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On Graph Polynomials and Their Applications of Nanostar Structures

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Abstract:
A Nanostars (nanostructure) refers to an entity that occupies an intermediate scale between microscopic and atomic structures. In this paper will used the nanostructure for two types of chemical components known as the fullerene dendrimer is represented as ,and Polypropylenimine octaamine dendrimer known as to get new form to redefine the third Zagrebpolynomial ,general Randicpolynomial , general sum-connectivity polynomial , fourth Zagreb, fifth Zagreb, and harmonic polynomials.

Keywords: Graph Polynomials, chemical graph, Nanostars, Dendrmiers.

1. Introduction

Mathematical chemistry is a specialised field within the realm of chemistry that employs mathematical theories and concepts to analyses and explain chemical structures[1]. Chemical graph theory is a subfield of mathematical chemistry that establishes a connection between graph theory and chemical structures. The concept of the chemical graph emerged throughout the eighteenth century as a result of the intellectual contributions of Isaac Newton. John Dalton, in 1805, introduced the initial model for representing different types of atoms using distinct circles[2][3].

A Nanostars (nanostructure) refers to an entity that occupies an intermediate scale between microscopic and atomic structures. This phenomenon occurs as a result of a physical measurement that is less than 100 nanometers [4].

The first synthesis of dendrimers was accomplished by Fritz Vogtle in 1978 , who employed several synthetic techniques[5].

These approaches included the contributions of RG Denkewalter and Donald Tomalia in 1980[6][7]. In1990[8] George R. Newkome, Craig Hawker, and Jean Frechet proposed a fusion synthesis technique. The prevalence of dendrimers has experienced a significant surge. Prior to 2005, a substantial number of scholarly articles and patents, exceeding 5000 papers, were produced[9].

Consider a graph G that is both simple and linked (molecular graph),The symbols V(G) and E(G) denote the vertex set and edge set of graph G, respectively. Let v be any vertex in the set V (G). We define $d_G(v)$ as the degree of vertex v and N(v) as the set of vertices that are neighbours of v, such that $\left|N(v)=d_G(v)\right|$. The $Z_1(G)$ and $Z_2(G)$ indices, also referred to as the first and second Zagreb indices, these indices were introduced in a publication in 1971 [10].

Also in (2011) Fath –Tabar [11] were defined the third Zagreb polynomial

$$Z_3(G)=\sum_{vu\in E(G)} x^{|d_u-d_v|}$$

In (2022) P.Gladyis et. al [12] used the fourth and fifth Zagreb polynomials of nanostar dendrimer *Dn* , which are depends on cutting number of the vertices

$$Z_4(G)=\sum_{vu\in E(G)} x^{d_v(d_v+d_u)}$$

$$Z_5(G)=\sum_{vu\in E(G)} x^{d_u(d_v+d_u)}$$

In (2021) Abdul Jalil M. Khalaf [13] used some polynomials to compute cellulose's chemical structure like as

The general sum-connectivity polynomial

$$\chi_\alpha(G,x)=\sum_{vu\in E(G)} x^{[dv+du]^\alpha}$$

Randic polynomial

$$R_\alpha(G,x)=\sum_{vu\in E(G)} x^{[dv\times du]^\alpha}$$

In (2018) Juan C. Hernández-Gómezused et. al [14], used harmonic polynomial H(G) to obtain several properties of the harmonic polynomial to obtain omany classical symmetric operations of graphsand . In (2020) ALI AHMAD et. al [15], compute any degree-based topological polynomials for Optical Transpose Interconnection System swapped network.The harmonic polynomial is given as

$$H(G)=\sum_{vuE(G)} x^{d_v+d_u-1}$$

2. Main Results:

2.1 First Result (Fullerene dendrimer) :

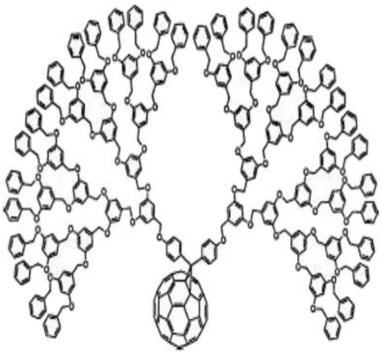

Fig 1. Fullerene dendrimer $NS_1[i]$.

