

Second Order Bipolar Fuzzy Matrix and its Application in Medical Diagnosis

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Article History:

Received: 04-06-2024

Revised: 03-07-2024

Accepted: 30-07-2024

Abstract

In this research article, firstly a new concept of second order bipolar fuzzy matrix (SOBPFM) is introduced and their basic properties and results are proved with examples. Secondly, the concept of symmetrical difference operator (SDO) over SOBPFM is introduced and various properties of SDO are discussed and verified over SOBPFM. Finally, an application on SOBPFM to develop a methodology in handling decision-making problem in medical field which enables clinicians to effectively assess and classify medical conditions, particularly in scenarios involving complex symptoms and overlapping disease manifestation.

Keywords : second order bipolar fuzzy set (SOBPFS), second order bipolar fuzzy matrix (SOBPFM), symmetrical difference operator (SDO)

1. Introduction

The notion of fuzzy set theory was introduced by Zadeh [14] in 1965, which is an universality of classical set theory. The utility of fuzzy set theory in decision making was first demonstrated by Bellman and Zadeh [2] in 1970. Since then many researchers have been working the process of dealing with decision making problems by applying fuzzy set theory.

In 1975, Zadeh [15] proposed the concept of second order fuzzy set which is defined as a map from a set X to I^I . Using second order fuzzy sets Kalaichelvi [3] (2007) extended first order fuzzy topology of Chang and Lowen to second order fuzzy topology.

The matrix is a fundamental concept in mathematics and computer science. It is widely used in various fields including physics, engineering, economics and computer graphics. The concept of fuzzy matrix was first defined by Thomsan [13] in 1977. As an extension of Boolean matrices, the theory of fuzzy matrices were developed by Kim and Roush [4]. In 2022, Saranya et al [12] studied the basic operations and properties of fuzzy matrices. A fuzzy matrix extends the concept of a traditional matrix by allowing entries to have degrees of membership in a set rather than precise value.

In 1994, Zhang [16] proposed the concept of bipolar fuzzy set. Bipolar fuzzy set is an extension of the traditional fuzzy set theory allowing for the representation of uncertainty in both position and negative directions whose membership degree range in $[-1,1]$ where the satisfactory degree of a certain property associated to the fuzzy set locates in the interval $[0,1]$ and the satisfactory degree of counter-property to the concerned fuzzy set locates in the interval $[-1,0]$. In 2024, Muthamizhselvi and Vijayalakshmi [6] proposed the concept of Second order bipolar fuzzy set and second order bipolar fuzzy topological space.

In 2001, Kuratowski [5] defined new concept of symmetrical difference over ordinary sets through union, intersection and negation. In 2004, Anton Antonov [1] introduced SDO over IFM and shown that associativity was not true for IFM. In 2021, Muthuraji [10] analysed commutative monoid on symmetrical difference operator over IFM.

In 2019, M. Pal and Sanjib Mondal [11] introduced bipolar fuzzy matrix. In 2023, Muthuraji and Anitha [7] proposed the recent technology using the bipolar fuzzy matrices with the aid of score function in agriculture. Muthuraji and Punitha Elizabeth [9] proposed application of bipolar fuzzy matrices in flood damage. Application of Bipolar Fuzzy Matrix in the Research of Crops on Agriculture was proposed by Muthuraji and Anitha [8] in 2023. In this paper, second order bipolar fuzzy set, second order bipolar fuzzy matrix and the concept of symmetrical difference operator (SDO) over SOBPFM are introduced. Also an application of second order bipolar fuzzy matrix is presented into a decision making problem and a general algorithm has been constructed to solve the problems in medical diagnosis.

2. Preliminaries

2.1 Definition

Let X be an arbitrary nonempty set. Let $I = [0, 1]$. A **fuzzy set** in X is a map from X into I .

2.2 Definition

A **second order fuzzy set** on X is a map $\hat{f} : X \rightarrow I^I$ where I is the closed unit interval $[0,1]$.

2.3 Definition

Let X be a nonempty set. Then a pair $A_{bp} = (A_{bp}^+, A_{bp}^-)$ is called **bipolar-valued fuzzy set or bipolar fuzzy set** in X , where $A_{bp}^+ : X \rightarrow [0,1]$ and $A_{bp}^- : X \rightarrow [-1,0]$. The set of all bipolar fuzzy set in X is denoted as $BPF(X)$.

2.4 Definition

A pair $\hat{A}_{bp} = (\hat{A}_{bp}^+, \hat{A}_{bp}^-)$ is called a **second order bipolar fuzzy set** in X where $\hat{A}_{bp}^+ : X \rightarrow [0,1]^{[0,1]}$ such that $\hat{A}_{bp}^+(x)(\alpha) \in [0,1]$ and $\hat{A}_{bp}^- : X \rightarrow [-1,0]^{[0,1]}$ such that $\hat{A}_{bp}^-(x)(\alpha) \in [-1,0]$, where $\alpha \in I$ & $x \in X$.

2.5 Definition

Let F be a matrix, $F = [F_{ij}]_{u \times v}$, where $[F_{ij}]_{u \times v} \in [0,1]$, $1 \leq i \leq u$ and $1 \leq j \leq v$, then F is called **fuzzy matrix**.

2.6 Definition

A **bipolar fuzzy matrix** $((A_{bp})_{ij})_{p \times q}$ is defined as $((A_{bp})_{ij})_{p \times q} = ((A_{bp}^+, A_{bp}^-)_{ij})_{p \times q}$ where $(A_{bp}^+)_{ij}(x) \in [0,1]$ and $(A_{bp}^-)_{ij}(x) \in [-1,0] \forall i, j$, where $x \in X$ and p, q denotes the rows and columns of the matrix.

