

Optimizing Reliability of Parallel Systems: Prioritized Preventive Maintenance and Inspection

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Abstract

The present study focuses on enhancing the reliability of a system comprising two identical units operating in parallel. Initially, both units are in operative mode. A single serviceman is available to perform all repair activities. Upon failure, a unit undergoes inspection to determine if it is repairable. If repairable, the unit is repaired by the serviceman; otherwise, it is replaced with a new unit. Preventive maintenance is conducted after a specified maximum operation time, with priority given to preventive maintenance over repair activities. Mathematical expressions for various reliability measures, such as Mean Time to System Failure (MTSF), availability, and profit analysis, are derived using the semi-Markov process and regenerative point technique. Tables and graphs are provided to visualize the efficiency of the model. The main contribution of this paper is to provide system designers with insights on how to enhance the profitability of parallel systems by implementing a priority-based approach.

Keywords: Reliability, Inspection, Parallel system, Priority, Preventive maintenance, Repair.

1. Introduction

In today's competitive business marketing, managers have understood that there is a huge demand to improve the reliability of the entire process. Redundancy is considered as the best way to make the system more reliable. Gaver (1993) and Chakravarthy (1983) analyzed the reliability of the system by using redundancy concept to make the system more reliable for use. Different techniques like maintenance, inspection, repairs, replacement are used to improve reliability of the system model. Sherbeny (2013) analyze a parallel system stochastically with preventive maintenance of the unit. In addition to these techniques a good statistical model is consider for providing a more reliable system. Gupta & Goel (1990), Papageorgiou & Kokolakis (2007), Zhang & Yanjun (2016) investigates cost benefit of system models by using standby configuration. To save the time and money, sometimes it is essential to develop a system in which preference is given to a particular repair activity over the other. The concept of priority has been introduced by many researchers in their field of work to make the system more available and efficient with less operational costs. Rathee & Malik (2015) analyze a two identical unit parallel system by giving priority to preventive maintenance over repair subject to

maximum operation and repair times to make the study more reliable. Kumar & Goel (2016), Nandal & Anand (2018) and Abo- Youssef & Assed (2018) obtained the availability and profit analysis of reliability models using the concept to preventive maintenance and priority to enhance the availability as well as profit of the system. Reliability techniques are powerful tools to maintain and increase market share and profitability. Manufacturers in various industrial sectors have been making every effort to improve the reliability of the system. Barak et.al (2018) and Kumar et al. (2022) investigate the different techniques of configuration reliability models with the concept of priority. Keeping in mind the demand of parallel system recently Dabas & Rathee (2022) analyzed the parallel system having two identical units with giving the priority to preventive maintenance over repair.

In the present study a new reliability model is comprises with two identical units works in parallel. Priority is given to preventive maintenance over repair using inspection technique. Single server is used to do the repair activities. Unit is failed by constant rate, and it works as new after repair. Recursive relations for certain reliability measures like MTSF, Availability and profit analysis etc. are obtained by semi-Markov process and regenerative-point technique. To relate the present model with real life systems particular cases are considered for various reliability parameters. Graphs are drawn for arbitrary values of parameters to visualize the efficiency of the model under such situations. The main contribution of this paper is to give the idea to the system designers that how they make a parallel system more profitable by using concept of priority.

2. Standard Notations

Table 1. Nomenclature

| | |
|------------------------------|--|
| λ | : Constant Failure Rate |
| $a/b/\alpha_0$ | : Rate by which system goes for Repair / Replacement / Preventive Maintenance respectively |
| $\alpha/\beta/\gamma/\theta$ | : Repair / Replacement / Inspection / Preventive Maintenance rate respectively done by the server |
| $h(t)/f(t)/r(t)/g(t)$ | : pdf of the Inspection / Repair / Replacement / Preventive Maintenance time respectively |
| $H(t)/F(t)/R(t)/G(t)$ | : cdf of the Inspection / Repair / Replacement / Preventive Maintenance time respectively |
| p_{ij} | : Transition probability from state S_i to state S_j |
| $p_{ij.kr}$ | : Transition probability from state S_i to state S_j via state S_k, S_r |
| $Q_{ij}(t)/q_{ij}(t)$ | : Cdf/pdf of passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in $(0, t]$ |
| $Q_{ij.kr}(t)/q_{ij.kr}(t)$ | : pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k, S_r once in $(0, t]$ |
| μ_i | : Mean sojourn time in state S_i |
| m_{ij} | : Contribution to mean sojourn time in state S_i when the system transits directly to state S_j |
| $*/**$ | : Symbol for Laplace transformation/ Laplace Stieltjes Transformation |
| \odot/\otimes | : Symbol for Laplace transformation/Laplace Stieltjes convolution |

