

Investigating The Impact of Inflation on Inventory Systems: Time-Dependent Quadratic Demand, Time-Variable Deterioration, and Shortage

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Abstract

This paper describes the effect of inflation on an inventory system in which the deterioration rate of items is time-dependent and demand is a time-dependent quadratic function of time. Here, shortages are permitted with a partial backlogging rate with exponentially decreasing function. This research aims to determine the ideal ordering amount and optimize the inventory cost. The sensitivity analysis of the optimal solution for various parameters is carried out. Finally, the relationship between parameters and total inventory cost is shown in the figure.

Key Words: Inflation Rate, Time-Dependent Quadratic Demand, Deterioration Rate, and Partial Backlogging.

1. Introduction

Inventory is a crucial asset for businesses. It keeps production running smoothly, saves money on orders, gets discounts on bulk purchases, and makes factories more efficient. Every business organization requires inventory to maintain the efficiency of its production process. The Economic Order Quantity (EOQ) represents the optimal quantity of goods a company should order to minimize inventory expenses, including holding costs, shortage costs, and order costs. The concept was initially introduced by Harris [9], who introduced a basic EOQ model with constant demand and no allowance for shortages.

Inflation refers to the rate at which the general level of prices for goods and services is rising, and subsequently, the purchasing power of currency is falling. This is a crucial factor in inventory management because it directly impacts all inventory costs. When there is inflation, the cost of carrying inventory rises due to the increased prices of goods. This affects various aspects of inventory management, including ordering quantities, reorder points, and inventory control policies. For instance, in the Economic Order Quantity (EOQ) model, which aims to minimize the total inventory costs, the inflation rate would be considered in calculating these costs. Higher inflation rates would lead to higher carrying costs, affecting the optimal order quantity and reorder point. Inflation can be caused by various factors, including, first is demand-pull inflation: When demand for goods and services exceeds supply, it can increase prices as producers and sellers capitalize on

the higher demand. Second is cost-push inflation: When the cost of production increases, such as rising wages or the price of raw materials, producers may pass on these higher costs to consumers, leading to inflation. Third is Built-in inflation: This occurs when people expect prices to rise in the future, leading to demands for higher wages or increased prices by businesses, thus creating a self-perpetuating cycle of inflation.

Several research studies have investigated the inflationary impact on inventory management policies. Buzacott [3] was the first author to integrate the concept of inflation into inventory modeling. He developed a minimum cost model for single-item inventory with inflation. Simultaneously, Misra [15] considered both inflation and the time value of money, including internal and external inflation rates, and analyzed their influence on replenishment strategy about interest rates. Expanding on Misra's work, Chandra and Bahner [4] extended the model to incorporate shortages. Sarker and Pan [21] explored a finite replenishment model, studying the effects of inflation and the time value of money on order quantity while allowing for shortages. Hariga [8] further extended this study to analyze the impact of inflation and the time value of money on an inventory model with time-dependent demand rates and shortages. Moon and Lee [17] explored various inventory models considering the impact of inflation. Jaggi et al. [10] formulated an inventory model to determine the optimal inventory replenishment policy for deteriorating items in the presence of inflation, employing a discounted cash flow (DCF) approach across a finite planning horizon. Kumar et al. [13] investigated an inventory model addressing time-dependent demand and limited storage capacity in the presence of inflation. The primary goal of their paper is to construct a two-warehouse inventory model incorporating partial back ordering and deterioration modeled by the Weibull distribution. They incorporate inflation into the analysis by applying discounted cash flow (DCF) techniques. Singh and Sharma [20] formulated an inventory model that accounts for deteriorating items, considering price-dependent demand and time-dependent holding costs while factoring in the effects of inflation. Arora et al. [1] devised an inventory model for Materials in an Inflationary Environment.

Other researchers have discussed inventory strategies for perishable items. Ghare [7] analyzed the perishable inventory problem and devised a straightforward economic order quantity model with a consistent decay rate. Deterioration is the process of decay or damage that renders an item unusable for its intended purpose. The inventory management of deteriorating items was first studied by Whitin [22], who examined fashion items deteriorating near the end of their shortage period. Bansal [2] developed an inventory model for degradation items that consider the impact of inflation over a fixed planning horizon. Padiyar et al. [18] developed an integrated inventory management system specifically for perishable items.