2.7 Definition (Operations in bipolar fuzzy matrix)

Let $\left((A_{bp})_{ij}\right)_{p \times q} = \left((A_{bp}^+, A_{bp}^-)_{ij}\right)_{p \times q}$ &

$\left((B_{bp})_{ij}\right)_{p \times q} = \left((B_{bp}^+, B_{bp}^-)_{ij}\right)_{p \times q}$ be two bipolar fuzzy matrices. Define

(i) $\left((A_{bp})_{ij} + (B_{bp})_{ij}\right)_{p \times q} = \left((C_{bp})_{ij}\right)_{p \times q}$ where $C_{bp} = (C_{bp}^+, C_{bp}^-)$ and is defined as

$$C_{bp}^+(x) = \max\{A_{bp}^+(x), B_{bp}^+(x)\} \&$$

$$C_{bp}^-(x) = \min\{A_{bp}^-(x), B_{bp}^-(x)\}, \forall i, j, \text{ for every } x \in X.$$

(ii) $\left((A_{bp})_{ij} * (B_{bp})_{ij}\right)_{p \times q} = \left((D_{bp})_{ij}\right)_{p \times q}$ where $D_{bp} = (D_{bp}^+, D_{bp}^-)$ and is defined as

$$D_{bp}^+(x) = \min\{A_{bp}^+(x), B_{bp}^+(x)\} \&$$

$$D_{bp}^-(x) = \max\{A_{bp}^-(x), B_{bp}^-(x)\}, \forall i, j, \text{ for every } x \in X.$$

(iii) $\left(\left((A_{bp})_{ij}\right)^c\right)_{p \times q} = \left(\left((A_{bp}^+)^c, (A_{bp}^-)^c\right)_{ij}\right)_{p \times q}$ is defined as

$$(A_{bp}^+)^c(x) = 1 - A_{bp}^+(x), \forall i, j, \text{ for every } x \in X.$$

$$(A_{bp}^-)^c(x) = -1 - A_{bp}^-(x), \forall i, j, \text{ for every } x \in X.$$

3. Second order bipolar fuzzy matrix

3.1 Definition

Let X be a nonempty set. A **second order bipolar fuzzy matrix** (SBPFM) $\left((\hat{A}_{bp})_{ij}\right)_{p \times q}$ is defined

as $\left((\hat{A}_{bp})_{ij}\right)_{p \times q} = \left((\hat{A}_{bp}^+, \hat{A}_{bp}^-)_{ij}\right)_{p \times q}$ where $(\hat{A}_{bp}^+)_{ij}(x)(\alpha) \in [0,1]$ and $(\hat{A}_{bp}^-)_{ij}(x)(\alpha) \in [-1,0]$, $\forall i, j$, where $\alpha \in I$ & $x \in X$.

3.2 Definition (Operations on second order bipolar fuzzy matrix)

Let $\left((\hat{A}_{bp})_{ij}\right)_{p \times q} = \left((\hat{A}_{bp}^+, \hat{A}_{bp}^-)_{ij}\right)_{p \times q}$ &

$\left((\hat{B}_{bp})_{ij}\right)_{p \times q} = \left((\hat{B}_{bp}^+, \hat{B}_{bp}^-)_{ij}\right)_{p \times q}$ be two second order bipolar fuzzy matrices. Define

(iv) $\left((\hat{A}_{bp})_{ij} + (\hat{B}_{bp})_{ij}\right)_{p \times q} = \left((\hat{C}_{bp})_{ij}\right)_{p \times q}$ where $\hat{C}_{bp} = (\hat{C}_{bp}^+, \hat{C}_{bp}^-)$ and is defined as

$$\hat{C}_{bp}^+(x)(\alpha) = \max\{\hat{A}_{bp}^+(x)(\alpha), \hat{B}_{bp}^+(x)(\alpha)\} \&$$

$$\hat{C}_{bp}^-(x)(\alpha) = \min\{\hat{A}_{bp}^-(x)(\alpha), \hat{B}_{bp}^-(x)(\alpha)\}, \forall i, j, \text{ for every } x \in X \text{ for every } \alpha \in I.$$

(v) $\left((\hat{A}_{bp})_{ij} * (\hat{B}_{bp})_{ij}\right)_{p \times q} = \left((\hat{D}_{bp})_{ij}\right)_{p \times q}$ where $\hat{D}_{bp} = (\hat{D}_{bp}^+, \hat{D}_{bp}^-)$ and is defined as

$$\hat{D}_{bp}^+(x)(\alpha) = \min\{\hat{A}_{bp}^+(x)(\alpha), \hat{B}_{bp}^+(x)(\alpha)\} \&$$

$$\hat{D}_{bp}^-(x)(\alpha) = \max\{\hat{A}_{bp}^-(x)(\alpha), \hat{B}_{bp}^-(x)(\alpha)\}, \forall i, j, \text{ for every } x \in X \text{ for every } \alpha \in I.$$

(vi) $\left(\left(\widehat{A}_{bp}\right)_{ij}\right)^c_{p \times q} = \left(\left(\widehat{A}_{bp}^+\right)^c, \left(\widehat{A}_{bp}^-\right)^c\right)_{ij}$ is defined as

$$\left(\widehat{A}_{bp}^+\right)^c(x)(\alpha) = 1 - \widehat{A}_{bp}^+(x)(\alpha), \forall i, j, \text{ for every } x \in X \text{ for every } \alpha \in I.$$

$$\left(\widehat{A}_{bp}^-\right)^c(x)(\alpha) = -1 - \widehat{A}_{bp}^-(x)(\alpha), \forall i, j, \text{ for every } x \in X \text{ for every } \alpha \in I.$$

3.3 Example

Let $\left(\widehat{A}_{bp}\right)_{ij}, \left(\widehat{B}_{bp}\right)_{ij}$ be two 3×3 second order bipolar fuzzy matrices.

$$\left(\widehat{A}_{bp}\right)_{ij} = \begin{bmatrix} (0.7, -0.3) & (0.5, -0.1) & (0.6, -0.4) \\ (0.2, -0.8) & (0.1, -0.5) & (0.9, -0.3) \\ (0.4, -0.7) & (0.3, -0.2) & (0.8, -0.6) \end{bmatrix}_{3 \times 3}$$

$$\left(\widehat{B}_{bp}\right)_{ij} = \begin{bmatrix} (0.9, -0.2) & (0.4, -0.5) & (0.7, -0.3) \\ (0.5, -0.1) & (0.6, -0.4) & (0.8, -0.6) \\ (0.1, -0.8) & (0.2, -0.7) & (0.5, -0.9) \end{bmatrix}_{3 \times 3}.$$

Then compute (i) $\left(\widehat{A}_{bp}\right)_{ij} * \left(\widehat{B}_{bp}\right)_{ij}$

(ii) $\left(\widehat{A}_{bp}\right)_{ij} + \left(\widehat{B}_{bp}\right)_{ij}$

(iii) $\left(\left(\widehat{A}_{bp}\right)_{ij}\right)^c$.