4. Transition Probabilities & Mean Sojourn Times (μ_i)

Transitions probabilities for the system model are given as

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \text{ and } dQ_{ij}(t) = q_{ij}(t)dt$$

Taking laplace stieltjes transformation of above expression we get

$$Q_{ij}^*(s) = \int_0^\infty e^{-st}q_{ij}(t)dt$$

Now taking $\lim_{s \rightarrow 0} Q_{ij}(s)$ we get p_{ij} .

$$p_{01} = \frac{2\lambda}{2\lambda + \alpha_0}, p_{02} = \frac{\alpha_0}{2\lambda + \alpha_0}, p_{13} = bh^*(\lambda + \alpha_0), p_{14} = \frac{\lambda}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0)),$$

$$p_{15} = ah^*(\lambda + \alpha_0), p_{16} = \frac{\alpha_0}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0)), p_{30} = r^*(\lambda + \alpha_0)$$

$$p_{38} = p_{31.8} = \frac{\lambda}{\lambda + \alpha_0}(1 - r^*(\lambda + \alpha_0)), p_{3,15} = p_{37.15} = \frac{\alpha_0}{\lambda + \alpha_0}(1 - r^*(\lambda + \alpha_0))$$

$$p_{49} = p_{6,12} = a, p_{4,10} = p_{6,13} = b, p_{50} = f^*(\lambda + \alpha_0)$$

$$p_{5,11} = p_{51.11} = \frac{\lambda}{\lambda + \alpha_0}(1 - f^*(\lambda + \alpha_0)), p_{5,12} = \frac{\alpha_0}{\lambda + \alpha_0}(1 - f^*(\lambda + \alpha_0))$$

$$p_{70} = g^*(\lambda), p_{7,14} = p_{71.14} = 1 - g^*(\lambda), p_{11.49} = \frac{\lambda a}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0))$$

$$p_{11.4,10} = \frac{\lambda b}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0)), p_{17.6,13} = \frac{\alpha_0 b}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0))$$

$$p_{1,12.6} = \frac{\alpha_0 a}{\lambda + \alpha_0}(1 - h^*(\lambda + \alpha_0))$$

It verified that

$$p_{01} + p_{02} = p_{13} + p_{15} + p_{11.49} + p_{11.4,10} + p_{17.6,13} + p_{1,12.6} = p_{50} + p_{51.11} + p_{5,12} =$$

$$p_{30} + p_{31.8} + p_{37.15} = p_{49} + p_{4,10} = p_{6,12} + p_{6,13} = p_{70} + p_{71.14} =$$

$$p_{27} = p_{81} = p_{91} = p_{10,1} = p_{11,1} = p_{12,5} = p_{13,7} = p_{14,1} = p_{15,7} = 1$$

And μ_i^{s} are given by the formula

$$\mu_i = E(t) = \int_0^\infty P(T > t) dt = \sum_j m_{ij} \text{ and } m_{ij} = \frac{d[Q_{ij}^*(s)]}{ds} |_{s=0}$$

$$\mu_0 = \frac{1}{2\lambda + \alpha_0}, \mu_3 = \frac{1}{\lambda + \alpha_0}(1 - r^*(\lambda + \alpha_0)), \mu_5 = \frac{1}{\lambda + \alpha_0}(1 - f^*(\lambda + \alpha_0))$$

$$\mu_7 = \frac{1}{\lambda}(1 - g^*(\lambda)), \mu'_1 = \left[\frac{1}{\lambda + \alpha_0} + \frac{\lambda a}{\alpha(\lambda + \alpha_0)} + \frac{1}{\gamma} + \frac{b}{\beta} \right] (1 - h^*(\lambda + \alpha_0))$$

$$\mu'_3 = \frac{1}{\beta}, \mu'_5 = \frac{(\alpha + \lambda)}{\alpha(\lambda + \alpha_0)}(1 - f^*(\lambda + \alpha_0)), \mu'_7 = \frac{1}{\theta}$$

5. Reliability & Mean Time To System Failure (MTSF)

Let $\Phi_i(t)$ represent the cumulative distribution function (CDF) of the first passage time from state S_i to the failure state, with the failure state being considered as an absorbing state. Thus, the expressions for $\Phi_i(t)$ from which the Mean Time to System Failure (MTSF) of the system is derived, are given as follows:

$$\Phi_i(t) = \sum_{i,j} Q_{ij}(t) \otimes \Phi_j(t) + \sum_{i,k} Q_{ik}(t) \tag{1}$$

Here, $\begin{cases} j = 1, 2, 3, 5, 0 & \text{for } i = 0, 1, 3, 5, 7 \text{ respectively} \\ k = (4 \text{ and } 6), (8 \text{ and } 15) & \text{for } i = 1, 3 \text{ respectively} \\ k = (11 \text{ and } 12), 14 & \text{for } i = 5, 7 \text{ respectively} \end{cases}$

and $Q_{ik} = 0$ for $i = 0$.