In (Fig. 1), the fullerene dendrimer is nanostar represented as $NS_1[i]$, where n is the number of steps of growth, as illustrated. The molecular graph of $NS_1[i]$ has two branches in symmetrical arrangement contain six types of degrees of the end vertices (1,3), (2,2), (2,3), (3,3), (3,4), and (4,4) . Hence, by doing a direct calculation, we obtain

$$E_1(NS_1[i]) = e_{13} = \{e \in E : d_v = 1, d_u = 3\}$$

$$E_2(NS_1[i]) = e_{22} = \{e \in E : d_v = 2, d_u = 2\}$$

$$E_3(NS_1[i]) = e_{23} = \{e \in E : d_v = 2, d_u = 3\}$$

$$E_4(NS_1[i]) = e_{33} = \{e \in E : d_v = 3, d_u = 3\}$$

$$E_5(NS_1[i]) = e_{34} = \{e \in E : d_v = 3, d_u = 4\}$$

$$E_6(NS_1[i]) = e_{44} = \{e \in E : d_v = 4, d_u = 4\}$$

$e_{13} = 2^{i+1}$, $e_{22} = 2^{i+1} + 2$, $e_{23} = 32 \times 2^{i-1} - 8$

, $e_{33} = 86$, $e_{34} = 6$, and $e_{44} = 3$.

Table 1: the value of degree in $NS_1[i]$ where $d(v,u)$ = (1,3), (2,2), (2,3), (3,3),(3,4), and (4,4) .

stage / degree	i=1	i=2	i=3
d(1,3)	2	4	8
d(2,2)	4	6	10
d(2,3)	8	24	56
d(3,3)	86	86	86
d(3,4)	6	6	6

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d(4,4)	3	3	3

Theorem 1: Let $NS_1[i]$ be the nanostar with i={0,1,2,…} the redefine third Zagreb polynomial is

$$Re\,Z_3G(NS_1[i],x)=2^{i+1}x^{12}+(2^{i+1}+2)x^{16}+(32\times2^{i-1}-8)x^{30}$$
$$+86x^{54}+6x^{84}+3x^{128}$$

Proof: in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (table 1) so by using the definition we obtain

$$\operatorname{Re}Z_3G(NS_1[i],x)=\sum_{vu\in E(NS_1[i])}x^{(dv\times du)(dv+du)}$$

$$=\sum_{vu\in E(NS_1[i])}x^{(1\times3)(1+3)}+\sum_{vu\in E(NS_1[i])}x^{(2\times2)(2+2)}$$
$$+\sum_{vu\in E(NS_1[i])}x^{(2\times3)(2+3)}+\sum_{vu\in E(NS_1[i])}x^{(3\times3)(3+3)}$$
$$+\sum_{vu\in E(NS_1[i])}x^{(3\times4)(3+4)}+\sum_{vu\in E(NS_1[i])}x^{(4\times4)(4+4)}$$

$$Re\,Z_3G(NS_1[i],x)=2^{i+1}x^{12}+(2^{i+1}+2)x^{16}+(32\times2^{i-1}-8)x^{30}$$
$$+86x^{54}+6x^{84}+3x^{128}$$

Theorem 2: Let $NS_1[i]$ be the nanostar with i={0,1,2,…} and α is positive integer the general sum-connectivity polynomial is

$$\chi_\alpha(NS_1[i],x)=(2\times2^{i+1}+2)x^{(4)^\alpha}+86x^{(6)^\alpha}+6x^{(7)^\alpha}+3x^{(8)^\alpha}$$

Proof: in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (table 1) so by using the definition to the general sum-connectivity polynomial we obtain

$$\chi_\alpha(NS_1[i],x)=\sum_{vu\in E(NS_1[i])}x^{[dv+du]^\alpha}$$

$$=\sum_{vu\in E(NS_1[i])}x^{(1+3)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(2+2)^\alpha}$$
$$+\sum_{vu\in E(NS_1[i])}x^{(2+3)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(3+3)^\alpha}$$
$$+\sum_{vu\in E(NS_1[i])}x^{(3+4)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(4+4)^\alpha}$$

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$$=|E_1(NS_1[i]|x^{(1+3)^\alpha}+|E_2(NS_1[i]|x^{(2+2)^\alpha}+|E_3(NS_1[i]|x^{(2+3)^\alpha}$$
$$+|E_4(NS_1[i]|x^{(3+3)^\alpha}+|E_5(NS_1[i]|x^{(3+4)^\alpha}+|E_6(NS_1[i]|x^{(4+4)^\alpha}$$

$$=2^{i+1}x^{(4)^\alpha}+(2^{i+1}+2)x^{(4)^\alpha}+86x^{(6)^\alpha}+6x^{(7)^\alpha}+3x^{(8)^\alpha}$$

$$=(2\times 2^{i+1}+2)x^{(4)^\alpha}+86x^{(6)^\alpha}+6x^{(7)^\alpha}+3x^{(8)^\alpha}$$

