Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 8s (2024)

Nano C $_\tau$ & Nano*gα -compactness in Nts

K. Baby[1] , H. Aaminumariyam[2], A. Singaravelan[3]
[1,3]Assistant Professor, [2]Research Scholar
Department of Mathematics, Kongunadu Arts and Science College(Autonomous)
Coimbatore – 641 029, Tamilnadu, India.
[1]Email: babymanoharan31@gmail.com, [2]Email: aaminumariyam3@gmail.com
[3]Email: siveanand@gmail.com

Article History:

Received: *28-04-2024*

Revised: *17-06-2024*

Accepted: *30-06-2024*

Abstract:

The purpose of this paper is to introduce and study the concept of C τ -compactness in Nts and entrenched few of their accompanying features. Ferther We investigate the Nano*gα compact and connectedness in Nts.

Keywords: Nano*gα -closed set, Nano*gα continuous function, Nano C τ -compact, Nano*gα compact, Nano*gα connected.

1 Introduction

Connectedness and disconnectedness in topology is introduced by A.V.Arhangelskii and R.Wiegandt [7].ctness in general is an essential part of the topological space with regard to the property of closed and bounded subsets.The idea of compacttness and connectedness are beneficial for the basis ideas of general topology as well as for advanced branches of mathematics. M. Vigneshwaran and R. Devi [3] introduced the concepts of *gα-closed sets in topological spaces.In 1970,Levine [6] introduced the concept of generalized closed sets as a generalization of closed sets in Topological spaces. Lellis Thivagar [4]and Carmel Richard introduced the concept of Nano topology,which was defined in terms of approximations and boundry region of a universe using a equivalence relation on it.He also introduced nano continuous functions, nano open mappings,nano closed mappings and nano homeomorphisms in Nts.S.Krishnaprakash et.al [8] innovative some concept of nano compact space and nano connected in nano topology.The intension of this paper is to establish the conception of C $_\tau$ - compact set and find few of their features. It also established the conception of N ano -compact and N ano*gα -connected. The current study is about few of associated theorems and results.

2. Preliminaries

In this section, we recall some basic definitions and results in nano topological spaces.

Definition 2.1.[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as in discernibility relation. Then U is divided into equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

- The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = U\{R(X): R(X) \subseteq X, x \in U\}$
- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(X): R(X) \cap X \neq \phi, x \in U\}$.
- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not $-X$ with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2.[4] Let u be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$where $X \subseteq U$. Then it satisfies the following axioms:

1. U and ϕ belongs to $\tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of $\tau_R(X)$.

Definition 2.3.[9] A subset A of $(U, \tau_R(X))$is called Nano*gα-closed set if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Ngα open in $(U, \tau_R(X))$.

Definition 2.4.[10] A function $f: (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is said to be Nano star generalized α continuous (briefly Nano*gα-continuous), if the inverse image of every nano closed set is $(V, \sigma_R(Y))$ is Nano*gα closed set in $(U, \tau_R(X))$.

3 .Nano*gα Compact Space

Definition 3.1. A collection $\{A_i$: i ∈ I } of Nano*gα -open sets in Nts $(U, \tau_R(X))$ is called Nano∗ gα-open cover of a subset A in $(U, \tau_R(X))$ if A $\subseteq \sum_{i \subseteq 1}$(Ai).

Definition 3.2. A subset A of Nts $(U, \tau_R(X))$ is called Nano*gα -ct relative to U if for every Nano∗ gα -open cover of U has finite subcover.

Definition 3.3. A Nts $(U, \tau_R(X))$ is called Nano*gα -ct if every N ano∗ gα-open cover of U has finite subcover.

Definition 3.4. A subset A of a Nts $(U, \tau_R(X))$ is called Nano*gα ct if A is Nano*gα -ct of the subspace of $(U, \tau_R(X))$.

Theorem 3.5. A N ano∗ gα-closed subset of N ano∗ gα-ct space is Nano*gα -ct relative to $(U, \tau_R(X))$.

Proof. Let A be a Nano*gα -closed subset of a Nts U .Then U − A is Nano*gα -open in U . Let H = $\{A_i$: i ∈ I } be a Nano*gα -open cover of A by N ano∗ gα-open subsets in U . Then H ∪ {U − A } is a Nano*gα -open cover of U . Since U is Nano*gα -ct, then there exists a finite subcover say $\{A_1, A_2, \ldots A_n\}$ is finite N ano∗ gα-open cover of A . Hence A is Nano*gα -ct relative to U.