The word shortage means a state or situation in which the needed items cannot be obtained in sufficient amounts or are absent. To explain the situation more clearly, let us suppose that a company produces some necessary goods whose demand increases suddenly or rapidly, and the company is unable to supply the product due to some unavoidable circumstances. During this time, customers experience shortages of this specific product. Shortages are crucial for many models, especially when considering delayed payment options. If a shortage happens but the company allows delayed payment, it can attract more customer orders, leading to profitable outcomes. Therefore, shortages

can lead to significant profits by providing delayed payment options. In inventory models that account for shortages, the typical assumption is that unmet demand is either entirely lost or fully backlogged. Mishra et al. [16] constructed a partial backlog inventory model that focused on deteriorating products, incorporating time-dependent demand and fluctuating holding costs.

In an inventory system, demand is crucial. It's all about how much people want to buy at different prices. Two main things affect demand: what people like (taste) and what they can afford (ability). When prices are high, fewer people want to buy, but when prices drop, more people do. If prices are super low, lots of folks can afford to buy the product. Many researchers have focused on studying time-dependent demand functions in inventory management. It's generally recognized that constant demand rates should not be assumed for items like fashionable clothing, computer equipment, and similar products. This is because time factors heavily influence the demand for such items. Some researchers have proposed modeling demand as a linear function of time, which implies a steady change in the demand rate per unit of time. However, this linear approach may not accurately reflect real-world market dynamics at all times. On the other hand, an exponential time-dependent demand function, while capturing rapid rate changes, is often considered unrealistic due to the very high rates of change it implies. In practice, market demand for most products is unlikely to experience such extreme exponential changes.

Donaldson [5] was the first to analyze inventory replenishment strategies for a linear trend in demand functions. Dye and colleagues [6] investigated a deteriorating inventory model that includes time-varying demand and partial backlogging dependent on shortages. Kalpakam and Shanthi [11] conducted a study on perishable systems, focusing on a modified base stock policy and random supply quantities. Rahman et al. [19] studied an inventory model that incorporates a demand function that is quadratic time-dependent, and the deterioration rate is time-variable and does not involve shortages. Khare and Sharma [12] proposed an inventory model focusing on deteriorating goods, incorporating time-dependent quadratic demand, and variable holding and ordering costs were investigated in their study. Liao et al. [14] proposed an inventory model tailored for non-instantaneous deteriorating items with expiration dates, considering the combined impact of preservation technology and linearly time-dependent holding costs, with order size tied to advance payment. The primary objective of this research is to optimize replenishment strategies and investment in preservation technology to maximize profitability within this framework.

We have extended the work of Rahman et al. [19] by introducing an inventory model that incorporates a quadratic demand function over time. Additionally, we investigate the effects of inflation on the inventory model and consider time-dependent deterioration rates. Our model allows for shortages with partial backlogging and is solved to minimize total cost. We present a numerical example to validate the model and demonstrate parameter sensitivity graphically.

2. Notations and Assumptions

2.1 Notations

- C_p is the purchase cost per unit time.
- h is the holding cost per unit time.
- $I_1(t)$ is inventory level at a time t_1 .

- $I_2(t)$ is inventory level at a time T .
- A is the inventory order cost per order.
- C_D is the deterioration cost per unit time.
- C_S is the shortage cost per unit time.
- t_1 is stock exhausts time.
- T is the length of a process duration.
- I_b is the highest level of delay purchase.
- I_m is the maximum level of stock during the period $[0, T]$
- TC is the total inventory cost.
- Q is the total quantity of an order.
- SC is the shortage cost per cycle.
- PC is the purchase cost per cycle.
- HC is the holding cost.
- DC is the deterioration cost.
- $R(t)$ is the deterioration rate.
- OC is the ordering cost.

2.2 Assumptions

- Replenishment rate is infinite, i.e. Replenishment rate is instantaneous.
- There is no lead time.
- The demand rate of the item is considered by a quadratic and continuous function of time.

