Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 8s (2024)

Applications of Chromatic Numbers in Traffic Control Problems

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Article History:

Received: *26-04-2024*

Revised: *15-06-2024*

Accepted: *29-06-2024*

Abstract:

The primary objective of this work is to illustrate the application of chromatic numbers in traffic control problems. It is simple to depict most traffic lanes as graphs with points (vertices) and lines (edges). In order to determine the minimum number of phases needed for the traffic light to allow all cars to pass through the intersection during a specific phase of the light and those in lanes where the light is green to pass through the intersection safely, this article uses three examples of traffic lanes to demonstrate how chromatic number can be applied effectively to traffic management problems.

Keywords: Graph theory, Chromatic Number, Traffic Control Problem, Modelling.

1. Introduction

These days, graph theory is widely applied in all areas of research and technology. Numerous other fields, such as biology, sociology, computer science (computers and algorithms), operations research (scheduling), and biochemistry (genomics), also frequently use it. This is due to its widespread usage in electrical engineering as well. In chemistry, it encompasses the study of atoms, molecules, and bond formation. Resource allocation and scheduling are the two most important uses of graph colouring; a lengthy list of other uses is given in [5 to 9]. Graph theory's paths, walks, and circuits are also used in a wide range of applications, such as resource networking, database design concepts, and the travelling salesman problem. New techniques and theorems with a wide range of applications are therefore produced [1]. Graph theoretic methods such as edge and vertex connectivity are used to study problems related to traffic control at crossings.

There are more cars on the road, which increases participant time losses, noise and environmental pollution, and the frequency of traffic accidents. This is a result of significant population expansion and the urbanisation of small towns and cities.

Traffic congestion is a major hindrance to the growth of several urban regions, impacting millions of people.

Although constructing more roads could improve the problem, given the current infrastructure, doing so would be extremely costly and frequently unfeasible. Enhancing the utilization of the current road network is the sole solution for traffic flow management in this kind of situation.

In order to minimize waiting times for traffic participants, this study addresses the application of chromatic numbers in traffic control problems at junctions.

2. Preliminaries:

Modelling the traffic lanes at a junction with a graph G. The graph G is built with a vertex set of lanes where two vertices (lanes) are connected by an edge if two lanes cannot safely access the junction at the same time because of the risk of an accident.

Definition 2.1: Chromatic Number:

The minimum number of colors required to color the vertices of a graph G such that no two adjacent vertices share the same color is referred to as the graph's chromatic number, or $\chi(G)$.

3. Problem statement:

At a junction of two streets where traffic is frequently high. Vehicles access the junction of these two streets via a number of traffic lanes. At this crossroads is a traffic signal. Cars in lanes with a green light can cross the junction safely during a specific period of this traffic signal.

a)Using a graph to represent this circumstance

b) To figure out how many phases are needed for all lanes of traffic to pass through the junction safely.

The situation can be explained through three examples:

Example 1: An example of a traffic control issue at a four-leg junction with four lanes is taken into consideration. [4]

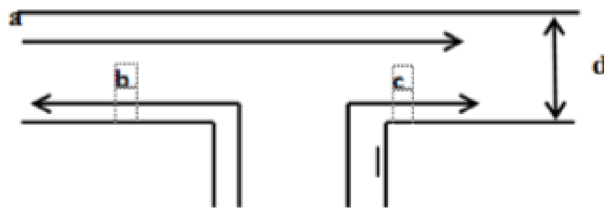

Figure 1: An Intersection with four Traffic Lanes

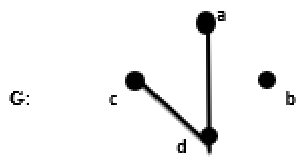

Figure 2: Modelling traffic lanes at an intersection by means of a graph G

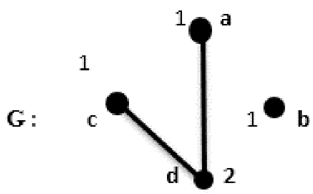

Figure 3: The Chromatic number of G is 2

a)Let G be a graph with the vertex set V={a,b,c,d}. If two vertices (lanes) in G are connected by an edge, then it is unsafe for the cars in these two lanes to reach the junction simultaneously because there is a risk of an accident (Fig. 2).

b) By figuring out graph G's chromatic number in Figure 3, two colours are required to colour these vertices. Hence, $\chi(G)=2$. Given that G may be properly coloured using the two colours 1, and 2, as seen in Fig. 3. Vehicles in lanes of the same colour may cross the junction simultaneously once the traffic light becomes green for that phase, meaning that the traffic signal has a minimum of two phases.

Example 2: A traffic control issue with seven lanes is another example taken into consideration here; it is represented in Figure 4 below[4].