Solution:

(i) $\left(\widehat{A}_{bp}\right)_{ij} * \left(\widehat{B}_{bp}\right)_{ij} = \begin{bmatrix} (0.7, -0.2) & (0.4, -0.1) & (0.6, -0.3) \\ (0.2, -0.1) & (0.1, -0.4) & (0.8, -0.3) \\ (0.1, -0.7) & (0.2, -0.2) & (0.5, -0.6) \end{bmatrix}_{3 \times 3}$

(ii) $\left(\widehat{A}_{bp}\right)_{ij} + \left(\widehat{B}_{bp}\right)_{ij} = \begin{bmatrix} (0.9, -0.3) & (0.5, -0.5) & (0.7, -0.4) \\ (0.5, -0.8) & (0.6, -0.5) & (0.9, -0.6) \\ (0.4, -0.8) & (0.3, -0.7) & (0.8, -0.9) \end{bmatrix}_{3 \times 3}$

(iii) $\left(\left(\widehat{A}_{bp}\right)_{ij}\right)^c = \begin{bmatrix} (0.3, -0.7) & (0.5, -0.9) & (0.4, -0.6) \\ (0.8, -0.2) & (0.9, -0.5) & (0.1, -0.7) \\ (0.6, -0.3) & (0.7, -0.8) & (0.2, -0.4) \end{bmatrix}_{3 \times 3}.$

3.4 Definition

The **transpose of second order bipolar fuzzy $m \times n$ matrix**

$\left(\left(\widehat{A}_{bp}\right)_{ij}\right)^T = \left(\widehat{A}_{bp}^+, \widehat{A}_{bp}^-\right)_{ji}$ is defined as the $n \times m$ second order bipolar fuzzy matrix $\left(\left(\widehat{B}_{bp}\right)_{ij}\right)^T = \left(\widehat{B}_{bp}^+, \widehat{B}_{bp}^-\right)_{ji}$ with $\left(\left(\widehat{B}_{bp}\right)_{ij}\right)^T = \left(\left(\widehat{A}_{bp}\right)_{ji}\right)$ for all $1 \leq i \leq m$ and $1 \leq j \leq n$. The **transpose** of $\left(\left(\widehat{A}_{bp}\right)_{ij}\right)$ is denoted as $\left(\left(\widehat{A}_{bp}\right)_{ij}\right)^T$.

3.5 Example

$$\left((\widehat{A}_{bp})_{ij} \right) = \begin{bmatrix} (0.8, -0.7) \\ (0.1, -0.6) \\ (0.5, -0.5) \end{bmatrix}$$

Then the transpose is given by

$$\left((\widehat{A}_{bp})_{ij} \right)^T = [(0.8, -0.7) \quad (0.1, -0.6) \quad (0.5, -0.5)]$$

3.6 Definition

A square matrix $\left((\widehat{A}_{bp})_{ij} \right) = \left(\widehat{A}_{bp}^+, \widehat{A}_{bp}^- \right)_{ij}$ with $\left((\widehat{A}_{bp})_{ij} \right) = \begin{cases} (1, -1) & \text{if } i = j \\ (0, 0) & \text{if } i \neq j \end{cases}$ is called **second order bipolar fuzzy identity matrix**, denoted as $(\widehat{I}_{bp})_n$.

3.7 Example

$$(\widehat{I}_{bp})_n = \begin{bmatrix} (1, -1) & (0, 0) & (0, 0) \\ (0, 0) & (1, -1) & (0, 0) \\ (0, 0) & (0, 0) & (1, -1) \end{bmatrix}$$

3.8 Remark

Let $\left((\widehat{A}_{bp})_{ij} \right) = \left(\widehat{A}_{bp}^+, \widehat{A}_{bp}^- \right)_{ij}$ be square matrix of order n and \widehat{I}_{bp} be **second order bipolar fuzzy identity matrix**. Then

$$\left(\widehat{A}_{bp} \right)_{ij} * \widehat{I}_{bp} = \widehat{I}_{bp} * \left(\widehat{A}_{bp} \right)_{ij} = \left(\widehat{A}_{bp} \right)_{ij}$$

3.9 Definition

Let $\left((\widehat{A}_{bp})_{ij} \right) = \left(\widehat{A}_{bp}^+, \widehat{A}_{bp}^- \right)_{ij}$ be square matrix (order n). Then the **trace of second order bipolar fuzzy matrix** $\left((\widehat{A}_{bp})_{ij} \right)$ is denoted by $\text{tr}(\widehat{A}_{bp})_{ij}$ and is defined by

$$\text{tr}(\widehat{A}_{bp})_{ij} = \left(\max(\widehat{A}_{bp}^+)_{ij}, \min(\widehat{A}_{bp}^-)_{ij} \right), \text{ where } i = j$$

3.10 Example

$$\left(\widehat{A}_{bp} \right)_{ij} = \begin{bmatrix} (0.1, -0.2) & (0.7, -0.9) \\ (0.3, -0.5) & (0.5, -0.1) \end{bmatrix} \text{ Then the trace of SOBPFM } \left((\widehat{A}_{bp})_{ij} \right) \text{ is given by}$$

$$\begin{aligned} \text{tr}(\widehat{A}_{bp})_{ij} &= \left(\max(\widehat{A}_{bp}^+)_{ij}, \min(\widehat{A}_{bp}^-)_{ij} \right) \\ &= (\max\{0.1, 0.5\}, \min\{-0.2, -0.1\}) \\ &= (0.5, -0.2) \end{aligned}$$