Theorem 3: Let $NS_1[i]$ be the nanostar with i={0,1,2,…} and α is positive integer the general Randic polynomial is

$$R_\alpha(NS_1[i],x)=2^{i+1}x^{(3)^\alpha}+(2^{i+1}+2)x^{(4)^\alpha}+(32\times 2^{i+1}-8)x^{(6)^\alpha}$$
$$+86x^{(9)^\alpha}+4x^{(12)^\alpha}+3x^{(16)^\alpha}$$

Proof: in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (Table 1) so by using the definition the general Randic polynomial

$$R_\alpha(NS_1[i],x)=\sum_{vu\in E(NS_1[i])}x^{[dv\times du]^\alpha}$$

$$=\sum_{vu\in E(NS_1[i])}x^{(1\times 3)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(2\times 2)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(2\times 3)^\alpha}$$
$$+\sum_{vu\in E(NS_1[i])}x^{(3\times 3)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(3\times 4)^\alpha}+\sum_{vu\in E(NS_1[i])}x^{(4\times 4)^\alpha}$$

$$=|E_1(NS_1[i]|x^{(1\times 3)^\alpha}+|E_2(NS_1[i]|x^{(2\times 2)^\alpha}+|E_3(NS_1[i]|x^{(2\times 3)^\alpha}$$
$$+|E_4(NS_1[i]|x^{(3\times 3)^\alpha}+|E_5(NS_1[i]|x^{(3\times 4)^\alpha}+|E_6(NS_1[i]|x^{(4\times 4)^\alpha}$$

$$=2^{i+1}x^{(3)^\alpha}+(2^{i+1}+2)x^{(4)^\alpha}+(32\times 2^{i+1}-8)x^{(6)^\alpha}$$
$$+86x^{(9)^\alpha}+4x^{(12)^\alpha}+3x^{(16)^\alpha}$$

Theorem 4: Let $NS_1[i]$ be the nanostar with i={0,1,2,…} the fourth Zagreb polynomial is

$$Z_4(NS_1[i],x)=2^{i+1}x^4+(2^{i+1}+2)x^8+(32\times 2^{i-1}-8)x^{10}$$
$$+86x^{18}+6x^{21}+3x^{32}$$

Proof :in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (table 1) so by using the definition to the fourth Zagreb polynomial

$$Z_4(NS_1[i],x)=\sum_{vu\in E(NS_1[i])}x^{d_v(d_v+d_u)}$$

$$= \sum_{vu\in E(NS_1[i])} x^{(1+3)} + \sum_{vu\in E(NS_1[i])} x^{2(2+2)}$$
$$+ \sum_{vu\in E(NS_1[i])} x^{2(2+3)} + \sum_{vu\in E(NS_1[i])} x^{3(3+3)}$$
$$+ \sum_{vu\in E(NS_1[i])} x^{3(3+4)} + \sum_{vu\in E(NS_1[i])} x^{4(4+4)}$$

$$= \left|E_1(NS_4[i]\right|x^4 + \left|E_2(NS_4[i]\right|x^8 + \left|E_3(NS_4[i]\right|x^{10}$$
$$+ \left|E_4(NS_4[i]\right|x^{18} + \left|E_5(NS_4[i]\right|x^{21} + \left|E_6(NS_4[i]\right|x^{32}$$

$$= 2^{i+1}x^4 + (2^{i+1}+2)x^8 + (32\times 2^{i-1} - 8)x^{10}$$
$$+86x^{18} + 6x^{21} + 3x^{32}$$

Theorem 5: Let $NS_1[i]$ be the nanostar when i={0,1,2,…} the fifth Zagreb polynomial is

$$Z_5(NS_1[i],x) = (2^{i+1}+2)x^8 + (2^{i+1})x^{12} + (32\times 2^{i-1} - 8)x^{15}$$
$$+86x^{18} + 6x^{28} + 3x^{32}$$

