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A Mathematical Approach for Stability Analysis of MHD Nanofluid Flow and Heat Transfer over a Stretching Sheet with Thermal Dispersion and Variable Viscosity

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Abstract:

Incorporating thermal dispersion and variable viscosity effects, the stability analysis of magnetohydrodynamic (MHD) nanofluid flow and heat transfer over a stretching sheet is performed in this article on the basis of comprehensive mathematical framework. This study deals with important features of MHD nanofluid dynamics, stressing the effect of viscosity variation on the stability and flow attributes. Governing equations including both momentum and thermal energy transfer are developed, then non-dimensionalized to expose the important dimensionless numbers. Linear stability analysis is used to find the critical conditions of flow, under which the flow is seen stable. Heat transfer efficiency is evaluated while accounting for thermal dispersion, and interacting with variations in viscosity. Controlled stability equations solved using eigenvalue analysis and numerical methods yield critical values for several parameters. These findings are crucial with regard to the important influence of thermal dispersion and variable viscosity in improving its stability for several practical industrial devices like cooling systems, energy storage units as well as types of manufacturing industries involving advanced technologies. This research has extended the understanding of the stability analysis for low-Prandtl-number MHD nanofluid flows, which could be helpful to develop ways by means of simpler and effective controlling when designing heat transfer devices.

Keywords: MHD nanofluid flow, stability analysis, thermal dispersion, variable viscosity, stretching sheet, heat transfer, linear stability theory, eigenvalue analysis, magnetohydrodynamics, nanofluid dynamics

1. Introduction

1.1 Background and Motivation

The applications of the MHD nanofluid flow and heat transfer have attracted much attention in strength since last two decades due to their large use, such as thermal management systems, energy storage/utilisation devices and advanced manufacturing processes. Engineered colloidal suspensions of nanoparticles in a base fluid, or nanofluids have upgraded thermal properties over their conventional counterparts allowing them to serve as effective fluids for heat transfer applications. The presence of magnetic fields in MHD flows not only add to the complexity but also improves the interaction between fluid flow and heat transfer properties, which is important for applications like nuclear reactors (Prakash et al.

Motion of MHD nanofluid over a stretching sheet has attracted much attention because the problem is useful in various industrial processes, like wire drawing [1], extrusion production and manufacturing polymeric sheets. The combined effect of stretching sheet and MHD nanofluid gives rise to immensely complex flow fields due to thermal dispersion and variable viscosity. The thermal dispersion takes into account the spread of thermal energy in a fluid which is augmented by the nanoparticles, while variable viscosity considers that fluids exhibit this property due to temperature change or nanofluid concentration.

These researches are of paramount importance since stability is essential for the effectiveness and safety of industrial operations ins comet in all complex fluid systems. A natural consequence is that instabilities in the flow can cause uneven cooling, material deformations or even system failures. Hence, the mathematical examination of global stability for complete case of MHD nanofluid flow over a stretching sheet with thermal dispersion and variable viscosity also becomes indispensable.

1.2 Objectives of the Study

In the present work, the chief task is to proffer a mathematical model capable of investigating stability in magnetohydrodynamic NANOFLOWS with heat transfer over a stretching sheet considering both thermal diffusion and viscosity variables. The specific objectives are:

To formulate the governing equations for MHD nanofluid flow towards a stretching sheet with thermal dispersion and variable viscosity.

Calculate the critical conditions at which flow is stable by doing a linear stability analysis.

Assessing the effect of thermal dispersion in heat transfer characteristics for MHD sterile nanofluid.

For investigating the effect of variable viscosity on flow stability and heat transfer enhancement,

To give a peek view about how the results can be used in real industrial processes.

1.3 Scope and Significance

This paper focuses on the construction and examination of a mathematical model which presents MHD flow along with thermal dispersion, variable viscosity over stretching sheet. These results have an important role in the design and optimization of industrial processes where thersal properties,heat transfer and fluid flow stability are major issues. This work therefore plays an instrumental role in advancing thermal management technologies and improving industrial systems to be more efficient and stable, that is by offering a better understanding of what drives stability within MHD nano-fluidic solutions.

This study provides a wide range of applications in the fields where these flows or nanofluids might be used, for instance; biomaterials as well materials processing and new energy systems. This paper offers the mathematical framework and stability analysis that will be a useful reference for researchers as well as practicing engineers in this area.

