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Zagreb Indices of an Undirected graphs G_n and ${G_{m,n}}^M$

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Abstract

A Topological index is a molecular structure descriptor of a molecular graph and it has many applications in real world issues. The Zagreb Indices based on degree of vertices of a graph are widely studied in chemical graph theory in over the past few years. Degree based Zagreb indices of undirected graphs ${G_{m,n}}^M$ and G_n graphs are computed in this paper.

Conclusions: The authors of this paper have studied the Zagreb indices of undirected graphs G_n when $n = 2^\alpha$, $\alpha > 2$ and undirected graph ${G_{m,n}}^M$ for some cases that is when $n = 2p$, p is prime,$m > n$, m is prime and when $m > n$, m, n are odd primes.

Keywords: Zagreb indices of a graph, undirected graph ${G_{m,n}}^M$, undirected graph G_n.

1. Introduction

Topological index is a numerical quantity which plays vital role in QSAR or QSPR studies. Different types of topological indices are introduced and studied so far. In 1972 Gutman [1], [2] introduced Zagreb indices to describe the properties of chemical compounds, the chemical structures are described by molecular graphs**.** This formula in graph theory represents atoms and chemical bonds as vertices and edges, respectively. Some bounds for the Zagreb indices discussed in [3-5]. The topological indices of graph operations are computed in [8,9]. New types of Zagreb indices are presented in [10].Siva Parvathi et al [11] calculated the Zagreb indices of selected chemical compounds of natural products.

Ivy Chakrabarthy et al [12] introduced the undirected graph G_n and proved some basic properties. Ivy Chakrabarthy et al [13] introduced the undirected graph ${G_{m,n}}^M$ and proved some basic properties of ${G_{m,n}}^M$ graph. Recently Eswaramma et al [14] calculated energy of this ${G_{m,n}}^M$ graph. Anusha et al[15]calculated the Zagreb indices of Arithmetic graphs.

Motivated by these, we calculate the Zagreb indices of undirected ${G_{m,n}}^M$graph, undirected $G_n$graph.

Definitions

Consider the simple graph G. Let d_i, d_j be the degrees of the vertices v_i, v_j and e_{ij} be the edges joining the vertices v_i, v_j respectively.Then

The First Zagreb index of graph G is defined as

$$M_1(G) = \sum_{v_i, v_j \in E(G)} (d_i + d_j)$$

The Second Zagreb index of graph G is defined as

$$M_2(G) = \sum_{v_i, v_j \in E(G)} (d_i \times d_j)$$

The hyper-Zagreb index was introduced by Shirdel et al [6] in 2013.The hyper Zagreb index of graph G is defined as

$$M_H(G) = \sum_{v_i, v_j \in E(G)} (d_i + d_j)^2$$

In [7], M.Ghorbani and N.Azimi defined different versions of the Zagreb indices based on the degree of vertices.

First multiple Zagreb index of the graph G is defined as

$$PM_1(G) = \prod_{v_i, v_j \in E(G)} (d_i + d_j)$$

Second multiple Zagreb index of the graph G is defined as

$$PM_2(G) = \prod_{v_i, v_j \in E(G)} (d_i \times d_j)$$

2. Zagreb indices of an undirected graph $G_{m,n}{}^M$

The Undirected simple graph $G_{m,n}{}^M$ is $G_{m,n}{}^M = (V, E)$ where the vertex set $V = \{1,2, \dots . n\}$ and two distinct vertices $u, v \in V$ are adjacent if and only if $u \neq v$and $u.v$ is not divisible by m on natural numbers subset which is finite where $m, n \in N$. Some of the properties are

1. Let $m = 1$ then the graph $G_{m,n}{}^M$ is a null graph with n vertices.
2. For $1 < m \leq n$,the graph $G_{m,n}{}^M$ is disconnected.
3. The graph $G_{m,n}{}^M$is connected for $m > n$
4. The graph $G_{m,n}{}^M$ has the Maximum degree $n - 1$.

Results on various Zagreb indices of the graph $G_{m,n}{}^M$ are presented in this section.

Theorem 2.1: If $G_{m,n}{}^M$ be an undirected graph where $n = 2p, p$ is prime, $m > n$, m is prime. Then

(i) First Zagreb index $M_1(G_{m,n}{}^M)$ is $n(n-1)^2$.

(ii)Second Zagreb index $M_2(G_{m,n}{}^M)$ is $\frac{n(n-1)^3}{2}$.

(iii)Hyper Zagreb index $M_H(G_{m,n}{}^M)$ is $2n(n-1)^3$.

