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The Carmichael Function in Graph Theory

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Abstract:

One of the most flourishing branches of modern Mathematics is the application of graph theory in graph theory. This work presents innovative graph which is an application of some arithmetical functions and this called the Carmichael Function Graph \lambda(G). Using a new technique in order to calculate the results to be found in this research. Moreover this work considered a new kind of application of some arithmetical functions in graph theory. So for this purpose, many basic properties in graph theory, including finding the characteristics of the independence number, domination number, clique number and chromatic number of this graph have been calculated.

Keywords: independence number, domination number,\ clique number and chromatic number , Carmichael Function Graph\ G\ ,\ clique number and chromatic number .

1. Introduction

This paper introduce an application of some arithmetical functions in graph theory. Specially were us the application arithmetical functions has been applied on the Carmichael function Graph $\lambda(G)$ in simple, nontrivial, finite an undirected. In general the application of some arithmetical functions has been studied from several authors, for instant the reader can see (M. A. Seoud, Essam EL-Seidy and Ahmed A. Omran studied Independence in Isosceles Triangular Chessboard,in 2012, Essam EL-Seidy ,Ahmed A. Omran studied DOMINATION IN RHOMBUS CHESSBOARD in 2014 and Sanaa Kadum Kamel Yaseen, Faez A.AL-maamori and Ahmed Abed Ali Omran studied Some Kinds of Mobius Function Graphs in 2022) . The graph G is consists of a non-empty finite set V(G) of elements called vertices, and a finite family E(G) of unordered pairs of (not necessarily distinct) elements of V(G) called edges [6].we called that a set $D \subseteq V$ is a dominating set of G if every vertex in $V - D$ is adjacent to a vertex in D. And the domination number of graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in graph G [4]. The independent set is a set of vertices in a graph such that are said to be independent if no two of them are adjacent. The cardinality of such a biggest independent set is called the independence number of the graph and is denoted by β[2]. We called that a clique of a graph is its maximal complete sub graph. The clique number $\omega(G)$ of a graph is the number of graph vertices in the largest clique of G[3]. The set of coloring is called the chromatic number $\chi(G)$ of a graph is the least number of colors required for a proper vertex coloring of G [1]. The Carmichael function in is a well-known function in Number theory and it is defined as $\lambda(1) = 1$ and if $n > 1$, we write If

$n = p_1^{\alpha_1}, p_2^{\alpha_2} \ldots. \; p_k^{\alpha_k}$ then $\lambda(p^\alpha) = \begin{cases} p^\alpha\,(p-1)\,, if\ p \geq 3\ or\ \alpha \leq 2 \\ 2^{\alpha-2}\ \ , if\ p = 2\,, and\ \alpha\ \geq 3 \end{cases}$ We called the graph G (V,E) by defining on the solid and tight arithmetic function called Carmichael Function Graph $\lambda(G)$[7] .The Features of Number theory were applied in Graph theory to design a graph is introduced by Nathonson In 1980 [5].

This section stats the main results and started with :

2. Materials and Methods

Theorem 2.1 If G be a Carmichael Function Graph $\lambda(G)$ of order n and $n(G)$ is the number of components , then each component in graph G is complete.

Proof.

Since every vertex is adjacent to all vertices in every component , and this graph is divided to many components, and every part in this graph is complete then we can say that each component in graph G is complete . This graph is not divided where the numbers of vertices one or two.

Corollary 2.2 If G be a Carmichael Function Graph $\lambda(G)$ of order n, and $n(G)$ is the number of components, then the dominations numbers as following : $\gamma\ (G) = n(G)$.

Proof:

Depend on the previse theorem that each component in graph G is complete then we can say that the domination set in this graph is the number of components.

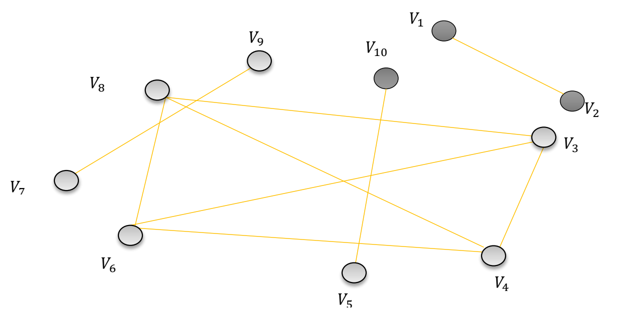


Figure 2.1 The Function Carmichael Graph (G) of order 10.