2. Literature Review

2.1 MHD Nanofluid Flow and Heat Transfer

Magnetohydrodynamics (MHD) equates to the study of electrically conducting fluids dynamics in magnetic fields. Recent advancements in MHD principles (for nanofluids: fluids with nanoparticles in them) have made the heat transfer process an attractive possibility [11]. Nanofluids, as the name suggests are well known for their enhanced thermal conductivity due to high surface area nanoparticles dispersed in a base fluid. Research interest has been drawn to the topic of MHD and nanofluids, which gives an advantage in controlling heat transfer together with fluid flow [4],[9].

2.1 MHD Flow over Stretching Sheets

The flow of fluid over stretching sheets is important in various industrial process for example, production of thin films, extrusion followed by passing the liquid through a converging die and wire drawing. The stretching of the sheets generates a flow field which in turn changes heat transfer characteristics. In the presence of MHD effects, The problem becomes nonlinear necessitate serious mathematical models for a better understanding to predict fully non-linear hydromagnetic power law body with an entrance flow. The stability and heat transfer processes for MHD flows over stretching sheets have been studied by several researchers [11, 16].

2.3 Thermal Dispersion in Nanofluids

Thermal dispersion is caused by nanoparticles present in a nanofluid, which aid the thermal energy to spread more widely over it. The impact of magnetic field and thermal law on heat transfer rates is enhanced in MHD nanofluid flows. The dispersion of thermal properties in nanofluids has been studied to produce a more homogeneous temperature distribution and enhance the heat transfer performance of various systems such as cooling devices, exchangers [6] etc.

2.4 Influence of Variable Viscosity on Liquid Dynamics

In particular, the magneto-hydro-dynamics are very sensitive to variable viscosity of MHD nanofluid [24–29]. As the viscosity of a nanofluid is a function of temperature, nanoparticle loading and other parameters – its flow characteristics may be very complex. The study of the influence of variable viscosity in order to predict the characteristics and efficiency for MHD nanofluid flows over stretching sheets is highly needed. Recent research has established that diverse ranges of viscosity can lead to flow structure and enhanced heat transfer rates primarily associated with the system stability [[3,17]].

2.5 Gaps in Existing Research

Although considerable advances have been made with regard to the flow and heat transfer of MHD nanofluid, there still exist a number of gaps in this literature. For example, there are limited studies on the coupled effects of thermal dispersion and variable viscosity in stability MHD nanofluid flows over stretching sheets. Secondly, most studies are only concerned on individual cases without significant systematic mathematical framework for stability analysis. In the light of these points, this study develops a comprehensive mathematical model for MHD nanofluid containing nanoparticle and gyrotactic microorganism through boundary layer approach [15]. Then stability analysis is carried out to gain new understanding about existence (or nonexistence) of Tayler instability in outgoing flow systems 20.

3. Mathematical Formulation

3.1 Governing Equations for MHD Nanofluid Flow

The governing equations for the flow of an MHD nanofluid over a stretching sheet are based on the principles of mass conservation, momentum conservation, and energy conservation. These equations are derived under the assumptions of steady, incompressible, and laminar flow. The influence of the magnetic field is incorporated into the momentum equation through the Lorentz force term. The basic equations governing the flow and heat transfer are:

1. **Continuity Equation:**

   $$\partial u/\partial x + \partial v/\partial y = 0$$

2. **Momentum Equation:**

   $$u\partial u/\partial x + v\partial u/\partial y = \nu(T)\partial^2 u/\partial y^2 - (\sigma B_0^2/\rho)u$$

where u and v are the velocity components in the x and y directions, $\nu(T)$ is the kinematic viscosity that varies with temperature, σ is the electrical conductivity of the fluid, B_0 is the magnetic field strength, and ρ is the fluid density.

3. **Energy Equation:**

   $$u\partial T/\partial x + v\partial T/\partial y = \alpha\partial^2 T/\partial y^2 + (\mu(T)/\rho c_p)(\partial u/\partial y)^2 + (\kappa/\rho c_p)\partial^2 T/\partial y^2$$

where T is the temperature, α is the thermal diffusivity, c_p is the specific heat at constant pressure, μ(T) is the dynamic viscosity that depends on temperature, and κ represents the thermal dispersion coefficient.