$D(t)$ is the time-dependent demand function which is defined by

$D(t) = (a + bt + ct^2)$. where $a, b, c > 0$. Here a is the initial rate of demand, b is the rate at which the demand rate increases, and c is the rate at which the change in the demand rate itself increases.

- The deterioration rate is variable at time t and its parameter is $R(t) = \theta t^2$.
- Shortage is allowed, with partial backlogging and the backlogging rate is $e^{-\lambda t}$. Unsatisfied demand is backlogged at a rate; the backlogging parameter λ is a positive constant. $\lambda > 0$ is the backlogging parameter and $t_1 \leq t \leq T$.

2. Mathematical Model

Figure 1, In our analysis of the deteriorating inventory model with quadratic demand, we start replenishment at $t = 0$ when the inventory reaches its maximum level, denoted as I_m . As time progresses from 0 to t_1 , the inventory decreases due to demand and deterioration, ultimately reaching zero at t_1 . Subsequently, during the time interval $[t_1, T]$, shortages occur, and the demand in this period is partially backlogged.

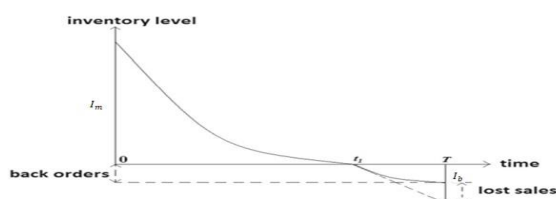


Figure 1: Inventory Level Over Time T

The stock exhausts during the period $[0, t_1]$ as a result of cumulative effects from decay. In this sense, the differential condition represents the stock level at any given time during $[0, t_1]$.

$$\frac{dI_1(t)}{dt} + R(t)I_1(t) = -D(t); \quad 0 \leq t \leq t_1 \quad \dots (1)$$

$$\frac{dI_2(t)}{dt} = -D(t)e^{-\lambda t}; \quad t_1 \leq t \leq T \quad \dots$$

(2)

Using the Deterioration rate $R(t) = \theta t^2$ and Demand rate $D(t) = (a + bt + ct^2)$ with the condition $t = t_1$, and $I_1(t_1) = 0$ in equation (1)

By equation (1) we get,

$$\frac{dI_1(t)}{dt} + \theta t^2 I_1(t) = -(a + bt + ct^2); \quad 0 \leq t \leq t_1$$

$$I_1(t) = \left\{ a(t_1 - t) + \frac{b}{2} (t_1^2 - t^2) + \frac{c}{3} (t_1^3 - t^3) + \frac{a\theta}{12} (3t^4 + t_1^4) + \frac{b\theta}{30} (3t^5 + 2t_1^5) + \frac{c\theta}{18} (t^6 + t_1^6) - \theta t^3 \left(\frac{at_1}{3} + \frac{bt_1^2}{6} + \frac{ct_1^3}{9} \right) \right\}$$

... (3)

Using the boundary condition $t = t_1$, and $I_2(t_1) = 0$ in equation (2), we get

By equation (2) we get,

$$\frac{dI_2(t)}{dt} = -D(t)e^{-\lambda t}; \quad t_1 \leq t \leq T$$

$$\frac{dI_2(t)}{dt} = -(a + bt + ct^2)e^{-\lambda t}$$

$$I_2(t) = \left\{ e^{-\lambda t} \left[\frac{(a+bt+ct^2)}{\lambda} + \frac{(b+2ct)}{\lambda^2} + \frac{(2c)}{\lambda^3} \right] - e^{-\lambda t_1} \left[\frac{(a+bt_1+ct_1^2)}{\lambda} + \frac{(b+2ct_1)}{\lambda^2} + \frac{(2c)}{\lambda^3} \right] \right\}$$

(4) ...

The maximum level of optimistic inventory is $I_m = I_1(0)$

$$I_m = \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] \right\}$$

(5) ...

The maximum negative inventory (back-ordered unit) is $I_b = -I_2(T)$

$$I_b = \left\{ e^{-\lambda t_1} \left[\frac{(a+bt_1+ct_1^2)}{\lambda} + \frac{(b+2ct_1)}{\lambda^2} + \frac{(2c)}{\lambda^3} \right] - e^{-\lambda T} \left[\frac{(a+bT+cT^2)}{\lambda} + \frac{(b+2cT)}{\lambda^2} + \frac{(2c)}{\lambda^3} \right] \right\}$$

(6) ...