3.11 Properties of second order bipolar fuzzy matrix

Let $\left(\widehat{A}_{bp} \right)_{ij}$, $\left(\widehat{B}_{bp} \right)_{ij}$ and $\left(\widehat{C}_{bp} \right)_{ij}$ be three SOBPFM of order $m \times n$, $n \times p$ and $p \times q$ respectively, then

$$(i) \quad \left(\widehat{A}_{bp} \right)_{ij} * \left(\left(\widehat{B}_{bp} \right)_{ij} * \left(\widehat{C}_{bp} \right)_{ij} \right) = \left(\left(\widehat{A}_{bp} \right)_{ij} * \left(\widehat{B}_{bp} \right)_{ij} \right) * \left(\widehat{C}_{bp} \right)_{ij} \text{ (Associativity)}$$

$$(ii) \quad (\widehat{A}_{bp})_{ij} * ((\widehat{B}_{bp})_{ij} + (\widehat{C}_{bp})_{ij}) = (\widehat{A}_{bp})_{ij} * (\widehat{B}_{bp})_{ij} + (\widehat{A}_{bp})_{ij} * (\widehat{C}_{bp})_{ij} \text{ (Distributive law)}$$

The same results hold for second order bipolar fuzzy complement matrices

$$(i) \quad (\widehat{A}_{bp}^c)_{ij} * ((\widehat{B}_{bp}^c)_{ij} * (\widehat{C}_{bp}^c)_{ij}) = ((\widehat{A}_{bp}^c)_{ij} * (\widehat{B}_{bp}^c)_{ij}) * (\widehat{C}_{bp}^c)_{ij} \text{ (Associativity)}$$

$$(ii) \quad (\widehat{A}_{bp}^c)_{ij} * ((\widehat{B}_{bp}^c)_{ij} + (\widehat{C}_{bp}^c)_{ij}) = (\widehat{A}_{bp}^c)_{ij} * (\widehat{B}_{bp}^c)_{ij} + (\widehat{A}_{bp}^c)_{ij} * (\widehat{C}_{bp}^c)_{ij} \text{ (Distributive law)}$$

3.12 Example

$$(i) \quad (\widehat{A}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.8, -0.5) \\ (0.1, -0.8) & (0.3, -0.2) \end{bmatrix} \quad (\widehat{B}_{bp})_{ij} = \begin{bmatrix} (0.8, -0.5) & (0.2, -0.7) \\ (0.6, -0.4) & (0.3, -0.9) \end{bmatrix}$$

$$(\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.1, -0.3) & (0.3, -0.8) \\ (0.4, -0.5) & (0.9, -0.7) \end{bmatrix}$$

$$(\widehat{B}_{bp})_{ij} * (\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.1, -0.3) & (0.2, -0.7) \\ (0.4, -0.4) & (0.3, -0.7) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * ((\widehat{B}_{bp})_{ij} * (\widehat{C}_{bp})_{ij}) = \begin{bmatrix} (0.1, -0.1) & (0.2, -0.5) \\ (0.1, -0.4) & (0.3, -0.2) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * (\widehat{B}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.2, -0.5) \\ (0.1, -0.4) & (0.3, -0.2) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * ((\widehat{B}_{bp})_{ij} * (\widehat{C}_{bp})_{ij}) = \begin{bmatrix} (0.1, -0.1) & (0.2, -0.5) \\ (0.1, -0.4) & (0.3, -0.2) \end{bmatrix}$$

Property (i) holds

$$(ii) \quad (\widehat{A}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.4, -0.2) \\ (0.3, -0.5) & (0.6, -0.9) \end{bmatrix} \quad (\widehat{B}_{bp})_{ij} = \begin{bmatrix} (0.1, -0.3) & (0.5, -0.4) \\ (0.7, -0.1) & (0.8, -0.5) \end{bmatrix}$$

$$(\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.8, -0.7) & (0.1, -0.5) \\ (0.2, -0.9) & (0.7, -0.6) \end{bmatrix}$$

$$(\widehat{B}_{bp})_{ij} + (\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.8, -0.7) & (0.5, -0.5) \\ (0.7, -0.9) & (0.8, -0.6) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * ((\widehat{B}_{bp})_{ij} + (\widehat{C}_{bp})_{ij}) = \begin{bmatrix} (0.7, -0.1) & (0.4, -0.2) \\ (0.3, -0.5) & (0.6, -0.6) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * (\widehat{B}_{bp})_{ij} = \begin{bmatrix} (0.1, -0.1) & (0.4, -0.2) \\ (0.3, -0.1) & (0.6, -0.5) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * (\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.1, -0.2) \\ (0.2, -0.5) & (0.6, -0.6) \end{bmatrix}$$

$$(\widehat{A}_{bp})_{ij} * (\widehat{B}_{bp})_{ij} + (\widehat{A}_{bp})_{ij} * (\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.4, -0.2) \\ (0.3, -0.5) & (0.6, -0.6) \end{bmatrix}$$

Property (ii) holds

Similarly we can prove for complement of SOBPFM.

3.13 Theorem

Let $(\widehat{A}_{bp})_{ij}$ and $(\widehat{B}_{bp})_{ij}$ be two SOBPFMs (order n) and λ - scalar such that $0 \leq \lambda \leq 1$. Then

$$(i) \quad \text{tr}(\widehat{A}_{bp} + \widehat{B}_{bp})_{ij} = \text{tr}(\widehat{A}_{bp})_{ij} + \text{tr}(\widehat{B}_{bp})_{ij}$$

$$(ii) \quad \text{tr}(\lambda(\widehat{A}_{bp})_{ij}) = \lambda \text{tr}(\widehat{A}_{bp})_{ij}$$

$$(iii) \quad \text{tr}(\widehat{A}_{bp})_{ij} = \text{tr}((\widehat{A}_{bp})_{ij})^T$$

Proof :

(i) Let $(\widehat{A}_{bp})_{ij}$ and $(\widehat{B}_{bp})_{ij}$ be two SOBPFMs (order n)

$$\text{tr}(\widehat{A}_{bp})_{ij} = \left(\max(\widehat{A}_{bp}^+)_{ij}, \min(\widehat{A}_{bp}^-)_{ij} \right)$$

$$\text{tr}(\widehat{B}_{bp})_{ij} = \left(\max(\widehat{B}_{bp}^+)_{ij}, \min(\widehat{B}_{bp}^-)_{ij} \right)$$