Proof : in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (table 1) so by using the definition to the fifth Zagreb polynomial

$$Z_5(NS_1[i],x) = \sum_{uv\in E(NS_1[i])} x^{d_u(d_v+d_u)}$$

$$= \sum_{vu\in E(NS_1[i])} x^{3(1+3)} + \sum_{vu\in E(NS_1[i])} x^{2(2+2)}$$
$$+ \sum_{vu\in E(NS_1[i])} x^{3(2+3)} + \sum_{vu\in E(NS_1[i])} x^{3(3+3)}$$
$$+ \sum_{vu\in E(NS_1[i])} x^{4(3+4)} + \sum_{vu\in E(NS_1[i])} x^{4(4+4)}$$

$$= \left|E_1(NS_1[i]\right|x^{3(1+3)} + \left|E_2(NS_1[i]\right|x^{2(2+2)} + \left|E_3(NS_1[i]\right|x^{3(2+3)}$$
$$+ \left|E_4(NS_1[i]\right|x^{3(3+3)} + \left|E_5(NS_1[i]\right|x^{4(3+4)} + \left|E_6(NS_1[i]\right|x^{4(4+4)}$$

$$= 2^{i+1}x^{12} + (2^{i+1}+2)x^8 + (32\times 2^{i-1} - 8)x^{15}$$
$$+86x^{18} + 6x^{48} + 3x^{32}$$

$$= (2^{i+1}+2)x^8 + (2^{i+1})x^{12} + (32\times 2^{i-1} - 8)x^{15}$$
$$+86x^{18} + 6x^{28} + 3x^{32}$$

Theorem 6: Let $NS_1[i]$ be the nanostar with i={0,1,2,…} by using the harmonic polynomial is

$$=(2\times 2^{i+1}+2)x^3+(32\times 2^{i-1}-8)x^4$$
$$+86x^5+6x^6+3x^7$$

Proof : in (Fig.1) we can see there are two similar branches depends on the degree of the end vertices that has six types of end degree we refers to it by (table 1) so by using the definition to the harmonic polynomial

$$H(G,x)=\sum_{vu\in E(x)} x^{d_v+d_u-1}$$

$$=\sum_{vu\in E(NS_1[i])} x^{(1+3)-1}+\sum_{vu\in E(NS_1[i])} x^{(2+2)-1}$$
$$+\sum_{vu\in E(NS_1[i])} x^{(2+3)-1}+\sum_{vu\in E(NS_1[i])} x^{(3+3)-1}$$
$$+\sum_{vu\in E(NS_1[i])} x^{(3+4)-1}+\sum_{vu\in E(NS_1[i])} x^{(4+4)-1}$$

$$=|E_1(NS_1[i]|x^3+|E_2(NS_1[i]|x^3+|E_3(NS_1[i]|x^4$$
$$+|E_4(NS_1[i]|x^5+|E_5(NS_1[i]|x^6+|E_6(NS_1[i]|x^7$$

$$=(2\times 2^{i+1}+2)x^3+(32\times 2^{i-1}-8)x^4$$
$$+86x^5+6x^6+3x^7$$

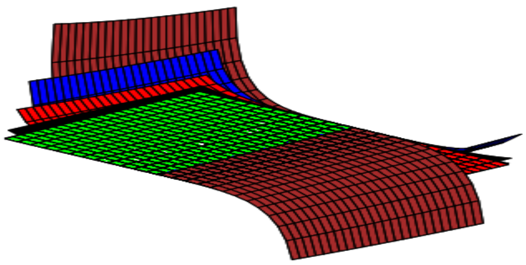

Fig.2 : Comparison between 3rd Zagreb (green), sum conn. (blue), 4th Zageb (black), 5th Zagreb (white), and harmonic (brown) plolynomials .

2.2 Second Results :

In this section we will study all polynomials that we remember in the (First Result) by using the another chemical setructuer in (Fig.3) known as (**Polypropylenimine octaamine dendrimer**) will be denoted as $NS_2[i]$ .where n is the number of steps of growth, as illustrated. The molecular graph of $NS_2[i]$ has three types of degrees of the end vertices (1,2), (2,2),and (2,3) . In symmetrical arrangement with four analogous branches. Hence, by doing a direct calculation, we obtain