By taking the Laplace-Stieltjes Transform (LST) of the above relation (1) and solving for (s), we obtain:

$$R^*(s) = \frac{1 - \Phi^{**}(s)}{s} \tag{2}$$

The system reliability is obtained by taking Inverse Laplace transform of (2) and MTSF is given by the formula

$$MTSF = \lim_{s \rightarrow 0} \frac{1 - \Phi^{**}(s)}{s} = \frac{N}{D}$$

Where, $N = \mu_0 + \mu_1 p_{01} + \mu_3 p_{01} p_{13} + \mu_5 p_{01} p_{15}$ and $D = 1 - p_{01} p_{13} p_{30} - p_{01} p_{15} p_{50}$

6. Analysis Of Availability

Let $A_i(t)$ be the probability that the system is operational at time t , given the condition that the system entered the regenerative state S_i at $t = 0$. The relationships for $A_i(t)$ can be expressed as follows:

$$A_i(t) = M_i(t) + \sum_{i,j} q_{ij}(t) \odot A_j(t) \tag{3}$$

Here,

$$\begin{cases} j = 1, 2 & \text{for } i = 0 \\ j = 3, 5, 1, 7, 12 & \text{for } i = 1 \\ j = 7 & \text{for } i = 2 \\ j = 0, 1, 7 & \text{for } i = 3 \end{cases} \quad \text{and} \quad \begin{cases} j = 0, 1, 12 & \text{for } i = 5 \\ j = 0, 1 & \text{for } i = 7 \\ j = 5 & \text{for } i = 12 \end{cases}$$

$M_i(t)$ is the probability that the system in up state S_i up to the time t without visiting to any other regenerative state.

$$M_0(t) = e^{-(2\lambda + \alpha_0)t}, \quad M_1(t) = e^{-(\lambda + \alpha_0)t} \overline{H(t)},$$

$$M_3(t) = e^{-(\lambda + \alpha_0)t} \overline{R(t)}, \quad M_5(t) = e^{-(\lambda + \alpha_0)t} \overline{F(t)}, \quad M_7(t) = e^{-(\lambda)t} \overline{G(t)}$$

Now, if we use LT of (3) and solved it for $A_0^*(s)$. We get the result for steady state availability as

$$A_0(\infty) = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{N_1}{D_1} \tag{4}$$

Where, $N_1 = \mu_0 A + (\mu_1 + \mu_3 p_{13})B + \mu_5 C + \mu_7 D$ and
 $D_1 = (\mu_0 + \mu_2 p_{02})A + (\mu'_1 + \mu'_3 p_{13})B + \mu'_5 C + \mu'_7 D + \mu_{12} E$

7. Busy Period Analysis For Server

Let $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$ represent the probabilities that the server is busy with inspection, repair, replacement, and preventive maintenance, respectively, at time t , given that the system entered the regenerative state S_i at $t=0$. The recursive relations for $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$ are as follows:

$$B_i^I(t) = W_i(t) + \sum_{i,j} q_{ij}(t) \odot B_j^I(t) \quad \text{and} \quad B_i^R(t) = W_i(t) + \sum_{i,j} q_{ij}(t) \odot B_j^R(t)$$

$$B_i^{Rp}(t) = W_i(t) + \sum_{i,j} q_{ij}(t) \odot B_j^{Rp}(t) \quad \text{and} \quad B_i^P(t) = W_i(t) + \sum_{i,j} q_{ij}(t) \odot B_j^P(t) \tag{5}$$

$$\begin{cases} j = 1, 2 & \text{for } i = 0 \\ j = 3, 5, 1, 7, 12 & \text{for } i = 1 \\ j = 7 & \text{for } i = 2 \\ j = 0, 1, 7 & \text{for } i = 3 \end{cases} \quad \text{and} \quad \begin{cases} j = 0, 1, 12 & \text{for } i = 5 \\ j = 0, 1 & \text{for } i = 7 \\ j = 5 & \text{for } i = 12 \end{cases}$$

$W_i(t)$ is the probability that the server is busy with repair activities at state S_i without making any transitions to other regenerative states or returning to the same state via one or more non-regenerative states.