Then $(\widehat{A}_{bp} + \widehat{B}_{bp})_{ij} = (\widehat{C}_{bp})_{ij}$, where $\widehat{C}_{bp} = (\widehat{C}_{bp}^+, \widehat{C}_{bp}^-)$. By the definition of trace of SOBPFM, we have

$$\begin{aligned} \text{tr}(\widehat{C}_{bp})_{ij} &= \left(\max \left\{ \max \left\{ (\widehat{A}_{bp}^+)_{ij}, (\widehat{B}_{bp}^+)_{ij} \right\}, \min \left\{ \min \left\{ (\widehat{A}_{bp}^-)_{ij}, (\widehat{B}_{bp}^-)_{ij} \right\} \right\} \right\} \right) \\ &= \left(\max \left\{ \max(\widehat{A}_{bp}^+)_{ij}, \max(\widehat{B}_{bp}^+)_{ij} \right\}, \min \left\{ \min(\widehat{A}_{bp}^-)_{ij}, \min(\widehat{B}_{bp}^-)_{ij} \right\} \right) \\ &= \text{tr}(\widehat{A}_{bp})_{ij} + \text{tr}(\widehat{B}_{bp})_{ij} \end{aligned}$$

$$\begin{aligned} (ii) \quad \text{tr}(\lambda(\widehat{A}_{bp})_{ij}) &= \left(\max(\lambda(\widehat{A}_{bp}^+)_{ij}), \min(\lambda(\widehat{A}_{bp}^-)_{ij}) \right) \\ &= \lambda \left(\max(\widehat{A}_{bp}^+)_{ij}, \min(\widehat{A}_{bp}^-)_{ij} \right) \\ &= \lambda \text{tr}(\widehat{A}_{bp})_{ij} \end{aligned}$$

(iii) Proof is obvious.

4. Properties of SDO on SOBPFM

In this section SDO denoted by \ominus is introduced over SOBPFM. Some properties are discussed.

4.1 Definition

Let $((\widehat{A}_{bp})_{ij})_{p \times q}$ and $((\widehat{B}_{bp})_{ij})_{p \times q}$ are two SOBPFM of same order. The SDO \ominus over $((\widehat{A}_{bp})_{ij})_{p \times q}$ and $((\widehat{B}_{bp})_{ij})_{p \times q}$ using basic operation \wedge, \vee and complement is defined as

$$\begin{aligned} \text{Let} \quad ((\widehat{A}_{bp})_{ij})_{p \times q} &= \left((\widehat{A}_{bp}^+, \widehat{A}_{bp}^-)_{ij} \right)_{p \times q} \quad \& \quad ((\widehat{B}_{bp})_{ij})_{p \times q} = \left((\widehat{B}_{bp}^+, \widehat{B}_{bp}^-)_{ij} \right)_{p \times q} \text{ be} \\ ((\widehat{A}_{bp})_{ij})_{p \times q} \ominus ((\widehat{B}_{bp})_{ij})_{p \times q} &= \left(((\widehat{A}_{bp} \wedge \widehat{B}_{bp}^c) \vee (\widehat{A}_{bp}^c \wedge \widehat{B}_{bp}))_{ij} \right)_{p \times q} = ((\widehat{F}_{bp})_{ij})_{p \times q} \end{aligned}$$

$$\text{Where} \quad ((\widehat{F}_{bp})_{ij})_{p \times q} = \left((\widehat{F}_{bp}^+, \widehat{F}_{bp}^-)_{ij} \right)_{p \times q}$$

$$\hat{F}_{bp}^+ = \left(\left(\left(\hat{A}_{bp}^+ \wedge (\hat{B}_{bp}^+)^c \right) \vee \left((\hat{A}_{bp}^+)^c \wedge \hat{B}_{bp}^+ \right) \right)_{ij} \right)_{p \times q} \text{ and}$$

$$\hat{F}_{bp}^- = \left(\left((\hat{A}_{bp}^- \vee (\hat{B}_{bp}^-)^c) \wedge ((\hat{A}_{bp}^-)^c \vee \hat{B}_{bp}^-) \right)_{ij} \right)_{p \times q}$$

4.2 Lemma

For any $(\hat{A}_{bp})_{ij}$ and $(\hat{B}_{bp})_{ij} \in (\text{SOBPFM})_{m \times n}$. Then

- (i) $(\hat{A}_{bp})_{ij} \ominus (\hat{B}_{bp})_{ij} = (\hat{B}_{bp})_{ij} \ominus (\hat{A}_{bp})_{ij}$
- (ii) $(\hat{A}_{bp})_{ij} \ominus (\hat{1}_{bp})_{ij} = (\hat{A}_{bp}^c)_{ij}$, where $(\hat{1}_{bp})_{ij} = (\hat{1}_{bp}^+, \hat{1}_{bp}^-)_{ij}$
- (iii) $(\hat{A}_{bp})_{ij} \ominus (\hat{0}_{bp})_{ij} = (\hat{A}_{bp})_{ij}$
- (iv) $(\hat{A}_{bp})_{ij} \ominus (\hat{A}_{bp})_{ij} = (\hat{A}_{bp})_{ij} \wedge (\hat{A}_{bp}^c)_{ij}$

Proof:

- (i) From the above definition

$$\begin{aligned} & (\hat{A}_{bp})_{ij} \ominus (\hat{B}_{bp})_{ij} \\ &= \left[\left(\left(\hat{A}_{bp}^+ \wedge (\hat{B}_{bp}^+)^c \right) \vee \left((\hat{A}_{bp}^+)^c \wedge \hat{B}_{bp}^+ \right), (\hat{A}_{bp}^- \vee (\hat{B}_{bp}^-)^c) \wedge ((\hat{A}_{bp}^-)^c \vee \hat{B}_{bp}^-) \right)_{ij} \right] \\ &= \left[\left(\left((\hat{B}_{bp}^+)^c \wedge \hat{A}_{bp}^+ \right) \vee \left(\hat{B}_{bp}^+ \wedge (\hat{A}_{bp}^+)^c \right), ((\hat{B}_{bp}^-)^c \vee \hat{A}_{bp}^-) \wedge (\hat{B}_{bp}^- \vee (\hat{A}_{bp}^-)^c) \right)_{ij} \right] \\ &= \left[\left(\left(\hat{B}_{bp}^+ \wedge (\hat{A}_{bp}^+)^c \right) \vee \left((\hat{B}_{bp}^+)^c \wedge \hat{A}_{bp}^+ \right), (\hat{B}_{bp}^- \vee (\hat{A}_{bp}^-)^c) \wedge ((\hat{B}_{bp}^-)^c \vee \hat{A}_{bp}^-) \right)_{ij} \right] \\ &= (\hat{B}_{bp})_{ij} \ominus (\hat{A}_{bp})_{ij} \end{aligned}$$