Theorem 3.6. Let $f: (U, \tau_R(X)) \to (V, \sigma_R(Y))$ be surjective,N ano∗ gα-continuous function. If U is N ∗ gα-ct, then V is nano ct.

Proof. Let $\{A_i$: i ∈ I} be nano open cover of V . Since f is Nano*gα -continuous function, then { $f^{-1}(A_i)$: i ∈ I } is Nano*gα -open cover of U . Since U is Nano*gα -ct, { $f^{-1}(A_i)$: i ∈ I } contains a finite subcover say { $f^{-1}(A_i)$: i ∈ I }Since f is surjective, then $\{A_1, A_2, \ldots A_n\}$ is finite subcover of $\{A_i$: i ∈ I}, for V . Therefote V is nano ct.

Theorem 3.7. Every Nano*gα -compact space is nano compact.

Proof. Let U be Nano*gα -ct. Let $\{A_j$: j ∈ J} is a Nano*gα -open cover of U . Since every nano open set is Nano*gα -open. Since U is Nano*gα -ct, then Nano*gα -open cover $\{A_j$: j ∈ J} of U has a finite subcover, say $\{A_j$: j ∈ J} for U . Hence U is nano ct.

Theorem 3.8. If a function $f: (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is Nano*gα -irresolute and a subset A of U is Nano*gα ct relative to U , then the image f(A) is Nano*gα -ct relative to V .

Proof. Let $\{A_i$: i ∈ I} be any collection of Nano*gα -open sets in V such that $f(A) = \bigcup_{i \in I}\{f^{-1}(A_i)$. Then A ⊆ $\bigcup_{i \in I}\{f^{-1}(A_i)$. where { $f^{-1}(A_i)$: i ∈ I } is N ano∗ gα-open sets in U .Since A is Nano*gα ct relative to U , there is a finite subcollection $\{A_1, A_2, \ldots A_n\}$ such that $A \subseteq \bigcup_{i \in I}\{f^{-1}(A_i)$.. Therefore $f(A) = \bigcup_{i \in I}\{f^{-1}(A_i)$. Hence $f(A)$ is Nano*gα ct relative to V .

4 .Nano C τ -Compact Space

Definition 4.1. A subset A of a Nts $(U, \tau_R(X))$is called a NanoC τ -set if there are two sets $G, F \in U$such that $G \neq U$ and $A \neq G - F$.

Definition 4.2. A collection R of subset of nano generalized Nts $(U, \tau_R(X))$is said to be a cover of U if the union of the elements R is equal to U . It is called a NanoC τ -cover of U if its elements are NanoC τ -subsets of U . The nano generalized NTS $(U, \tau_R(X))$is called NanoC τ -ct if every NanoC τ -Cover of U has finite subcover.

Definition 4.3. A space $(U, \tau_R(X))$ is called nano T_2 space if for any pair of distinct points α_1, α_2 of U there exists disjoint NanoC τ -set G and H of U containing α_1, α_2respectively.

Theorem 4.4. If $(U, \tau_R(X))$ is finite nano generalized NTS. Then U is NanoC τ -ct.

Proof. Let $\{A_i$: i ∈ U} be a nano cover of U .Let R be a NanoC τ -covering of U . Then the element in U belongs to one of the members of R say $\{A_1, A_2, \ldots A_n\}$ ∈ H . Where every $\{G_i$: i ∈ R}, $G \neq U$, i = 1, 2, ..n. Since each G is NanoC τ -set the collection $\{A_1, A_2, \ldots A_n\}$ is finite subcollection of NanoC τ -set which covers U . Hence U is NanoC τ –ct.

Theorem 4.5. Let A be NanoC τ -ct subsets of nano T_2 space in $(U, \tau_R(X))$ and α ∈ U is not in A , then there is a NanoC τ -set G such that A ⊂ G .

Proof. Let A be NanoC $_\tau$ -ct subsets of nano T_2space in $(U,\tau_R(X))$.Since $(U,\tau_R(X))$is NanoC $_\tau$ -set, for each β ∈ U , there exists NanoC $_\tau$ -set $A_\alpha \in$ α and $A_\beta \in$ β then $A_\alpha \cap A_\beta = A_\psi$are NanoC $_\tau$ -set. The collection $\{A_\beta$: β ∈ U} is NanoC $_\tau$ -covering of U . There exist is a finite subcollection $\{A_{\alpha_1}, A_{\alpha_2}, .. A_{\alpha_n}\} \in A$ is a NanoC$_\tau$ covering of U . Thus $A \subseteq \bigcup_{i=1}^{n} A_{\alpha_i} = \bigcup_{i=1}^{n} A_{\beta_i} - G_{\beta_i} \subset \bigcup_{i=1}^{n} A_{\beta_i}$ Since A_{β_i} is NanoC $_\tau$ -open.