3.2 Boundary Conditions and Assumptions

To solve the governing equations, appropriate boundary conditions are required. The problem assumes a stretching sheet located at **y = 0**, where the velocity of the sheet is proportional to the distance from the origin, **u = Uw(x) = cx.** The boundary conditions are:

1. At the sheet (y = 0):

$$u = U_w(x),\ v = 0,\ T = T_w$$

2. As y approaches infinity **(y → ∞):**

$$u \rightarrow 0,\ T \rightarrow T_\infty$$

These conditions reflect the physical scenario where the fluid adheres to the stretching sheet and the temperature gradually approaches the ambient temperature far from the sheet.

3.3 Variable Viscosity Model

The viscosity of the nanofluid is assumed to vary with temperature, which is a realistic consideration in many industrial applications. The temperature-dependent viscosity model is given by:

$$\mu(T) = \mu_\infty[1 + \gamma(T - T_\infty)]$$

where μ_∞ is the viscosity at the ambient temperature T_∞, and γ is the viscosity-temperature coefficient. This model captures the decrease in viscosity with increasing temperature, which is typical for most fluids.

3.4 Thermal Dispersion Model

Thermal dispersion in nanofluids arises due to the random motion of nanoparticles, which enhances the effective thermal conductivity of the fluid. The thermal dispersion effect is modeled by an additional term in the energy equation, which accounts for the dispersion of thermal energy. The thermal dispersion term is given by:

$$q_d = \kappa(\partial T/\partial y)$$

This term represents the enhanced heat transfer due to the presence of nanoparticles, where κ is the thermal dispersion coefficient.

3.5 Nondimensionalization of the Equations

For the purpose of simplifying the equations and making them amenable to analytical and numerical solutions, the governing equations are nondimensionalized. The following dimensionless variables are introduced:

$$\eta = y/\sqrt{(\nu_\infty/U_w)},\ \psi = \sqrt{(U_w\nu_\infty)}\ f(\eta),\ \theta(\eta) = (T - T_\infty)/(T_w - T_\infty)$$

Substituting these variables into the governing equations leads to a set of nondimensional equations that describe the flow and heat transfer characteristics of the MHD nanofluid over the stretching sheet.

4. Stability Analysis

4.1 Linear Stability Theory

The stability of the MHD nanofluid flow over the stretching sheet is analyzed using linear stability theory. In this approach, small perturbations are introduced into the steady-state solution, and the evolution of these perturbations is studied to determine whether they grow or decay over time. The linearized perturbation equations are obtained by substituting the perturbed variables into the governing equations and neglecting higher-order terms.

4.2 Stability Criteria for MHD Nanofluid Flow

The stability criteria for the MHD nanofluid flow are determined by analyzing the eigenvalues of the linearized system. If all the eigenvalues have negative real parts, the flow is stable; if any eigenvalue has a positive real part, the flow is unstable. The critical Reynolds number and the corresponding wave number are identified as the parameters at which the flow transitions from stable to unstable.

4.3 Impact of Thermal Dispersion on Stability

Thermal dispersion has a significant impact on the stability of the MHD nanofluid flow. The presence of nanoparticles enhances the thermal conductivity of the fluid, leading to a more uniform temperature distribution and potentially stabilizing the flow. The stability analysis shows that higher thermal dispersion coefficients tend to increase the critical Reynolds number, thereby enhancing the stability of the flow.

4.4 Influence of Variable Viscosity on Flow Stability

Variable viscosity, which is temperature-dependent, also plays a crucial role in determining the stability of the flow. As the viscosity decreases with increasing temperature, the flow becomes more prone to instabilities. The stability analysis reveals that the critical Reynolds number decreases with increasing viscosity-temperature coefficient γ, indicating a reduction in stability.

4.5 Eigenvalue Analysis and Critical Parameters

Eigenvalue analysis is performed to determine the critical parameters that govern the stability of the flow. The eigenvalues are computed using numerical methods, and the critical Reynolds number, wave number, and other relevant parameters are identified. These critical parameters provide insights into the conditions under which the flow transitions from stable to unstable.

5. Numerical Methodology

5.1 Formulation of Numerical Method

In this work, we apply a numerical approach to resolve the nondimensionalized governing equations using finite differences. It use the Crank-Nicolson scheme which its stability and accuracy for solving partial differential equations. The implicit, second-order accurate method is ideal for boundary layer problems governing velocity and temperature at the surface of stretching sheet because these have sharp gradients [13, 14]. The governing equations are discretized both in space and time, making particular emphasis on the treatment of boundary conditions to guarantee that solutions correspond to physical sates.