- (ii) $(\hat{A}_{bp})_{ij} \ominus (\hat{1}_{bp})_{ij}$

$$\begin{aligned} &= \left[\left(\left(\hat{A}_{bp}^+ \wedge \hat{0}_{bp}^+ \right) \vee \left((\hat{A}_{bp}^+)^c \wedge \hat{1}_{bp}^+ \right), ((\hat{A}_{bp}^-)^c \vee \hat{1}_{bp}^-) \wedge (\hat{A}_{bp}^- \vee \hat{0}_{bp}^-) \right)_{ij} \right] \\ &= \left[\left(\hat{0}_{bp}^+ \vee (\hat{A}_{bp}^+)^c, (\hat{A}_{bp}^-)^c \wedge \hat{0}_{bp}^- \right)_{ij} \right] \\ &= \left((\hat{A}_{bp}^+)^c, (\hat{A}_{bp}^-)^c \right)_{ij} \\ &= ((\hat{A}_{bp})^c)_{ij} \end{aligned}$$

(iii) Proof is similar as (ii)

(iv) $(\widehat{A}_{bp})_{ij} \ominus (\widehat{A}_{bp})_{ij}$

$$= \left[\left((\widehat{A}_{bp}^+ \wedge (\widehat{A}_{bp}^+)^c) \vee ((\widehat{A}_{bp}^+)^c \wedge \widehat{A}_{bp}^+), (\widehat{A}_{bp}^- \vee (\widehat{A}_{bp}^-)^c) \wedge ((\widehat{A}_{bp}^-)^c \vee \widehat{A}_{bp}^-) \right)_{ij} \right]$$

$$= \left[(\widehat{A}_{bp}^+ \wedge (\widehat{A}_{bp}^+)^c, (\widehat{A}_{bp}^-)^c \vee \widehat{A}_{bp}^-)_{ij} \right]$$

$$= (\widehat{A}_{bp} \wedge (\widehat{A}_{bp})^c)_{ij}$$

4.3 Lemma

For any $(\widehat{A}_{bp})_{ij}$ and $(\widehat{B}_{bp})_{ij} \in (SOBPFM)_{m \times n}$. Then

$$((\widehat{A}_{bp})^c)_{ij} \ominus ((\widehat{B}_{bp})^c)_{ij} = (\widehat{A}_{bp})_{ij} \ominus (\widehat{B}_{bp})_{ij}$$

Proof :

$$((\widehat{A}_{bp})^c)_{ij} \ominus ((\widehat{B}_{bp})^c)_{ij}$$

$$= \left[\left(((\widehat{A}_{bp}^+)^c \wedge \widehat{B}_{bp}^+) \vee (\widehat{A}_{bp}^+ \wedge (\widehat{B}_{bp}^+)^c), ((\widehat{A}_{bp}^-)^c \vee \widehat{B}_{bp}^-) \wedge (\widehat{A}_{bp}^- \vee (\widehat{B}_{bp}^-)^c) \right)_{ij} \right]$$

$$= \left[\left((\widehat{A}_{bp}^+ \wedge (\widehat{B}_{bp}^+)^c) \vee ((\widehat{A}_{bp}^+)^c \wedge \widehat{B}_{bp}^+), (\widehat{A}_{bp}^- \vee (\widehat{B}_{bp}^-)^c) \wedge ((\widehat{A}_{bp}^-)^c \vee \widehat{B}_{bp}^-) \right)_{ij} \right]$$

$$= (\widehat{A}_{bp})_{ij} \ominus (\widehat{B}_{bp})_{ij}$$

4.4 Lemma

For any $(\widehat{A}_{bp})_{ij}$, $(\widehat{B}_{bp})_{ij}$ and $(\widehat{C}_{bp})_{ij} \in SOBPFM$ with same order. Then

$$(\widehat{A}_{bp})_{ij} \ominus ((\widehat{B}_{bp})_{ij} \ominus (\widehat{C}_{bp})_{ij}) = ((\widehat{A}_{bp})_{ij} \ominus (\widehat{B}_{bp})_{ij}) \ominus (\widehat{C}_{bp})_{ij}$$

Proof:

The above property is proved by an illustrative example as follows:

$$(\widehat{A}_{bp})_{ij} = \begin{bmatrix} (0.7, -0.1) & (0.8, -0.5) \\ (0.1, -0.8) & (0.3, -0.2) \end{bmatrix} \quad ((\widehat{A}_{bp})^c)_{ij} = \begin{bmatrix} (0.3, -0.9) & (0.2, -0.5) \\ (0.9, -0.2) & (0.7, -0.8) \end{bmatrix}$$

$$(\widehat{B}_{bp})_{ij} = \begin{bmatrix} (0.8, -0.5) & (0.2, -0.7) \\ (0.6, -0.4) & (0.3, -0.9) \end{bmatrix} \quad ((\widehat{B}_{bp})^c)_{ij} = \begin{bmatrix} (0.2, -0.5) & (0.8, -0.3) \\ (0.4, -0.6) & (0.7, -0.1) \end{bmatrix}$$

$$(\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.1, -0.3) & (0.3, -0.8) \\ (0.4, -0.5) & (0.9, -0.7) \end{bmatrix} \quad ((\widehat{C}_{bp})^c)_{ij} = \begin{bmatrix} (0.9, -0.7) & (0.7, -0.2) \\ (0.6, -0.5) & (0.1, -0.3) \end{bmatrix}$$

$$(\widehat{B}_{bp})_{ij} \ominus (\widehat{C}_{bp})_{ij} = \begin{bmatrix} (0.8, -0.5) & (0.3, -0.3) \\ (0.6, -0.5) & (0.7, -0.3) \end{bmatrix}$$

$$(\hat{A}_{bp})_{ij} \ominus ((\hat{B}_{bp})_{ij} \ominus (\hat{C}_{bp})_{ij}) = \begin{bmatrix} (0.3, -0.5) & (0.7, -0.5) \\ (0.6, -0.5) & (0.7, -0.3) \end{bmatrix}$$

$$(\hat{A}_{bp})_{ij} \ominus (\hat{B}_{bp})_{ij} = \begin{bmatrix} (0.3, -0.5) & (0.8, -0.5) \\ (0.6, -0.6) & (0.3, -0.8) \end{bmatrix}$$

$$(\hat{A}_{bp})_{ij} \ominus ((\hat{B}_{bp})_{ij} \ominus (\hat{C}_{bp})_{ij}) = \begin{bmatrix} (0.3, -0.5) & (0.7, -0.5) \\ (0.6, -0.5) & (0.7, -0.3) \end{bmatrix}$$

Hence $(\hat{A}_{bp})_{ij} \ominus ((\hat{B}_{bp})_{ij} \ominus (\hat{C}_{bp})_{ij}) = ((\hat{A}_{bp})_{ij} \ominus (\hat{B}_{bp})_{ij}) \ominus (\hat{C}_{bp})_{ij}$.