Thus, the total quantity in the inventory $[0, T]$ is

$$Q = I_m + I_b$$

$$Q = \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] \right\} + \left\{ e^{-\lambda t_1} \left[\frac{(a+bt_1+ct_1^2)}{\lambda} + \frac{(b+2ct_1)}{\lambda^2} + \frac{2c}{\lambda^3} \right] - e^{-\lambda T} \left[\frac{(a+bT+cT^2)}{\lambda} + \frac{(b+2cT)}{\lambda^2} + \frac{(2c)}{\lambda^3} \right] \right\} \dots$$

(7)

Now,

The Holding cost during the interval $[0, t_1]$ is given by:

$$HC = \int_0^{t_1} h(t) I_1(t) e^{-rt} dt$$

$$HC = h \left\{ \frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{a\theta t_1^5}{20} + \frac{b\theta t_1^6}{24} + \frac{c\theta t_1^7}{28} - r \left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10} + \frac{a\theta t_1^6}{60} + \frac{b\theta t_1^7}{70} + \frac{c\theta t_1^8}{80} \right] \right\} \dots$$

(8)

The Shortage cost during $[t_1, T]$ is evaluated as follows.

$$SC = -C_S \int_{t_1}^T I_2(t) e^{-rt} dt$$

$$SC = \left\{ C_S \frac{c}{\lambda} \left[\frac{e^{-(\lambda+r)T}}{(\lambda+r)^3} (T^2(\lambda+r)^2 + 2T(\lambda+r) + 2) - \frac{e^{-(\lambda+r)t_1}}{(\lambda+r)^3} (t_1^2(\lambda+r)^2 + 2t_1(\lambda+r) + 2) \right] + C_S \frac{(b\lambda+2c)}{\lambda^2} \left[\frac{e^{-(\lambda+r)T}}{(\lambda+r)^2} (T(\lambda+r) + 1) - \frac{e^{-(\lambda+r)t_1}}{(\lambda+r)^2} (t_1(\lambda+r) + 1) \right] + C_S \frac{(a\lambda^2+b\lambda+2c)}{\lambda^3(\lambda+r)} \left[e^{-(\lambda+r)T} - e^{-(\lambda+r)t_1} \right] - C_S \frac{(a\lambda^2+b\lambda+2c)e^{-\lambda t_1}}{\lambda^3 r} [e^{-rT} - e^{-rt_1}] - C_S \frac{(\lambda t_1 c + b\lambda + 2c)e^{-\lambda t_1} t_1}{\lambda^2 r} [e^{-rT} - e^{-rt_1}] \right\} \dots$$

(9)

The Deterioration cost is given by:

$$DC = C_D \left\{ I_1(0) - \int_0^{t_1} D(t) e^{-rt} dt \right\}$$

$$DC = C_D \left\{ r \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} \right] + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] \right\} \dots$$

(10)

The Purchase cost is given by:

$$PC = C_P Q$$

$$PC = C_P \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] + e^{-\lambda t_1} \left[\frac{(a+bt_1+ct_1^2)}{\lambda} + \frac{(b+2ct_1)}{\lambda^2} + \frac{2c}{\lambda^3} \right] - e^{-\lambda T} \left[\frac{(a+bT+cT^2)}{\lambda} + \frac{(b+2cT)}{\lambda^2} + \frac{2c}{\lambda^3} \right] \right\} \dots$$

(11)

Now, the ordering cost is $OC = A$.