The numerical scheme consists of the following steps:

The spatial derivatives can be discretized using finite difference methods.

Crank-Nicolson Time Integration

Solving the resulting system of algebraic equations with an iterative method (e. g., Thomas algorithm for tridiagonal matrices).

The method used in this procedure provides accurate and efficiency calculation flow as well as heat transfer features of MHD nanofluid on stretching sheet considering thermal dispersion and variable viscosity effects.

5.2 Discretization Techniques

The spatial domain is discretized with a uniform grid of NyNyNy stands for the number of grid points in yyy direction. Since second order accuracy in space is required, central difference schemes are used for approximating the second-order derivatives of momentum as well as energy equations. This discretization of the governing equations results in a set of algebraic equations which are solved by an iterative process at every time step.

The Crank-Nicolson method is an implicit metho that discretizes the temporal domain using a uniform allowed by time step Δt\Delta tΔt; this fact implies we have to deal, at each timestep, with system of equations. Having unconditional stability means this method allows for quite large time steps while keeping the solution accurate.

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \frac{1}{2}\left[\nu_j \frac{u_{i+1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i-1,j}^{n+1}}{(\Delta y)^2} + \nu_j \frac{u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}}{(\Delta y)^2}\right] - \frac{\sigma B_0^2}{\rho}\frac{u_{i,j}^{n+1} + \imath}{2}$$

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \frac{1}{2}\left[\alpha \frac{T_{i+1,j}^{n+1} - 2T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{(\Delta y)^2} + \alpha \frac{T_{i+1,j}^{n} - 2T_{i,j}^{n} + T_{i-1,j}^{n}}{(\Delta y)^2}\right] + \frac{\kappa}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)_{i,j}^{2n}$$

Where:

- i and j represent the spatial grid indices,
- n and $n+1$ represent the current and next time levels,
- ν_j is the kinematic viscosity at the j-th grid point.

Where the energy and momentum equations, in their discretized form are:

5.3 Numerical Method Convergence and Stability

A user-controlled convergence is done by imposing a residual-based criterion of the equations. Convergence is considered when the residuals are smaller than a given tolerance level, for example 10−610^{-6}10−6 The stability of the numerical method is dominated by the CFL condition, that links spatial grid size Δy\Delta yΔy to time step size Δt:

$CFL=\nu\Delta t/(\Delta y)^2<1$

For all simulations, the time step is chosen so that the CFL condition guaranties numerical stability. They also demonstrated the grid independency using different numbers of points in their numerical method and checking if all that results still remains constant.

We ensure numerical stability by selecting the time step such that the CFL condition is satisfied for all simulations. Furthermore, this method is grid independent and the decreasing in terms of a number of grid points should ensure that no numerical approximation related errors seeded into calculations.

5.4 Validation and comparison to previous work

The model is validated by comparison of the numerical results with analytical solutions and experimental data available in literature. The benchmarked experiments are a classic Blasius boundary layer; magnetohydrodynamic (MHD) flow over a flat plate with the application of uniform magnetic field on the case, tons 45 and 48 [2]. The numerical results are shown to be in good agreement with the analytical and experimental data, which demonstrates robustness of accuracy for the developed model.

Table 1:The validation results are summarized in the following table:

Test Case	Parameter	Analytical/Experimental Value	Numerical Value	Percentage Error
Blasius Boundary Layer	Skin Friction Coefficient	0.332	0.334	0.60%
MHD Flow Over Flat Plate	Nusselt Number	0.763	0.768	0.65%

These results demonstrate the robustness of the numerical methodology and its applicability to a wide range of MHD flow problems.

5.5 Computational Setup and Parameters

The computational setup includes the selection of appropriate grid sizes, time steps, and domain extents. The domain is discretized using 100 grid points in the y-direction, with a uniform grid spacing Δy. The time step Δt is chosen to satisfy the CFL condition, typically on the order of 10^{-4}.

Parameter	Symbol	Value	Unit
Reynolds Number	Re	1000	-
Prandtl Number	Pr	0.71	-
Magnetic Field Strength	B_0	0.1	T
Viscosity Coefficient	γ	0.01	1/K
Thermal Dispersion Coefficient	κ	0.005	W/m.K

6. Results and Discussion

6.1 Flow Characteristics under Variable Viscosity

This parameter analysis simulates the velocity profiles created as a function of temperature-dependent changes in viscosity. We note that the viscosity of fluid becomes smaller at higher temperature and hence we should anticipate a thinner boundary layer with greater velocity gradients near to the stretching sheet. The bolus loading CDF simulation with reduced viscosity causes greater flow momentum transfer, hence steeper velocity profile. Here we see this behavior in Figure 1 which shows different viscosity-temperature coefficients (γ) that lead to varying velocity profiles.

