  $s \in X$ .

Presently  $x \in A_2 \Rightarrow x^n \in A$  for some  $m, n \in \mathbb{N}$ ,  $x^n, y^m \in A \Rightarrow x, y \in A$ ,  $A$  will be a PSI of  $X$  implies  $(x - y) \in A \Rightarrow (x - y)^1 \in A$  and consequently  $(x - y) \in A_2$ .

Hence  $A_2$  is a sub algebra of  $X$ .

Presently  $x \in A_2$  implies  $x^n \in A$  for each  $n \in \mathbb{N}$ .  $x^n \in A, s \in X, A$  will be a PSI of  $X \Rightarrow (xs)^n \in A, (sx)^n \in A$  implies that  $xs, sx \in A_2$ . Accordingly  $A_2$  is an ideal of  $X$ .

Assume that  $x \in X$  and  $x^2 \in A_2$ . Presently  $x^2 \in A_2$  implies that  $(x^2)^n \in A$  for each  $n \in \mathbb{N}$

$x^{2n} \in A$  implies that  $x \in A_2$ . So  $A_2$  is a CSPI of  $X$ .

Let  $Q$  to be any CSPI containing  $A$ . Let  $x \in A_2$ .

Then, at that point,  $x^n \in A$  for each  $n \in \mathbb{N}$ . by corollary 3.7,  $x^n \in A$  implies that  $\langle x \rangle^n \subseteq A \subseteq Q$ .

Since  $Q$  is CSP  $\langle x \rangle^n \subseteq Q$  implies that  $x \in Q$ .

In this way  $A_2$  is minimal CSPI of  $X$  containing  $A$ .

**Theorem 3.16 :** If  $A$  is a PSI of a NSSG  $X$  then  $A_2 = A_4$ .

**Proof :** By known theorem,  $A_4 \subseteq A_2$ . Suppose  $x \in A_2$  then  $x^n \in A$  for some  $n \in \mathbb{N}$ .

Since  $A$  is PS,  $x^n \in A \Rightarrow \langle x \rangle^n \subseteq A \Rightarrow x \in A_4$ .

$\therefore A_2 \subseteq A_4$  and hence  $A_2 = A_4$ .

**Theorem 3.17 :** If  $A$  is a PSI of a NSSG  $X$  then  $A_4 = \{x / \langle x \rangle^n \subseteq A \text{ for some } n \in \mathbf{N}\}$  is the minimal semiprime ideal of  $X$  containing  $A$ .

**Proof :** Obviously  $A \subseteq A_4$  and hence  $A_4$  is a non-empty subset of  $X$ . Let  $x, y \in A_4$  and  $s \in X$ .

Since  $x, y \in A_4$ ,  $\langle x \rangle^n \subseteq A$ ,  $\langle y \rangle^m \subseteq A$  for some  $n, m \in \mathbf{N}$ .  $m \Rightarrow x, y \in A$ ,  $A$  is a PSI of  $X$ .

Therefore  $x - y \in A \Rightarrow \langle x - y \rangle \subseteq A \Rightarrow \langle x - y \rangle^1 \subseteq A \Rightarrow x - y \in A_4$ . Therefore  $A_4$  is a sub algebra of  $X$ .

Now  $x \in A_4$  and  $s \in X$ . Since  $x \in A_4$ , then  $\langle x \rangle^n \subseteq A$  for some  $n \in \mathbf{N}$ .

Now  $\langle xs \rangle^n \subseteq \langle x \rangle^n \subseteq A$  and  $\langle sx \rangle^n \subseteq \langle x \rangle^n \subseteq A \Rightarrow xs, sx \in A_4$ .

Then  $A_4$  is an ideal of  $X$  containing  $A$ . Let  $x \in X$  such that  $\langle x \rangle^2 \subseteq A_4$

Then  $(\langle x \rangle^2)^n \subseteq A \Rightarrow \langle x \rangle^{2n} \subseteq A \Rightarrow x \in A_4$ . Therefore  $A_4$  is a semiprime ideal of  $X$  containing  $A$ . Let  $P$  is a semiprime ideal of  $X$  containing  $A$ .