(iv)First multiple Zagreb index $PM_1(G_{m,n}{}^M)$ is $2(n-1)^{\frac{n(n-1)}{2}}$.

(iv) Second multiple Zagreb index $PM_2(G_{m,n}{}^M)$ is $(n-1)^{n(n-1)}$.

Proof: Consider an undirected graph $G_{m,n}{}^M$ where $n = 2p, p$ is prime, $m > n$,m is prime with $V = \{1,2,3 \dots n\}$ as the vertex set, the vertex degree is $(n-1)$ for every $v \in V$and E is the edge set.

Let d_i, d_j be the degrees of the vertices v_i, v_j and e_{ij} be the edges joining the vertices v_i, v_j.

Then $\left|e_{ij}\right| = \frac{n(n-1)}{2}$

(i)First Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_1\left({G_{m,n}}^M\right) = \sum_{v_i,v_j \in E}(d_i + d_j)$$

$$= \left|e_{ij}\right|(d_i + d_j)$$

$$= \frac{n(n-1)}{2}[(n-1) + (n-1)]$$

$$= n(n-1)^2.$$

(ii)Second Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_2\left({G_{m,n}}^M\right) = \sum_{v_i,v_j \in E}(d_i \times d_j) = \left|e_{ij}\right|(d_i \times d_j)$$

$$= \frac{n(n-1)}{2}[(n-1) \times (n-1)]$$

$$= \frac{n(n-1)^3}{2}$$

(iii) Hyper Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_H\left({G_{m,n}}^M\right) = \sum_{v_i,v_j \in E(G)}(d_i + d_j)^2$$

$$= \left|e_{ij}\right|(d_i + d_j)^2$$

$$= \frac{n(n-1)}{2}[(n-1) + (n-1)]^2$$

$$= 2n(n-1)^3.$$

(iv)First multiple Zagreb index of the graph ${G_{m,n}}^M$ is

$$PM_1({G_{m,n}}^M) = \prod_{v_i,v_j \in E(G)}(d_i + d_j)$$

$$= (d_i + d_j)^{\left|e_{ij}\right|}$$

$$= [(n-1)+(n-1)]^{\frac{n(n-1)}{2}}$$

$$= (2(n-1))^{\frac{n(n-1)}{2}}.$$

(v) Second multiple Zagreb index of the graph ${G_{m,n}}^M$ is

$$PM_2({G_{m,n}}^M) = \prod_{v_i,v_j \in E}(d_i \times d_j)$$

$$= (d_i \times d_j)^{\left|e_{ij}\right|}$$

$$= [(n-1)(n-1)]^{\frac{n(n-1)}{2}}$$

$= (n-1)^{n(n-1)}$.

Theorem 2.2: If ${G_{m,n}}^M$ be an undirected graph where $m > n, m, n$ are odd primes. Then

(i) First Zagreb index $M_1({G_{m,n}}^M)$ is $n(n-1)^2$.

(ii)Second Zagreb index $M_2({G_{m,n}}^M)$ is $\frac{n(n-1)^3}{2}$.

(iii)Hyper Zagreb index $M_H({G_{m,n}}^M)$ is$2n(n-1)^3$.

(iv)First multiple Zagreb index $PM_1({G_{m,n}}^M)$ is $2(n-1)^{\frac{n(n-1)}{2}}$.

(iv) Second multiple Zagreb index $PM_2({G_{m,n}}^M)$ is $(n-1)^{n(n-1)}$.

Proof: Consider an undirected graph ${G_{m,n}}^M$ with $m > n, m, n$ are odd primes and $V = \{1,2,3 \ldots n\}$is the vertex set .Here the degree of the vertex is $(n-1)$ for every $v \in V$.

Let d_i, d_j be the degrees of the vertices v_i, v_j and e_{ij} be the edges joining the vertices v_i, v_j.

Then $|e_{ij}| = \frac{n(n-1)}{2}$.