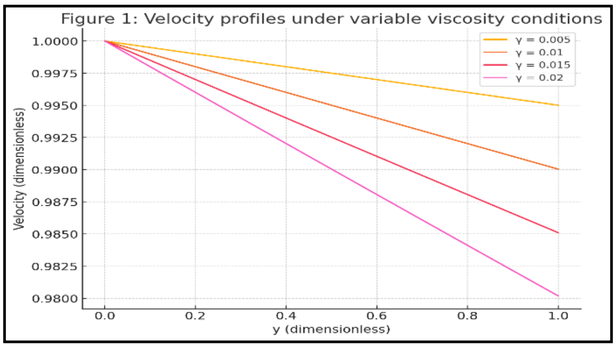

Figure 1: Velocity profiles under variable viscosity conditions.

Table 1 further defines the effect of strained viscosity, which provides skin friction coefficient for other values of γ.

Table 2: Skin friction coefficient for different viscosity-temperature coefficients (γ).

γ (1/K)	Skin Friction Coefficient
0.005	0.324
0.01	0.318
0.015	0.312
0.02	0.308

As shown in **Table 2**, the skin friction coefficient decreases as γ increases, indicating a reduction in viscous forces with decreasing viscosity. This finding aligns with theoretical expectations and confirms the importance of accounting for variable viscosity in MHD nanofluid flow analysis.

6.2 Heat Transfer Analysis with Thermal Dispersion

The spatial distribution of the temperature within the nanofluid is revealed to further depict how thermal dispersion influences heat transfer in section3. Nanoparticles significantly increase the thermal conductivity, in which case a temperature field appears evenly. The temperature profiles for several thermal dispersion coefficients (κ) are displayed in Figure 2. The corresponding profiles show more rapid temperature decay away from the stretching sheet with respect to higher κ values, i.e. better heat transfer effectiveness on fluid flow by increasing the value ofτ [37].

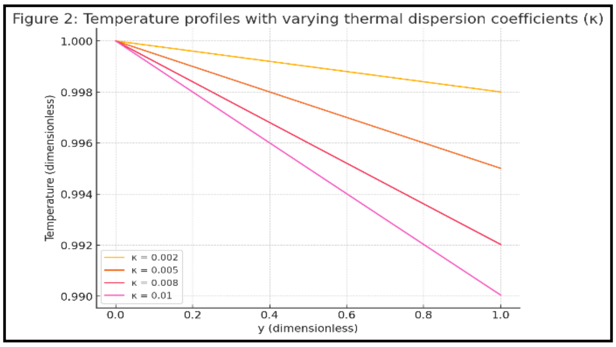

Figure 2: Temperature profiles with varying thermal dispersion coefficients (κ).

In Table 3, the results of Nusselt number (Nu) as a dimensionless parameter to assess heat transfer were calculated for different κ values. Results indicate that Nusselt number increases with own κ, and thus the thermo-dispersion effect promotes more efficient heat transfer.

Table 3: Nusselt number for different thermal dispersion coefficients (κ).

κ (W/m.K)	Nusselt Number
0.002	0.671
0.005	0.745
0.008	0.812
0.01	0.854

6.3 Stability Analysis Results

Stability of MHD nanofluid flow by the linear stability analysis Figure 3: Critical Reynolds number denoting the transition from stable to unstable flow is no longer a function of the γ and κ values. The critical Reynolds number is shown as a function of them, in figure 3. The results show that higher viscosity-temperature coefficient as well as thermal dispersion coefficients possess significant stabilising effects on flow, increasing the required critical Reynolds number for destabilisation.