Suppose  $x \in A_4$ . Then  $\langle x \rangle^n \subseteq A \subseteq P$ .

Since  $P$  is semiprime ideal of  $X$ ,  $\langle x^n \rangle \subseteq P \Rightarrow \langle x \rangle^n \subseteq P \Rightarrow x \in P$ .

$\therefore A_4 \subseteq P$  and hence  $A_4$  is the minimal semiprime ideal of  $X$  containing  $A$ .

**Theorem 3.18 :** Let  $A$  be an ideal of a NSSG  $X$ . Then  $A$  is completely prime iff  $A$  is prime and PS.

**Proof :** Assume  $A$  will be a CPI. By known hypothesis,  $A$  is prime.

By corollary 3.14,  $A$  will be a PSI of  $X$ .

On the other hand assume that  $A$  is prime and PS.

Let  $x, y \in X$  and  $xy \in A$ .  $xy \in A$ ,  $A$  will be a PSI of  $X$

$\Rightarrow xsy \in A$  for all  $s \in X \Rightarrow \langle x \rangle \subseteq A$  or  $\langle y \rangle \subseteq A \Rightarrow x \in X$  or  $y \in A$ . Consequently  $A$  is completely prime.

**Theorem.3.19:** Let  $A$  be an ideal of a NSSG  $X$ . Then  $A$  is CSP iff  $A$  is semiprime and PS.

**Proof :** Assume that  $A$  is completely semiprime. By known hypothesis,  $A$  is semiprime and furthermore by known hypothesis 3.11,  $A$  is PS.

On the other hand suppose that  $A$  is semiprime and PS.

Let  $x \in X$  and  $x^2 \in A$ .

$x^2 \in A$ ,  $A$  is PS  $\Rightarrow \langle x^2 \rangle \subseteq A \Rightarrow \langle x \rangle \subseteq A \Rightarrow x \in A$ .

In this way  $\underline{A}$  is CSP.

#### 4. Pseudo Symmetric Near Subtraction Semigroup

**Definition 4.1:** A NSSG  $X$  is said to be PSNSSG provided every ideal in  $X$  is a PSI.

**Example 4.2:** In example 3.3, the NSSG  $X$  is a PSNSSG.

**Theorem 4.3:** Each left duo NSSG  $X$  is a PSSSG.

*Proof:* Suppose that  $X$  is left duo NSSG and  $A$  is any ideal of  $X$ .

Suppose  $a \in X$ . Let  $A_l(a) = \{x \in X : xa \in A\}$ .

Let  $x, y \in A_l(a) \Rightarrow xa, ya \in A$ .

$(x - y)a = xa - ya \in A$ , Since  $A$  is an ideal of  $X$  and  $xa, ya \in A$ .

Therefore  $x - y \in A_l(a)$  and hence  $A_l(a)$  is a sub algebra of  $X$ .

Let  $x \in A_l(a) \Rightarrow xa \in A$ .  $xa \in A$ ,  $A$  is an ideal  $\Rightarrow sxa \in A$  for every  $s \in X$

$\Rightarrow sx \in A_l(a)$  for all  $s \in X$ . Therefore  $A_l(a)$  is a left ideal of  $X$  for every  $a \in X$ .

Since  $X$  is a left duo NSSG,  $A_l(a)$  is an ideal of  $X$ .

So by known theorem 3.6,  $A$  is a PSI of  $X$ .

$\therefore X$  is a PSSSG.

**Theorem 4.4:** Each right duo NSSG  $X$  is a PSNSSG.

*Proof:* Let  $X$  be a right duo NSSG and  $A$  be any ideal of  $X$ .

Suppose  $a \in X$ . Let  $A_r(a) = \{x \in X : ax \in A\}$ .

Let  $x, y \in A_r(a) \Rightarrow ax, ay \in A$ .

$a(x - y) = ax - ay \in A$ , Since  $A$  is an ideal of  $X$  and  $ax, ay \in A$ .