(i)First Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_1({G_{m,n}}^M) = \sum_{v_i, v_j \in E} (d_i + d_j)$$

$$= |e_{ij}|(d_i + d_j)$$

$$= \frac{n(n-1)}{2}[(n-1) + (n-1)]$$

$$= n(n-1)^2.$$

(ii)Second Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_2({G_{m,n}}^M) = \sum_{v_i, v_j \in E} (d_i \times d_j)$$

$$= |e_{ij}|(d_i \times d_j)$$

$$= \frac{n(n-1)}{2}[(n-1) \times (n-1)]$$

$$= \frac{n(n-1)^3}{2}$$

(iii) Hyper Zagreb index of the graph ${G_{m,n}}^M$ is

$$M_H({G_{m,n}}^M) = \sum_{v_i, v_j \in E} (d_i + d_j)^2$$

$$= |e_{ij}|(d_i + d_j)^2$$

$$= \frac{n(n-1)}{2}[(n-1) + (n-1)]^2$$

$$= 2n(n-1)^3.$$

(iv)First multiple Zagreb index of the graph ${G_{m,n}}^M$ is

$PM_1({G_{m,n}}^M)= \prod_{v_i,v_j \in E}(d_i + d_j)$

$= (d_i + d_j)^{|e_{ij}|}$

$= [(n-1)+(n-1)]^{\frac{n(n-1)}{2}}$

$= (2(n-1))^{\frac{n(n-1)}{2}}.$

(v) Second multiple Zagreb index of the graph ${G_{m,n}}^M$ is

$PM_2({G_{m,n}}^M)= \prod_{v_i,v_j \in E}(d_i \times d_j)$

$= (d_i \times d_j)^{|e_{ij}|}$

$= [(n-1)(n-1)]^{\frac{n(n-1)}{2}}$

$= (n-1)^{n(n-1)}.$

3. Zagreb Indices of the undirected graph G_n

Let an undirected simple graph $G_n = (V, E)$ whose vertex set V is a subset of Natural numbers defined as $V = \{x \in N/(x, n) \neq 1, x < n\}$, where $n \in N$and n is not a prime number and a pair of vertices $x, y \in V$ is adjacent if and only if gcd $(x, y) > 1$.Some of the properties of graph G_n as follows

1. The graph G_n is complete if and only if $n = p^m$ where p is prime.
2. The graph G_n is disconnected if and only if $n = 2p$ where p is an odd prime.

Results on various Zagreb indices of the graph G_n are presented in this section.

Theorem 3.1: If G_n be an undirected graph where $n = 2^\alpha, > 2$. Then

(i) First Zagreb index $M_1(G_n)$ is $(2^{\alpha-1} - 1)(2^{\alpha-1} - 2)^2$.

(ii)Second Zagreb index $M_2(G_n)$ is $\frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)^3}{2}$.

(iii)Hyper Zagreb index $M_H(G_n)$ is $2(2^{\alpha-1} - 1)(2^{\alpha-1} - 2)^3$.

(iv)First multiple Zagreb index $PM_1(G_n)$ is $(2(2^{\alpha-1} - 2))^{\frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2}}$.

(iv) Second multiple Zagreb index $PM_2(G_n)$ is $(2^{\alpha-1} - 2)^{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}$.

Proof: Consider an undirected graph G_n where $n = 2^\alpha$, $\alpha > 2$ with the vertex set$V = \{2,2.2,3.2, \dots .(2^{\alpha-1} - 1). 2\}$and E is the edge set.Here the degree of the vertex is $(2^{\alpha-1} - 2)$ for every $v \in V$.

Let d_i, d_j be the degrees of the vertices v_i, v_j and e_{ij} be the edges joining the vertices v_i, v_j.

Then $|e_{ij}| = \frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2}$

(i)First Zagreb index of the graph G_n where $n = 2^\alpha, \alpha > 2$ is

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$$M_1(G_n) = \sum_{v_i, v_j \in E} (d_i + d_j)$$

$$= |e_{ij}|(d_i + d_j)$$

$$= \frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2} [(2^{\alpha-1} - 2) + (2^{\alpha-1} - 2)]$$

$$= (2^{\alpha-1} - 1)(2^{\alpha-1} - 2)^2.$$

(ii)Second Zagreb index of the graph G_n where $n = 2^\alpha, \alpha > 2$ is

$$M_2(G_n) = \sum_{v_i, v_j \in E} (d_i \times d_j)$$

$$= |e_{ij}|(d_i \times d_j)$$

$$= \frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2} [(2^{\alpha-1} - 2) \times (2^{\alpha-1} - 2)]$$

$$= \frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)^3}{2}$$

(iii) Hyper Zagreb index of the graph G_n where $n = 2^\alpha, \alpha > 2$ is

$$M_H(G_n) = \sum_{v_i, v_j \in E} (d_i + d_j)^2$$

$$= |e_{ij}|(d_i + d_j)^2$$

$$= \frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2} [(2^{\alpha-1} - 2) + (2^{\alpha-1} - 2)]^2$$

$$= 2(2^{\alpha-1} - 1)(2^{\alpha-1} - 2)^3.$$

(iv) First multiple Zagreb index of the graph G_n where $n = 2^\alpha, \alpha > 2$ is

$$PM_1(G_n) = \prod_{v_i, v_j \in E} (d_i + d_j)$$

$$= (d_i + d_j)^{|e_{ij}|}$$

$$= [(2^{\alpha-1} - 2) + (2^{\alpha-1} - 2)]^{\frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2}}$$

$$= (2(2^{\alpha-1} - 2))^{\frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2}}.$$