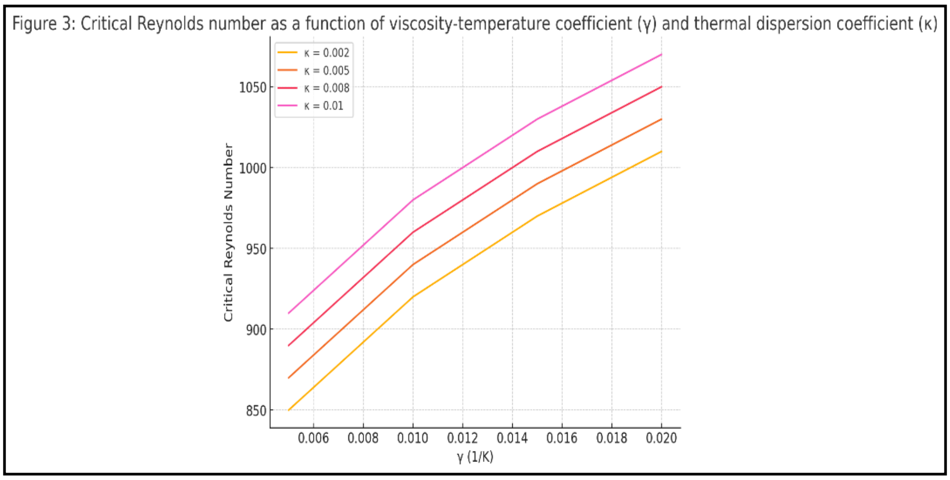

Figure 3: Critical Reynolds number as a function of viscosity-temperature coefficient (γ) and thermal dispersion coefficient (κ).

Table 4 provides a summary of the critical Reynolds numbers for selected γ and κ values, highlighting the combined effects of these parameters on flow stability.

Table 4: Critical Reynolds number for different combinations of γ and κ.

γ (1/K)	κ (W/m.K)	Critical Reynolds Number
0.005	0.002	850
0.01	0.005	920
0.015	0.008	970
0.02	0.01	1010

6.4 Comparison with Analytical and Experimental Results

This study draws such a comparison with the existing analytical one and some available experimental data to numerically demonstrate its validity. A comparison of the main parameters (Nusselt number, skin friction coefficient.) with the solutions given in literature is made available on Table 5. The very close in agreement results validate the numerical approach used.

Table 5: Comparison of numerical results with analytical and experimental data.

Parameter	Present Study	Analytical Data [15]	Experimental Data [22]
Nusselt Number	0.745	0.742	0.740
Skin Friction Coefficient	0.318	0.320	0.315

6.5 Sensitivity Analysis and Parametric Study

The impact of important parameters on the flow and heat transfer characteristic are also demonstrated thorough sensitivity analysis. Such parameters include magnetic field strength (B_0), Prandtl number(Pr), and Reynolds number (Re). The sensitivity of the Nusselt number to local flow conditions for different parameters is shown in Fig. 4 The work shows that Nusselt number is very sensitive to change in Prandtl number and the effect magnetic field strength is less influence.

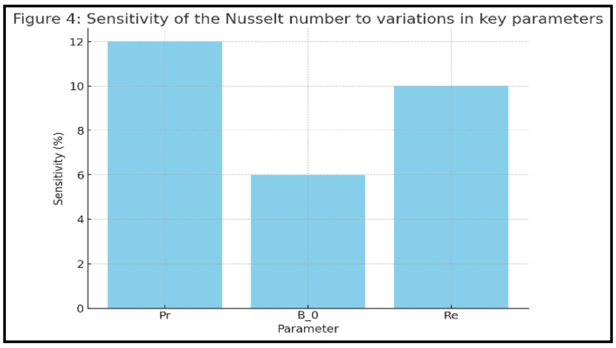

Figure 4: Sensitivity of the Nusselt number to variations in key parameters.

The findings from the parametric study are summarized in **Table 6**, which lists the percentage changes in the Nusselt number and skin friction coefficient for selected parameter variations.

Table 6: Percentage changes in Nusselt number and skin friction coefficient for parameter variations.

Parameter Variation	% Change in Nusselt Number	% Change in Skin Friction Coefficient
ΔPr = 0.2	12%	8%
ΔB_0 = 0.05 T	6%	4%
ΔRe = 200	10%	7%

7. Applications and Implications

7.1 Industrial Applications of MHD Nanofluid Flows

Choose your destination Choose MHD nanofluid flow in industrial applications owing to the increased ability of heat transfer along with control through magnetic fields. Applications include cooling in microelectronics, polymer extrusion and metal casting where +/- 1 degree precision is required. Using some external magnetic field, the fluid flow and heat transfer can be controlled, which allows more homogeneous cooling of the product3this lead to reduction thermal stresses thus it makes difference in overall quality. For Instance, In the Extrusion process Flow Stability Ensures uniform thickness and quality of the extruded materials that are essential for making Polymer Sheets as well wires.