$$E_1(NS_2[i])=e_{12}=\{e\in E:d_v=1,d_u=2\}$$

$$E_2(NS_2[i]) = e_{22} = \{e \in E : d_v = 2, d_u = 2\}$$

$$E_3(NS_2[i]) = e_{23} = \{e \in E : d_v = 2, d_u = 3\}$$

$$e_{12} = 2^{i+1}, e_{22} = (8 \times 2^i - 5), e_{23} = (6 \times 2^i - 6)$$

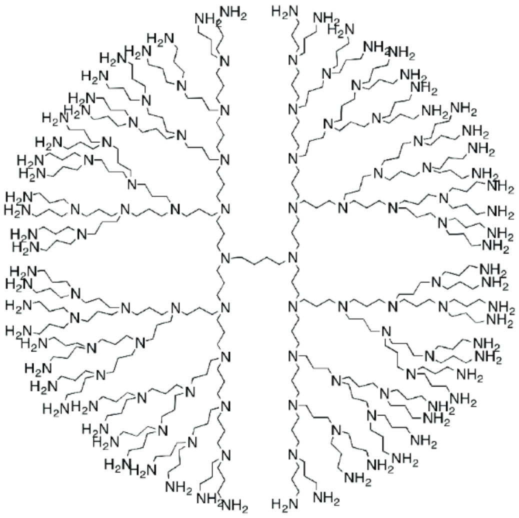

Figure 3. Polypropylenimine octaamine dendrimer $NS_2[i]$

Table **2**: the value of degree in $NS_2[p]$ where $d(v,u)$ = (1,2), (2,2), and (2,3) with stage i={0,1,2,…}

degree \ stage	i=o	i=1	i=2
d(1,2)	2	4	8
d(2,2)	3	11	27
d(2,3)	0	6	18

Theorem 1: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} the redefine third Zagreb polynomial is

$$RZ_3G(NS_2[i],x) = 2^{i+1}x^6 + (8 \times 2^i - 5)x^{16} + (6 \times 2^i - 6)x^{30}$$

Proof: in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using the definition to redefine the third Zagreb polynomial

$$\text{Re}\, Z_3G(NS_2[i], x) = \sum_{vu \in E(NS_2[i])} x^{(dv \times du)(dv + du)}$$

$$= \sum_{vu \in E(NS_2[i])} x^{(1\times 2)(1+2)} + \sum_{vu \in E(NS_2[i])} x^{(2\times 2)(2+2)} + \sum_{vu \in E(NS_2[i])} x^{(2\times 3)(2+3)}$$

$$=\left|E_1(NS_2[i]\right|\sum_{vu\in E(NS_2[i])}x^6+\left|E_1(NS_2[i]\right|\sum_{vu\in E(NS_2[i])}x^{16}$$

$$+\left|E_1(NS_2[i]\right|\sum_{vu\in E(NS_2[i])}x^{30}$$

$$=2^{i+1}x^6+(8\times 2^i-5)x^{16}+(6\times 2^i-6)x^{30}$$

Theorem 2: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} and α is positive integer the general sum-connectivity polynomial is

$$\chi_\alpha(NS_2[i],x)=2^{i+1}x^{(3)^\alpha}+(8\times 2^i-5)x^{(4)^\alpha}+(6\times 2^i-6)x^{(5)^\alpha}$$

Proof : in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using the definition the general sum-connectivity polynomial then we obtain

$$\chi_\alpha(NS_2[i],x)=\sum_{vu\in E(NS_2[i])}x^{[dv+du]^\alpha}$$

$$=\sum_{vu\in E(NS_2[i])}x^{(1+2)^\alpha}+\sum_{vu\in E(NS_2[i])}x^{(2+2)^\alpha}$$

$$+\sum_{vu\in E(NS_2[i])}x^{(2+3)^\alpha}$$

$$=\left|E_1(NS_2[i]\right|x^{(1+2)^\alpha}+\left|E_2(NS_2[i]\right|x^{(2+2)^\alpha}+\left|E_3(NS_2[i]\right|x^{(2+3)^\alpha}$$

$$=2^{i+1}x^{(3)^\alpha}+(8\times 2^i-5)x^{(4)^\alpha}+(6\times 2^i-6)x^{(5)^\alpha}$$

Theorem 3: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} and α is positive integer the general Randic polynomial is

$$R_\alpha(NS_2[i],x)=2^{i+1}x^{(2)^\alpha}+(8\times 2^i-5)x^{(4)^\alpha}+(6\times 2^i-6)x^{(6)^\alpha}$$

Proof: in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using the definition to the general Randic polynomial

$$R_\alpha(NS_2[i],x)=\sum_{vu\in E(NS_2[i])}x^{[dv\times du]^\alpha}$$

$$=\sum_{vu\in E(NS_2[i])}x^{(1\times 2)^\alpha}+\sum_{vu\in E(NS_2[i])}x^{(2\times 2)^\alpha}+\sum_{vu\in E(NS_2[i])}x^{(2\times 3)^\alpha}$$

$$=\left|E_1(NS_2[i]\right|x^{(1\times 2)^\alpha}+\left|E_2(NS_2[i]\right|x^{(2\times 2)^\alpha}+\left|E_3(NS_2[i]\right|x^{(2\times 3)^\alpha}$$

$$=2^{i+1}x^{(2)^\alpha}+(8\times 2^i-5)x^{(4)^\alpha}+(6\times 2^i-6)x^{(6)^\alpha}$$