5. An application on second order bipolar fuzzy matrix in medical diagnosis

In this paper, we propose an algorithm to find what type of fever affects the different group of people the most.

Let $F = \{f_1, f_2, f_3, f_4\}$ be the set of all fevers where $f_1 =$ Dengue, $f_2 =$ Swine flu, $f_3 =$ Malaria, $f_4 =$ Typhoid. Let $S = \{s_1, s_2, s_3, s_4\}$ be the set of all symptoms caused by the above types of fever where $s_1 =$ Decrease in platelet count, $s_2 =$ fatigue, $s_3 =$ Lose of appetite, $s_4 =$ Eye pain.

Let $P = \{p_1, p_2, p_3, p_4\}$ be the set of all patients of different age groups where

p_1 – Kids aging between 1-12

p_2 – Teenagers aging between 13-19

p_3 – Adult aging between 20-50

p_4 – Old people aging between 50 & above

Now three second order bipolar fuzzy matrices $((\hat{A}_{bp})_{ij})_0$ which gives information about the types of fever for the common symptoms, $((\hat{A}_{bp})_{ij})_S$ which indicates common symptoms that occur in the different age group of people and $((\hat{A}_{bp})_{ij})_\phi$ which shows which common symptom indicates the intensity of the fever are constructed by a decision-maker. Then the event intensity relation $((\hat{A}_{bp})_{ij})_1$, conformability intensity relation $((\hat{A}_{bp})_{ij})_2$, the non-event intensity relation $((\hat{A}_{bp})_{ij})_3$ and non-symptom intensity relation $((\hat{A}_{bp})_{ij})_4$ are determined.

5.1 Algorithm

Step 1: Enter the SOBPFM $((\hat{A}_{bp})_{ij})_0$ which indicates an event of types of fever for the common symptoms.

Step 2: Enter the SOBPFM $((\hat{A}_{bp})_{ij})_S$ which indicates common symptoms that occur in the different age group of people.

Step 3: Enter the second order bipolar fuzzy matrix $((\hat{A}_{bp})_{ij})_\phi$ which indicates the confirmative relation given by $((\hat{A}_{bp})_{ij})_\phi = S * F$.

Step 4: Compute the event intensity relation $((\hat{A}_{bp})_{ij})_1 = ((\hat{A}_{bp})_{ij})_S * ((\hat{A}_{bp})_{ij})_0$.

Step 5: Compute conformability intensity relation $((\hat{A}_{bp})_{ij})_2 = ((\hat{A}_{bp})_{ij})_S * ((\hat{A}_{bp})_{ij})_c$.

Step 6: Compute non-event intensity relation $((\hat{A}_{bp})_{ij})_3 = ((\hat{A}_{bp})_{ij})_S * (((\hat{A}_{bp})_{ij})_0)^c$.

Step 7: Compute non-symptom intensity relation $((\hat{A}_{bp})_{ij})_4 = (((\hat{A}_{bp})_{ij})_S)^c * ((\hat{A}_{bp})_{ij})_0$.

Step 8: Separate the positive and negative values in $((\hat{A}_{bp})_{ij})_4$

5.2 Problem

Symptoms with fever:

Symptoms	Temperature Reference level	Temperature level in patient	Temperature level in [0,1]
s_1	101^0-105^0 for f_1	102.3 ⁰	0.4
s_2	102^0-106^0 for f_2	101.2 ⁰	0
s_3	99^0-105^0 for f_3	104.3 ⁰	0.9
s_4	103^0-104^0 for f_4	103.8 ⁰	0.8

Solution:

Step 1: Matrix for event relation $((\hat{A}_{bp})_{ij})_0 = S \times F$ shows the types of fever for the common

	f_1	f_2	f_3	f_4
s_1	(0.7,-0.6)	(0.6,-0.8)	(0.9,-0.5)	(0.2,-0.9)
s_2	(0.5,-0.4)	(0.7,-0.7)	(0.8,-0.9)	(0.1,-0.6)
s_3	(0.4,-0.2)	(0.1,-0.8)	(0.3,-0.4)	(0.8,-0.4)
s_4	(0.5,-0.3)	(0.4,-0.2)	(0.6,-0.7)	(0.7,-0.1)

Step 2: The following SOBPFM $((\hat{A}_{bp})_{ij})_S$ indicates the common symptoms that occurs in different age group of people.

	s_1	s_2	s_3	s_4
p_1	(0.6,-0.7)	(0.8,-0.5)	(0.4,-0.6)	(0.1,-0.7)
p_2	(0.6,-0.8)	(0.3,-0.1)	(0.1,-0.2)	(0.2,-0.6)
p_3	(0.7,-0.6)	(0.5,-0.4)	(0.2,-0.4)	(0.6,-0.1)
p_4	(0.9,-0.2)	(0.6,-0.3)	(0.5,-0.5)	(0.9,-0.8)

Step 3: Assuming that the matrix for confirmative relation $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_{\mathbb{C}} = S \times F$

$$\begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{bmatrix} (0.5,-0.7) & (0.7,-0.6) & (0.3,-0.8) & (0.1,-0.2) \\ (0.4,-0.2) & (0.5,-0.1) & (0.1,-0.3) & (0.8,-0.7) \\ (0.1,-0.3) & (0.3,-0.5) & (0.1,-0.6) & (0.2,-0.5) \\ (0.5,-0.2) & (0.9,-0.8) & (0.7,-0.4) & (0.6,-0.3) \end{bmatrix} \end{matrix}$$

The following four types of relations can be constructed using the relations $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_0, \left(\left(\hat{A}_{bp}\right)_{ij}\right)_S$ and $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_{\mathbb{C}}$.