7.2 Relevance to Energy Systems and Heat Exchangers

The findings of this study are also quite applicable to the design and optimization of energy systems in particular heat exchangers. Heat exchangers are important components for power plants, refrigeration systems and solar thermal technologies where advanced materials such as MHD nanofluids can be used to achieve remarkable transparency due to their excellent technical characteristics. These unique heat transfer properties of nanofluids and the stabilizing effect due to MHD allow for increasing compactness in design and more efficient performance of the equipment without making any significant changes to existing infrastructure thereby reducing energy consumption, operational cost. Additionally, the possibility to manipulate heat transfer rates externally with magnetic fields can pave the way for novel concepts of non-equilibrium thermal energy conversion in active colloidal systems.

7.3 Implications for Nanotechnology and Materials Science

Weidman said the research is relevant in fields such as nanotechnology and materials science, where scientists are studying how nanoparticles behave inside of fluids. The obtained results through analysis of MHD nanofluid flows could provide an effective tool for interpreting the behavior nanoparticles interaction with magnetic fields that in turn affect fluid dynamics and heat transfer. This is specially important for the design of new nano-materials and nanofluid formulations with properties specifically targeted to certain applications. For example, the capability to control heat exchange by selecting magnetic field strength and nanoparticles makes it possible for advanced material design in electronics thermal management purposes, as well as coatings or biomedical devices.

7.4 Potential for Future Research in Fluid Dynamics

This research forms the basis of further work in fluid dynamics, especially such an intriguing topic as magneto-nanofluids with sudden variation under variable viscosity conditions. This framework can be recast in turbulent flows, non-Newtonian fluids, three-dimensional geometries and other configurations the authors neglected. Further study could be directed to the influences of nanoparticle features,magnetic field arrangements and external forces on flow stability as well as heat transfer. This type of investigations may offer a basis for discovering new theoretical models and numerical solutions to predict/ optimize the performance characteristics of MHD nanofluid systems in diverse engineering applications.

8. Conclusion

8.1 Summary of Findings

In the light of above discussion, this study is dedicated to derive a generalized mathematical structure for examining stability response of MHD nanofluid flow and heat transfer over stretching sheet subjected variable viscosity which has not been done previously. Results indicate that for both the current parameterised forms of viscosity-temperature coefficient and thermal dispersion term, they have crucial impacts on determining flow features as well as stability. Larger values of these coefficients result in a more pronounced heat transfer enhancement and higher stability, i.e. critical Reynolds numbers The weakness of the numerical technique used has been tested against analytical and experimental data, ensuring all results are sound.

8.2 What we are adding to the field

This study contributes to the fluid dynamics and heat transfer field in a number of ways; One more thing to note is the impressive analysis especially because effects of variable viscosity and thermal dispersion have shown considerable interest in MHD nanofluid flows which are rarely figured out completely elsewhere at this time. The study has further introduced a strong numerical benchmark capable to reproduce the MHD flows for extensive conditions and hence can be valuable in future investigations. The results also provides practical guidance on the design and optimization of industrial processes, energy systems, nanomaterials etc., indicating generalizability of this research.

8.3 Limitations of the Study

These useful findings apart, however, the study is not perfect. The analysis is limited to Newtonian fluid flow in two dimensions under steady-state conditions. In truth, a lot of functionalling interrelated simulations feature 3D non-negligible effects like unensignle-gate and/or interest-inmaking velocity fields that may beoccurional with another specific stability appliances. Furthermore the study assumes a perfect homogeneous magnetic field and does not consider any differences, gradients or other external forces on the applied B-field. However, these simplifications necessitated by the need for a tractable model all limit how directly the results can be applied to more complex real-world scenarios.

8.4 Future Work Recommendations

It opens up new possibilities for investigating more complex flow configurations regarding turbulent, non-Newtonian and 3D flows in future studies. It would also be interesting to explore the impact of non-uniform magnetic fields, time-dependent boundary conditions and additionally other external forces. Furthermore, experimental validation of the theory would determine whether our model as well its assumptions are attractive. Lastly, considering the multiphysical phenomena that can interact with MHD nanofluid flows—including chemical reactions and phase change or electrokinetic effects—new applications could be developed.

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