The Total Inventory cost is given by:

$$TC = \frac{1}{T} \{ HC + SC + DC + PC + OC \}$$

$$\begin{aligned}
 TC = & \frac{1}{T} \left\{ h \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{a\theta t_1^5}{20} + \frac{b\theta t_1^6}{24} + \frac{c\theta t_1^7}{28} - r \left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10} + \frac{a\theta t_1^6}{60} + \frac{b\theta t_1^7}{70} + \right. \right. \right. \\
 & \left. \left. \left. \frac{c\theta t_1^8}{80} \right] \right\} + \left\{ C_S \frac{c}{\lambda} \left[\frac{e^{-(\lambda+r)T}}{(\lambda+r)^3} (T^2(\lambda+r)^2 + 2T(\lambda+r) + 2) - \frac{e^{-(\lambda+r)t_1}}{(\lambda+r)^3} (t_1^2(\lambda+r)^2 + 2t_1(\lambda+r) + \right. \right. \right. \\
 & \left. \left. \left. 2) \right] + C_S \frac{(b\lambda+2c)}{\lambda^2} \left[\frac{e^{-(\lambda+r)T}}{(\lambda+r)^2} (T(\lambda+r) + 1) - \frac{e^{-(\lambda+r)t_1}}{(\lambda+r)^2} (t_1(\lambda+r) + 1) \right] + C_S \frac{(a\lambda^2+b\lambda+2c)}{\lambda^3(\lambda+r)} \left[e^{-(\lambda+r)T} - \right. \right. \\
 & \left. \left. e^{-(\lambda+r)t_1} \right] - C_S \frac{(a\lambda^2+b\lambda+2c)e^{-\lambda t_1}}{\lambda^3 r} [e^{-rT} - e^{-rt_1}] - C_S \frac{(\lambda t_1 c + b\lambda + 2c)e^{-\lambda t_1} t_1}{\lambda^2 r} [e^{-rT} - e^{-rt_1}] \right\} + \\
 & C_D \left\{ r \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} \right] + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] \right\} + C_P \left\{ at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \theta \left[\frac{at_1^4}{12} + \frac{bt_1^5}{15} + \frac{ct_1^6}{18} \right] + \right. \\
 & \left. e^{-\lambda t_1} \left[\frac{(a+bt_1+ct_1^2)}{\lambda} + \frac{(b+2ct_1)}{\lambda^2} + \frac{2c}{\lambda^3} \right] - e^{-\lambda T} \left[\frac{(a+bT+cT^2)}{\lambda} + \frac{(b+2cT)}{\lambda^2} + \frac{2c}{\lambda^3} \right] + A \right\} \dots (12)
 \end{aligned}$$

The necessary condition to be reduced is

$$\frac{dTc}{dt_1} = 0 \text{ i.e., And } \frac{d^2Tc}{dt_1^2} > 0$$

3. Numerical examples

Values of parameters used in our Inventory Model are as follows:

A=12, C_P =\$15, λ =0.02, r =7%, C_S =\$3, C_D =\$15, θ =0.87, a=10, b=8, c=5, T=1 year and h=\$1 in appropriate units. We obtained the optimal value $t_1=0.42$ year, $TC=255.81$, and $Q= 15.54$ units.

Table 1: Effect of Deterioration Rate (θ) and Inflation Rate (r) on The Total Inventory Cost

θ \ r	Total Cost	0.696	0.783	0.87	0.957	1.044
0.056	TC	255.44	255.56	255.66	255.76	255.85
0.063	TC	255.53	255.64	255.74	255.83	255.92
0.07	TC	255.62	255.72	255.81	255.90	255.98
0.077	TC	255.70	255.79	255.88	255.97	256.05
0.084	TC	255.77	255.86	255.95	256.03	256.10



Figure 2: Graphical Representation of The Effect of Deterioration Rate (θ) And Inflation Rate (r) on The Total Inventory Cost

4. Sensitivity analysis

By changing the values of parameters used in our model and reading out the effects on t_1 and TC . The rate of changes in parameter values are taken -20%, -10%, +10%, and 20%.