(v) Second multiple Zagreb index of the graph G_n where $n = 2^\alpha, \alpha > 2$ is

$$PM_2(G_n) = \prod_{v_i, v_j \in E} (d_i \times d_j)$$

$$= (d_i \times d_j)^{|e_{ij}|}$$

$$= [(2^{\alpha-1} - 2)(2^{\alpha-1} - 2)]^{\frac{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}{2}}$$

$$= (2^{\alpha-1} - 2)^{(2^{\alpha-1}-1)(2^{\alpha-1}-2)}.$$

4. Conclusion

The authors of this paper have studied the Zagreb indices of undirected graphs G_n when $n = 2^{\alpha}, \alpha > 2$ and undirected graph $G_{m,n}{}^{M}$ for some cases that is when $n = 2p$, p is prime, $m > n$, m is prime and when $m > n$, m, n are odd primes.

5. Refrences

[1] I.Gutman and N.Trinajstic, Graph theory and Molecular Orbitals,Total π-electron energy of alternant hydrocarbons,Chem.Phys.Lett.17(1972),533-538.

[2] Gutman.I.,Rusic.R., Trinajstic.N.,-Graph Theory and Molecular Orbitals,XII Acyclic polynes.J.Chem.Phys.62,1975,pp.3399-3405.

[3] K.C.Das,I.Gutman and B.Zhou.,"New upper bounds on Zagreb Indices",Journal of Mathematical Chemistry,vol.46,no.2,pp.514-521,2009.

[4] K.C.Das A.Yurttas,M,Togan,A.S.Cevik,I.N.Cangul,"The Multiplicative Zagreb Indices of graph operations",Journal of Inequalities and Applications ,vol.2013,Article ID 90,2013.

[5] G.H.Fath –Tabar.,"Old and new Zagreb Indices of graphs",MATCH Communications in Mathematical and in Computer Chemistry,vol.65,pp.79-84,2011.

[6] Shirdel GH,Rezapour H,AM sayadi-"The Hyper-Zagreb index of graph operations", Iranian Journal of Mathematical Chemistry,4(2),pp.213-220,2013.

[7] Ghorbani.M,Azimi A-"A note on multiplicative Zagreb indices" , Iranian Journal of Mathematical Chemistry,3(2),pp.137-143,2012.

[8] Das.K.C.,Gutman.I, "Some properties of the second Zagreb Index", MATCH Commun.Math.Comput.Chem.52, pp.103-112,2004.

[9] A.R.Ashrafi,T.Doslic,A.Hamzeh,-"The Zagreb coindices of graph operations",Discrete Appl,Math.158,1571-1578,2010.

[10] Zhou.B.-Zagreb Indices , MATCH Commun.Math.Comput.Chem.52,pp.113-118,2004.

[11] M.Siva Parvathi,D.Sujatha and T.Sukeerthi,"A Stastical Comparision between Zagreb Indices for correlation with toxicity predictions of natural products ",International Journal of Research in Pharmaceutical Sciences ,vol.13.No.1,2022.

[12] Ivy Chakrabarthy,Joseph Varghese Kureethara and MuktiAcharya,A Study of an Undirected graph on a finite subset of Natural numbers,South East Asian J.of Mathematics and Mathematical Sciences,Vol 18,No.3(2022),433-448.

[13] I.Chakrabarty , J. V. Kureethara , M. Acharya, the $G_{m,n}{}^{M}$ graph on a finite subset of natural numbers, proceedings of IAM, V.10, N.1, 2021, pp.45-59.

[14] L.Eswaramma and D.Venkata Lakshmi,"Energy and Spectrum of $G_{m,n}{}^{M}$graph",Eur.Chem.Bull.12,issue 2,pp.321-329,2023.

[15] M.Venkata Anusha,M.Siva Parvathi and G.S.Shanmuga Priya,"Zagreb Indices on Arithmetic graphs",Zeichen Journal,Vol 7,issue 1,pp.144-151,2021