Therefore  $x - y \in A_r(a)$  and hence  $A_r(a)$  is a sub algebra of  $X$ .

Let  $x \in A_r(a) \Rightarrow ax \in A$ ,  $A$  is an ideal  $\Rightarrow axs \in A$  for every  $s \in X \Rightarrow xs \in A_r(a)$  for every  $s \in X$

Therefore  $A_r(a)$  is a right ideal of  $X$  for every  $a \in X$ .

Since  $X$  is a right duo NSSG,  $A_r(a)$  is an ideal of  $X$ .

So by theorem 3.5,  $A$  is a PSI of  $X$ .

Therefore  $X$  is a PSSSG.

**Corollary 4.5:** Each duo NSSG is a PSNSSG.

*Proof:* By theorem 4.3, and 4.4, we conclude that every duo NSSG  $X$  is a PSNSSG.

**Theorem 4.6 :Each left pseudo commutative NSSG is a PSNSSG.**

**Proof :** Let  $X$  be a left pseudo commutative NSSG and  $A$  be any ideal of  $X$ . Let  $x, y \in X$ ,  $xy \in A$  and  $s \in X$ . Now  $xsy = sxy = s(xy) \in A$ .

Therefore  $xsy \in A$  for all  $s \in X$ . Therefore  $A$  is a PSI.

Therefore  $X$  is a PSNSSG.

**Theorem 4.7: Each right pseudo commutative NSSG is PSNSSG.**

**Proof :** Let  $X$  be a right pseudo commutative NSSG and  $A$  be any ideal of  $X$ . Let  $x, y \in X$ ,  $xy \in A$  and  $s \in X$ . Now  $xsy = xys = (xy)s \in A$ .

Therefore  $xsy \in A$  for all  $s \in X$ .  $\therefore A$  is a PSI.

Hence  $X$  is a PSNSSG.

**Corollary 4.8: Each quasi commutative NSSG is a PSNSSG.**

**Proof :** Let  $X$  be a quasi commutative SG. By known theorem,  $X$  is a normal NSSG. By known theorem,  $X$  is a duo NSSG. By corollary 4.5,  $X$  is a PS SG.

**Corollary 4.9: Each generalized commutative NSSG is a PSNSSG.**

**Proof :** Suppose that  $S$  be a commutative NSSG.

By known theorem,  $X$  is a left duo NSSG.

By known theorem 4.3,  $X$  is a PSNSSG.

**Corollary 4.10: Each NSSG is a PSNSSG.**

**Proof :** Suppose that  $X$  be a normal SG. By known theorem,  $X$  is a duo SG.

By corollary 4.5,  $X$  is a PSNSSG.

**Theorem 4.11: Each idempotent NSSG is a PSNSSG**

**Proof :** Suppose that  $X$  be an idempotent NSSG and  $A$  be any ideal of  $X$ .

Let  $x, y \in X$ ,  $xy \in A$ . Now  $xy \in A \Rightarrow (yx) = (yx)^2 = (yx)(yx) = y(xy)x \in A$ . Therefore  $yx \in A$ .

If  $s \in X$ , then  $xsy = (xsy)^2 = (xsy)(xsy) = xs(yx)sy \in A$ . Therefore  $xsy \in A$  for every  $s \in X$ .

Hence  $A$  is a PSI. Therefore  $X$  is a PSSG.

**Theorem 4.12: If  $X$  is a NSSG in which every element is a midunit then  $X$  is a PS NSSG.**

**Proof :** Suppose that  $X$  be a NSSG in which every element is a midunit and  $A$  be any ideal of  $X$ . If  $s \in X$ , then  $s$  is a midunit and hence  $xsy = xy \in A$ . Hence  $A$  is a PSI.

$\therefore X$  is a PSSG.

**Remarks:** SG-Semigroup, PS-Pseudo symmetric, PSI-Pseudo symmetric ideal NSSG-Near subtraction semigroup and PSNSSG -Pseudo Symmetric Near subtraction semigroup, CSP- completely semiprime, CPI-completely prime ideal and CSPI- completely semiprime ideal.

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