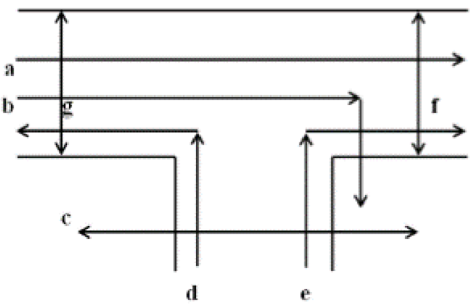

Figure 4: An Intersection with Seven Traffic Lanes

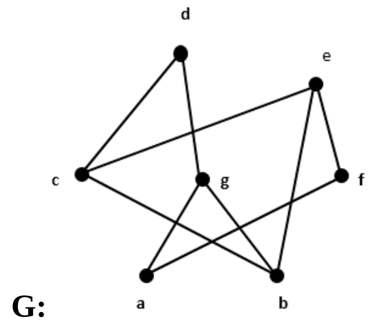

Figure 5: Modelling traffic lanes at an intersection by means of a graph G

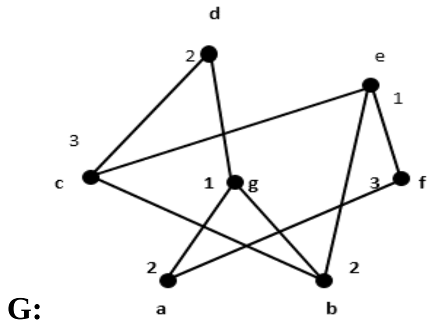

Figure 6: The Chromatic number of G is 3

a)Let G be a graph whose vertex set V={a,b,c,d, e,f,g}. If two vertices (lanes) in G are connected by an edge, then it is unsafe for the cars in these two lanes to cross at the same time because there might be an accident (Fig. 5).

b) By figuring out graph G's chromatic number in Figure 6, three colours are required to colour these vertices. Hence, $\chi(G)=3$. Given that G may be properly coloured using the three colours 1, 2, and 3 as seen in Fig. 6. Vehicles in lanes of the same colour may cross the junction simultaneously once the traffic light becomes green for that phase, meaning that the traffic signal has a minimum of three phases.

Example 3:

A traffic control problem at a seven-leg junction is the case under consideration. The seven lanes in Fig. 7 below are derived from [3], and the related graph is produced as indicated in **Fig. 8**

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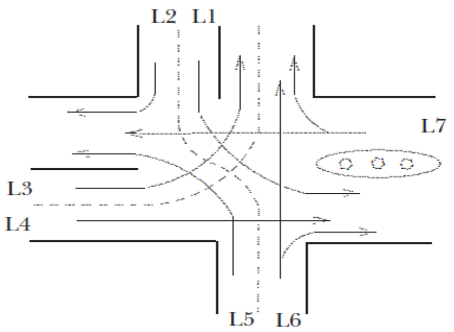

Fig 7: An intersection with seven traffic lanes

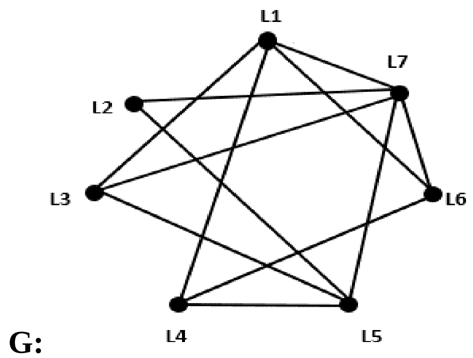

Fig 8: Modelling Traffic lanes at an intersection by means of Graph G

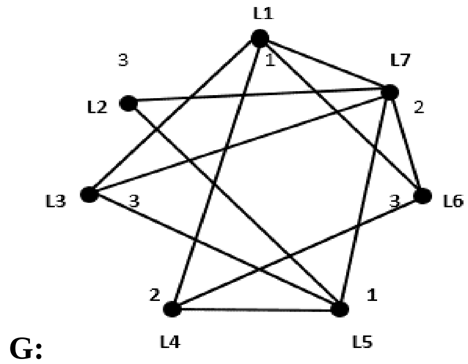

Figure 9: The Chromatic number of G is 3

a)Let G be a graph with the vertex set V={L1, L2, L3, L4, L5, L6, L7}. If two vertices (lanes) in these lanes cannot safely reach the junction at the same time due to the danger of an accident, then these two lanes are related by an edge (Fig. 8).

b) By figuring out graph G's chromatic number in Figure 9, three colours are required to colour these vertices. Hence, $\chi(G)=3$. Given that Fig. 9 illustrates the correct colouring of G with the three colours 1, 2, and 3. Vehicles in lanes of the same colour may cross the junction simultaneously once the traffic light becomes green for that phase, meaning that the traffic signal has a minimum of three phases.

4. Applications:

There are several applications of chromatic numbers in an intersection of traffic control issues, preparing the timetable, scheduling the task, Sudoku and finding minimum slots for an examination, etc.

5. Conclusions:

This work presents a few examples of applications of graph theory. Specifically, the notion of chromatic number may be used in many real-world contexts and utilised as a graph theoretic tool to explore the intersection of traffic management issues. The chromatic number may be used to determine how many phases are required for cars in all lanes to cross the junction safely. This will reduce waiting times for all traffic participants and offer thorough traffic network information. Therefore, graph theory is quite important in present-day today life.

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