Theorem 5: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} the fourth Zagreb polynomial is

$$Z_4(NS_2[i],x) = 2^{i+1}x^3 + (8\times 2^i - 5)x^8 + (6\times 2^i - 6)x^{10}$$

Proof : in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using definition to the fourth Zagreb polynomial

$$Z_4(NS_2[i],x) = \sum_{vu\in E(NS_2[i])} x^{d_v(d_v+d_u)}$$

$$= \sum_{vu\in E(NS_2[i])} x^{(1+2)} + \sum_{vu\in E(NS_2[i])} x^{2(2+2)}$$

$$+ \sum_{vu\in E(NS_2[i])} x^{2(2+3)}$$

$$= |E_1(NS_2[i]|x^3 + |E_2(NS_2[i]|x^8 + |E_3(NS_2[i]|x^{10}$$

$$= 2^{i+1}x^3 + (8\times 2^i - 5)x^8 + (6\times 2^i - 6)x^{10}$$

Theorem 6: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} the fifth Zagreb polynomial is

$$Z_5(NS_2[i],x) = 2^{i+1}x^3 + (8\times 2^i - 5)x^8 + (6\times 2^i - 6)x^{10}$$

Proof : in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using definition to the fifth Zagreb polynomial

$$Z_5(NS_2[i],x) = \sum_{uv\in E(NS_2[i])} x^{d_u(d_v+d_u)}$$

$$= \sum_{vu\in E(NS_2[i])} x^{2(1+2)} + \sum_{vu\in E(NS_2[i])} x^{2(2+2)}$$

$$+ \sum_{vu\in E(NS_2[i])} x^{3(2+3)}$$

$$= |E_1(NS_2[i]|x^6 + |E_2(NS_2[i]|x^8 + |E_3(NS_2[i]|x^{15}$$

$$= 2^{i+1}x^6 + (8\times 2^i - 5)x^8 + (6\times 2^i - 6)x^{15}$$

Theorem 7: Let $NS_2[i]$ be the nanostar with i={0,1,2,…} the harmonic polynomial is

$$H(NS_2[i],x) = 2^{i+1}x^2 + (8\times 2^i - 5)x^3 + (6\times 2^i - 6)x^4$$

Proof : in (Fig.3) we can see there are two similar branches depends on the degree of the end vertices that has three types of end degree we refers to it by (table 2) so by using definition to the harmonic polynomial

$$H(NS_2[i],x)=\sum_{uv\in E(NS_2[i])} x^{d_v+d_u-1}$$

$$=\sum_{vu\in E(NS_2[i])} x^{(1+2)-1}+\sum_{vu\in E(NS_2[i])} x^{(2+2)-1}$$

$$+\sum_{vu\in E(NS_2[i])} x^{(2+3)-1}$$

$$=|E_1(NS_2[i]|x^2+|E_2(NS_2[i]|x^3+|E_3(NS_2[i]|x^4$$

$$=2^{i+1}x^2+(8\times 2^i-5)x^3+(6\times 2^i-6)x^4.$$

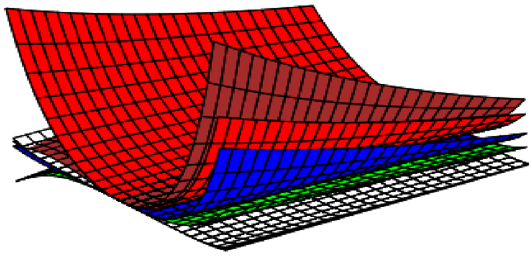

Fig4: Comparison between 3rd Zagreb (black), sum conn(blue), Randic (red), 4th Zagreb(green), 5th Zagreb(white),harmonic(brown) plolynomials .

3. Conclusion

A nanostar, or nanostructure, is a phenomena that occurs at a scale intermediate between microscopic and atomic formations. This research utilises nanostructures, notably fullerene dendrimers and polypropylenimine octaamine dendrimers, to redefine some polynomials like as : the third Zagreb polynomial , the general Randic polynomial , the general sum-connectivity polynomial , the fourth Zagreb polynomial, the fifth Zagreb polynomial, and the harmonic polynomial, the polynomials are also computed .

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