Table 2: Sensitivity Analysis of Parameters

Parameter	%	-20%	-10%	0%	+10%	+20%
λ	t_1	0.42	0.42	0.42	0.42	0.42
	TC	256.29	256.05	255.81	255.57	255.33
C_S	t_1	0.38	0.40	0.42	0.44	0.45
	TC	254.14	255.00	255.81	256.59	257.32
C_D	t_1	0.44	0.43	0.42	0.41	0.41
	TC	255.47	255.64	255.81	255.97	256.12
C_P	t_1	0.43	0.43	0.42	0.42	0.41
	TC	209.18	232.50	255.81	279.12	302.43
θ	t_1	0.44	0.43	0.42	0.41	0.41
	TC	255.62	255.72	255.81	255.90	255.98
A	t_1	0.42	0.42	0.42	0.42	0.42
	TC	253.41	254.61	255.81	257.01	258.21
h	t_1	0.43	0.43	0.42	0.42	0.41
	TC	255.58	255.70	255.81	255.92	256.03
a	t_1	0.42	0.42	0.42	0.42	0.42
	TC	224.62	240.22	255.81	271.41	287.01
b	t_1	0.42	0.42	0.42	0.42	0.42
	TC	243.38	249.60	255.81	262.03	268.24
c	t_1	0.42	0.42	0.42	0.42	0.42
	TC	250.67	253.24	255.81	258.38	260.95
r	t_1	0.43	0.43	0.42	0.42	0.41
	TC	255.66	255.74	255.81	255.88	255.95

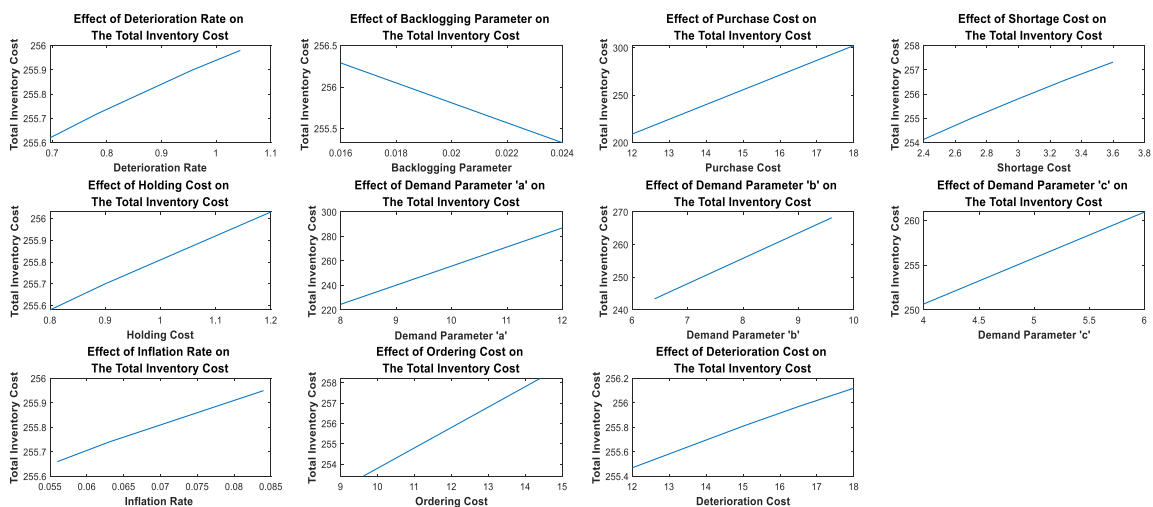


Figure 3: Graphical Representation of the Sensitivity Analysis of Parameters

5. Conclusion

We created an inventory model for a quadratic demand rate with a time-dependent deterioration rate and Shortages allowed with partially backlogged. Also, we investigate the effects of inflation on the inventory model. This given model is supported by a numerical example along with sensitivity analysis is carried out to measure the effect of parameters on the total average inventory cost. Table 1 illustrates how changes in the inflation rate and deterioration rate impact the total cost, as depicted in Figure 2. A sensitivity analysis is conducted to examine the effects of boundary values on the optimal arrangement. Based on the model analysis, it has been concluded that in Table 2, as depicted in Figure 3, these effects are illustrated.

From the analysis of the model, it has concluded that:

- Stock exhaust time (t_1) is not sensitive to changes in A, a, b, c and λ . But it decreases marginally for C_D, C_P, θ, h, r and increases marginally for C_S .
- The total inventory cost decreases marginally for backlogging parameter λ .
- The total inventory cost increases marginally for C_S, C_D, θ, h, r , and A .
- The total inventory cost increased significantly for a, b, c , and C_P .

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