Where,

$$W_1(t) = e^{-(\lambda+\alpha_0)t} \overline{H(t)} + (\lambda e^{-(\lambda+\alpha_0)t} \odot 1) \overline{H(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t} \odot 1) \overline{H(t)}$$

$$W_5(t) = e^{-(\lambda+\alpha_0)t} \overline{F(t)} + (\lambda e^{-(\lambda+\alpha_0)t} \odot 1) \overline{F(t)}$$

$$W_3(t) = e^{-(\lambda+\alpha_0)t} \overline{R(t)} + (\lambda e^{-(\lambda+\alpha_0)t} \odot 1) \overline{R(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t} \odot 1) \overline{R(t)}$$

$$W_2(t) = \overline{G(t)} = W_{12}(t), W_7(t) = e^{-\lambda t} \overline{G(t)} + (\lambda e^{-\lambda t} \odot 1) \overline{G(t)}$$

Take LT of (5) and solving it for $B_0^{I*}(s), B_0^{R*}(s), B_0^{Rp*}(s), B_0^{P*}(s)$. The busy time in inspection, repair, replacement and preventive maintenance for server is given by

$$B_0^I(\infty) = \lim_{s \rightarrow 0} sB_0^{I*}(s) = \frac{N_2}{D_1}, \quad B_0^R(\infty) = \lim_{s \rightarrow 0} sB_0^{R*}(s) = \frac{N_3}{D_1}$$

$$B_0^{Rp}(\infty) = \lim_{s \rightarrow 0} sB_0^{Rp*}(s) = \frac{N_4}{D_1}, \quad B_0^P(\infty) = \lim_{s \rightarrow 0} sB_0^{P*}(s) = \frac{N_5}{D_1}$$

Here, $N_2 = W_1^*(0)B, N_3 = W_5^*(0)C, N_4 = W_3^*(0)p_{13}B$

$N_5 = W_2^*(0)p_{02}A + W_7^*(0)D + W_{12}^*(0)E$ and D_1 is mentioned above.

8. Expected Number Of Visits By The Server

Consider, $I_0(t), R_0(t), Rp_0(t), Pm_0(t)$ as the expected number of official visit made by the server for inspection, repair, replacement and PM in $(0, t]$. The recursive relations for $I_0(t), R_0(t), Rp_0(t), Pm_0(t)$ are as follows:

$$I_i(t) = \sum_{i,j} Q_{ij}(t) \otimes (C + I_j(t)) \quad \text{and} \quad R_i(t) = \sum_{i,j} Q_{ij}(t) \otimes (C + R_j(t))$$

$$Rp_i(t) = \sum_{i,j} Q_{ij}(t) \otimes (C + Rp_j(t)) \quad \text{and} \quad Pm_i(t) = \sum_{i,j} Q_{ij}(t) \otimes (C + Pm_j(t)) \quad (6)$$

Where,

$$\begin{cases} j = 1, 2 & \text{for } i = 0 \\ j = 3, 5, 1, 7, 12 & \text{for } i = 1 \\ j = 7 & \text{for } i = 2 \\ j = 0, 1, 7 & \text{for } i = 3 \end{cases} \quad \text{and} \quad \begin{cases} j = 0, 1, 12 & \text{for } i = 5 \\ j = 0, 1 & \text{for } i = 7 \\ j = 5 & \text{for } i = 12 \end{cases}$$

C = 1 when j is regenerative state otherwise C = 0.

Taking the Laplace-Stieltjes Transform (LST) of the recursive relations and solving it for $I_0^{**}(s), R_0^{**}(s), Rp_0^{**}(s), Pm_0^{**}(s)$. The expected number of inspections, repairs, replacements and preventive maintenance by the server is given by (per unit time)

$$I_0(\infty) = \lim_{s \rightarrow 0} sI_0^{**}(s) = \frac{N_6}{D_1}, \quad R_0(\infty) = \lim_{s \rightarrow 0} sR_0^{**}(s) = \frac{N_7}{D_1}$$

$$Rp_0(\infty) = \lim_{s \rightarrow 0} sRp_0^{**}(s) = \frac{N_8}{D_1}, \quad Pm_0(\infty) = \lim_{s \rightarrow 0} sPm_0^{**}(s) = \frac{N_9}{D_1}$$