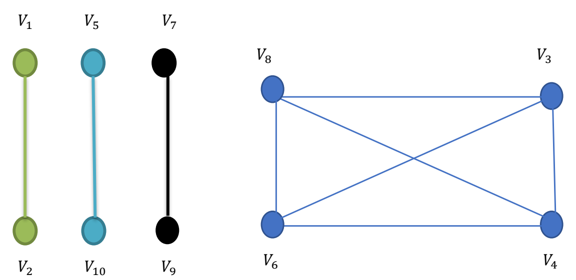


Figure 2.2 The components of Function Carmichael Graph (G) of order 10.

Corollary 2.3 If G be a Carmichael Function Graph $\lambda(G)$ of order n and $n(G)$ is the number of components, then the independence number as following : $\beta(G) = n(G)$.

Proof:

Depend on the previse theorem that each component in graph G is complete then we can say that the independence set in this graph is the number of components.

Theorem 2.4 If G be a Carmichael Function Graph $\lambda(G)$ of order n, then the clique number as following : $\omega(G) = |S\,|\ ,\ where\ \ S = \{\,u\,:\ \lambda(u) = 2\,\}$

Proof:

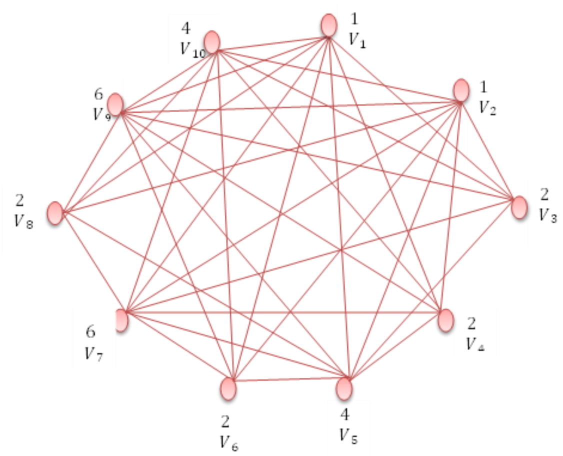
Since every component is a complete sub graph and this graph has many of components such that every component has vertices are adjacent and the clique number dependent on the largest component complete in this graph. The component of the image is equal two is the largest complete sub graph in this graph .

Theorem 2.5 If G be a Carmichael Function Graph $\lambda(G)$ of order n, then the chromatic number as following : $\chi(G) = |S|$, $where\ S = \{u : \lambda(u) = 2\}$

Proof:

According to the theorem (2.4) the clique number is equal $|S|, \{where\ S = \{u : \lambda(u) = 2\}$ since this component is the largest component in this graph is complete such that all vertices are adjacent.

Figure 2.3 The complement of Carmichael Function Graph $\lambda(G)$ of order 10.



Theorem 2.6 If G be a Carmichael Function Graph $\lambda(G^c)$ of order n, then the complement of domination number as :

$$\gamma(G^c) = \begin{cases} 1 & , \ if\ \ n = p\ \ where\ p\ is\ prime \\ 2 & , \ if\ \ there\ is\ not\ isolate\ vertex \end{cases}$$

Proof.

If f n = p for some p , where p is prime then the Graph $\lambda(G)$ has an isolate vertex. If there exist isolate vertex in this graph then there is vertex is called dominating vertex. If the graph has no dominating vertex then there is not isolate vertex. If there is not isolate vertex then the complement of domination number is 2. This graph is connected in every vertices except when the numbers of vertices is two be dis connected.

Theorem 2.7 If G be a Carmichael Function Graph $\lambda(G^c)$ of order n, then the complement of independence number as : $\beta(G^c) = |S|$, $where\ S = \{u : \lambda(u) = 2\}$.