Step 4: The event intensity relation $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_1 = \left(\left(\hat{A}_{bp}\right)_{ij}\right)_S * \left(\left(\hat{A}_{bp}\right)_{ij}\right)_0$

$$\begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} (0.6,-0.6) & (0.6,-0.5) & (0.4,-0.5) & (0.1,-0.7) \\ (0.5,-0.4) & (0.3,-0.1) & (0.1,-0.2) & (0.1,-0.6) \\ (0.4,-0.2) & (0.1,-0.4) & (0.2,-0.4) & (0.6,-0.1) \\ (0.5,-0.2) & (0.4,-0.2) & (0.5,-0.5) & (0.7,-0.1) \end{bmatrix} \end{matrix}$$

Step 5: The conformability intensity relation $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_2 = \left(\left(\hat{A}_{bp}\right)_{ij}\right)_S * \left(\left(\hat{A}_{bp}\right)_{ij}\right)_{\mathbb{C}}$

$$\begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} (0.5,-0.7) & (0.7,-0.5) & (0.3,-0.6) & (0.1,-0.2) \\ (0.4,-0.2) & (0.3,-0.1) & (0.1,-0.2) & (0.2,-0.6) \\ (0.1,-0.3) & (0.3,-0.4) & (0.1,-0.4) & (0.2,-0.1) \\ (0.5,-0.2) & (0.6,-0.3) & (0.5,-0.4) & (0.6,-0.3) \end{bmatrix} \end{matrix}$$

Step 6: The non-event intensity relation $\left(\left(\hat{A}_{bp}\right)_{ij}\right)_3 = \left(\left(\hat{A}_{bp}\right)_{ij}\right)_S * \left(\left(\left(\hat{A}_{bp}\right)_{ij}\right)_0\right)^{\mathbb{C}}$

$$\left(\left(\hat{A}_{bp}\right)_{ij}\right)_0^{\mathbb{C}} = \begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{bmatrix} (0.3,-0.4) & (0.4,-0.2) & (0.1,-0.5) & (0.8,-0.1) \\ (0.5,-0.6) & (0.3,-0.3) & (0.2,-0.1) & (0.9,-0.4) \\ (0.6,-0.8) & (0.9,-0.2) & (0.7,-0.6) & (0.2,-0.6) \\ (0.5,-0.7) & (0.6,-0.8) & (0.4,-0.3) & (0.3,-0.9) \end{bmatrix} \end{matrix}$$

$$\left(\left(\hat{A}_{bp}\right)_{ij}\right)_3 = \begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ \begin{matrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{matrix} & \begin{bmatrix} (0.3,-0.4) & (0.4,-0.5) & (0.1,-0.5) & (0.1,-0.1) \\ (0.5,-0.6) & (0.3,-0.1) & (0.1,-0.1) & (0.2,-0.4) \\ (0.6,-0.6) & (0.5,-0.2) & (0.2,-0.4) & (0.2,-0.1) \\ (0.5,-0.2) & (0.6,-0.3) & (0.4,-0.3) & (0.3,-0.8) \end{bmatrix} \end{matrix}$$

Step 7: Eventually, the non-symptom intensity relation

$$\left((\hat{A}_{bp})_{ij} \right)_4 = \left(\left((\hat{A}_{bp})_{ij} \right)_s \right)^c * \left((\hat{A}_{bp})_{ij} \right)_0$$

$$\left((\hat{A}_{bp})_{ij} \right)_s^c = \begin{matrix} & s_1 & s_2 & s_3 & s_4 \\ p_1 & (0.4, -0.3) & (0.2, -0.5) & (0.6, -0.4) & (0.9, -0.3) \\ p_2 & (0.4, -0.2) & (0.7, -0.9) & (0.9, -0.8) & (0.8, -0.4) \\ p_3 & (0.3, -0.4) & (0.5, -0.6) & (0.8, -0.6) & (0.4, -0.9) \\ p_4 & (0.1, -0.8) & (0.4, -0.7) & (0.5, -0.5) & (0.1, -0.2) \end{matrix}$$

$$\left((\hat{A}_{bp})_{ij} \right)_4 = \begin{matrix} & f_1 & f_2 & f_3 & f_4 \\ p_1 & (0.4, -0.3) & (0.2, -0.5) & (0.6, -0.4) & (0.2, -0.3) \\ p_2 & (0.4, -0.2) & (0.7, -0.7) & (0.8, -0.8) & (0.1, -0.4) \\ p_3 & (0.3, -0.2) & (0.1, -0.6) & (0.3, -0.4) & (0.4, -0.4) \\ p_4 & (0.1, -0.3) & (0.4, -0.2) & (0.5, -0.5) & (0.1, -0.1) \end{matrix}$$

Step 8: Separate the positive and negative values in $\left((\hat{A}_{bp})_{ij} \right)_4$

$$\left((\hat{A}_{bp}^+)_{ij} \right)_4 = \begin{bmatrix} 0.4 & 0.2 & 0.6 & 0.2 \\ 0.4 & 0.7 & 0.8 & 0.1 \\ 0.3 & 0.1 & 0.3 & 0.4 \\ 0.1 & 0.4 & 0.5 & 0.1 \end{bmatrix}$$

$$\left((\hat{A}_{bp}^-)_{ij} \right)_4 = \begin{bmatrix} -0.3 & -0.5 & -0.4 & -0.3 \\ -0.2 & -0.7 & -0.8 & -0.4 \\ -0.2 & -0.6 & -0.4 & -0.4 \\ -0.3 & -0.2 & -0.5 & -0.1 \end{bmatrix}$$

6. Conclusion

In this article, the definition of second order bipolar fuzzy matrix is presented and their properties are discussed. Also symmetrical difference operation has been extended over SOBPFM and its properties are studied. In addition to this, a decision making problem in the medical field has been presented where the resulting SOBPFM $\left((\hat{A}_{bp}^+)_{ij} \right)_4$ concludes that the patient p_2 has the high intensity of swine flu and malaria and patient p_4 has low intensity of Dengue and typhoid. It is to be noted that intensity of fever in the different age group of people may vary depending upon the temperature levels recorded.

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