Where,

$$N_6 = B, \quad N_7 = (p_{11.4,9} + p_{15} + p_{1,12.6})B, \quad N_8 = (p_{11.4,10} + p_{17.6,13} + p_{13})B,$$

$$N_9 = p_{02}A + D + E \quad \text{and} \quad D_1 \text{ is already mentioned.}$$

Here A, B, C, D & E are given as

$$A = (1 - p_{5,12})(p_{13}p_{30} + p_{70}(p_{13}p_{37.15} + p_{17.6,13})) + (p_{15} + p_{1,12.6})p_{50}$$

$$B = (1 - p_{5,12})(1 - p_{02}p_{70})$$

$$C = (p_{15} + p_{1,12.6})(1 - p_{02}p_{70})$$

$$D = (1 - p_{5,12})(p_{13} + p_{17.6,13} - p_{01}p_{13}p_{30} - p_{13}p_{31.8}) + p_{02}p_{50}(p_{15} + p_{1,12.6})$$

$$E = (p_{15}p_{5,12} + p_{1,12.6})(1 - p_{02}p_{70})$$

9. Profit Function

In the steady state, the profit function of the system model can be derived by considering the costs and revenues associated with the operation, repair, replacement, and preventive maintenance activities. The profit function, PP , can be expressed as the difference between the total revenue generated by the system and the total costs incurred. Which can be obtained as

$$P = k_0A_0 - k_1B_0^I - k_2B_0^R - k_3B_0^{Rp} - k_4B_0^{Pm} - k_5I_0 - k_6R_0 - k_7Rp_0 - k_8Pm_0$$

Here,

P = Profit function of system model

k_0 = Revenue per unit up – time of the system

$k_1, k_2, k_3, k_4 =$ Cost per unit time of the server when it is busy in
 inspection, repair, replacement, preventive maintenance
 $k_5, k_6, k_7, k_8 =$ Cost per unit time for inspection, repair,
 replacement, preventive maintenance

10. Results And Discussion

To analyze the availability and profit of the system, exponential distribution is considered for the repair actions. Tables and graphs for availability and profit function are drawn by using numerical values for all the parameters as shown below.

From figure 3, we obtained that availability of the system model is decreasing with increasing failure rate (λ). Availability of the system model is maximum when we increase the inspection rate (γ) from 1.3 to 3 when repair rate $a = 0.6$, replacement rate $b = 0.4$ and other parameters are constant.

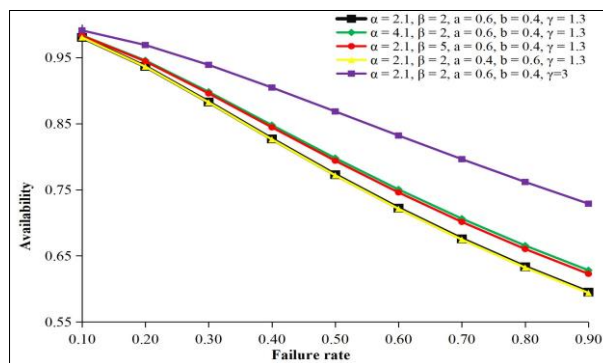


Figure 3. Availability Vs Failure Rate (λ)

From figure 4, we found that the profit of the system model is decreasing with increasing failure rate (λ) which can be enhanced by increasing repair rate (α), replacement rate (β) and inspection rate (γ) of the failed units. Profit is maximum when we increase γ from 1.3 to 3 and take repair rate more ($a = 0.6$) than replacement rate ($b = 0.4$) while other parameters are constant.

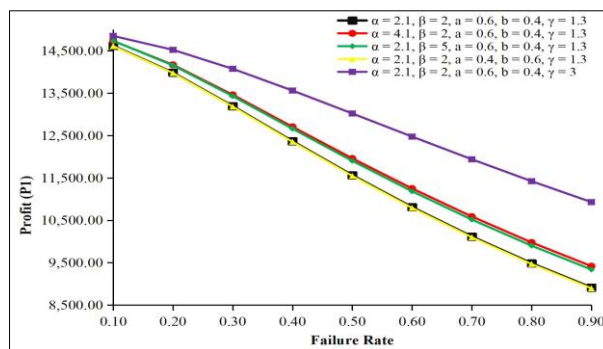


Figure 4. Profit (P1) Vs Failure Rate (λ)

11. Conclusion

From the graphical representation of MTSF, availability and profit of the system model we find that all the three reliability measures are decreasing with the increasing failure rate of the unit. We can optimize these reliability measures by keeping repair and inspection rates higher for constant values

of all other parameters. Therefore, we can conclude that inspection activity is favourable for a manufacturer to enhance the availability and profit of the system model.

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