Proof:

Since every vertex is adjacent to all vertices in other components, and these vertices are not adjacent in the same component. We will take the largest component in this graph that represent the image of vertices is equal two in this graph.

Theorem 2.8 If G be a Carmichael Function Graph (G^c) of order n, then the complement of clique number as :$\omega(G^c) = K_n(G)$

Proof:

The number of components is $n(G)$ and the largest component complete in this graph is $K_n(G)$. Since every vertex in any component is adjacent to all vertices in other components and these vertices are not adjacent in the same component then we take the complement of clique number is $K_n(G)$.

Thus the vertices which have numbers constitute an induced sub graph isomorphic to the complete.

Theorem 2.9 If G be a Carmichael Function Graph of order n, then the complement of chromatic number as following : $\chi(G^c) = K_n(G)$

Proof:

According to the theorem (2.8) that the complement of clique number is equal $\omega(G^c) = K_n(G)$ since this component is the largest component in this graph is complete such that all vertices are adjacent between them in all components and theses vertices are not connected in one component.

Theorem 2.10 If G be a Carmichael Function Graph $\lambda(G)$ of order n, then $\gamma^{-1}(G)$ is not exist if G has an isolated vertex , otherwise$\gamma^{-1}(G) = \gamma(G) = n(G)$.

Proof:

Depend on the theorem (2.1) that each component in graph G is complete then we can say that the inverse of domination set in this graph is the number of components. If the graph has only isolate vertex then there is not exist any inverse of dominations number of this graph.

Theorem 2.11 If G be a Carmichael Function Graph $\lambda(G)$ of order n, then the results of $\lambda(G)$ is even or one .

Proof:

There are two cases as following :

Case 1 : if $p \geq 3$ or $\alpha < 2$, then $p^{\alpha-1}(p-1)$, so there is two subcases as fallows :

Subcase 1: if $p = 2$, then $2^{\alpha-1}(1) = 2^{\alpha-1}$ and $\alpha = 1$ then the result is one.

Subcase 2: if $p = 2$, then $2^{\alpha-1}(1) = 2^{\alpha-1}$ and $\alpha > 1$ then the result is even.

Case 2 : if $p = 2$ and $\alpha \geq 3$ then $2^{\alpha-2}$ and the power of the number 2 is positive and hence the result is even.

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4.Conclusion

In this paper we gets that distinct kind of the graphs is called Carmichael Function Graph $\lambda(G)$.Dependence on the results that proved we obtained the dominance, independence, and clique number are determined. Also we calculated the complement the basic elements in this graph as domination number , independence number and clique number . Moreover, the relation between the independence number and domination number is discussed and determined

References

[1] Vince , Star Chromatic Number, Journal of Graph Theory, Vol. 12, No. 4, 551-559 (1988), 0 1988 by John Wiley & Sons, Inc .
[2] Gayathri , S. Kaspar , Connected Co-Independent Domination of a Graph, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 9, 423 – 429.
[3] E. EL-KholyN. El-Sharkawey, The Chromatic Number and Graph Folding, European Journal of Scientific Research ISSN 1450-216X / 1450-202X Vol.120 No.1 (2014), pp.138-144.
[4] G. KOKILAMBAL, A STUDY ON DOMINATING SETS , G. KOKILAMBAL (Reg. No. F9380) Research Scholar Post Graduate and Research Department of Mathematics Thiagarajar College, Madurai-625 009 Tamil Nadu.
[5] K. K. Srimitra1, Shaik Sajana2, D. Bharathi3 , Some Properties of Graph of Mobius Function for ‘0 , International Journal of Innovative Research in Science, Engineering and Technology (An ISO 3297: 2007 Certified Organization) Website: www.ijirset.com Vol. 6, Issue 8, August 2017 , ISSN(Online): 2319-8753, ISSN (Print): 2347-6710.
[6] Robin J. Wilson, Introduction to Graph Theory , Addison Wesley Longman Limited, Edinburgh Gate, Harlow, Essex CM20 2JE, England and Associated Companies throughout the world., Robin Wilson 1972, 1996, Fourth edition, 1996.
[7] Tom M. Apostol ,Introductionto Analytic Number Theory, S.Axler,F. W. Gehring ,K.A. Ribet .