Theorem 4.6. Let $(U,\tau_R(X))$ be strong nano generalized Nts. Then finite union of NanoC $_\tau$ -ct set.

Proof. Assume that G ⊆ U and F ⊆ U are any NanoC $_\tau$ -ct subset of U . Let R be NanoC $_\tau$ a cover of G∪F. Then R will also nano Cτ cover of both G and F. So by hypothesis,there exist a finite subcollection of R of NanoC $_\tau$ -set say $\{G_1, G_2, \ldots G_n\}$ and $\{F_1, F_2, \ldots F_n\}$ covering G and F respectively, Where G = A − B, A ≠ U and A and B are nano open.Clearly the collection $\{G_1, G_2, \ldots G_n\ ,\ F_1, F_2, \ldots F_n\}$ is a finite subcollection of R of NanoC $_\tau$ -sets covering G∪F. By induction, every finite union of NanoC $_\tau$ -compaact sets is NanoC $_\tau$ -compaact.

Theorem 4.7. Let $(U,\tau_R(X))$ strong nano generalized Nts. If R is a collection of all nano open set then the non-empty subset of a NanoC $_\tau$ space is NanoC $_\tau$ -ct.

Proof. A $(U,\tau_R(X))$ is nano generalized NTS and U be nano Cτ -ct space.Let G be non empty nano subset of U . By hypothesis there exist two nano open P and Q, P 6= Q such that G = P − Q. U − G = U − (P − Q) which implies U − G is NanoC $_\tau$ -set. Consider the collection R=Ai : i ∈ U are nano open sets be a NanoC $_\tau$ -cover of G . It is given that U is NanoC $_\tau$ -ct, then there exist a collection R of NanoC$_\tau$ -covering U . Which can be either $\{A_{\alpha_1}, A_{\alpha_2}, .. A_{\alpha_n}\}$ or $\{A_{\alpha_1}, A_{\alpha_2}, .. A_{\alpha_n}$, U − G }. Since $A \subseteq \bigcup_{i=1}^{n} A_{\alpha_i}$ = U and G ⊆ U,G = $\bigcup_{i=1}^{n} A_{\alpha_i}$Then the collection Aαi i=1,2...n of NanoC $_\tau$ -sets is finite subcollection of R covering G . Hence G is NanoC $_\tau$ -compact.

5 Nano*gα -connected

Definition 5.1. A NTS $(U,\tau_R(X))$ is said to be is Nano*gα -connected if U cannot be written as a union of two disjoint nonempty is Nano*gα -open sets.

Definition 5.2. A subset G of a Nts $(U,\tau_R(X))$ is said to be is Nano*gα is said to be Nano*gα -connected set in U if G cannot be expressed as the union of two disjoint nonempty Nano*gα open sets in $(U,\tau_R(X))$.

Theorem 5.3. For a Nts $(U,\tau_R(X))$ the following statements are equivalent.

(i)U is Nano*gα -connected.

(ii) The only subsets of U which are both Nano*gα -open and N ano∗ gα-closed are the empty set ϕ and U .

(iii) Each Nano*gα -continuous function of U into a discrete space V with atleast two points is a constant function.

Proof. (i) ⇒ (ii) Let U be a Nano*gα -connected space. Let A be Nano*gα -open and Nano*gα -closed subset of U . Then U − A is both Nano*gα -open and Nano*gα -closed in U . That implies, U is the union of disjoint Nano*gα -open sets A and U ⊆ A . Since U is Nano*gα -connected either A = ϕ OR U − A = ϕ. This is, A = ϕ or A = U .

(ii) ⇒ (i) Suppose that U = A ∪ B, where A and B are disjoint non empty Nano*gα open subsets of U . Then A and B are proper subsets of U .Since A = U − B, A is Nano*gα -closed subset of U ,Then A is both Nano*gα -open and Nano*gα -closed subset of U .Therefore , A = ϕ and A = U , which is contradiction. Thus U is Nano*gα.

(ii) ⇒ (iii) Let $f:\big(U,\tau_R(X)\big) \to \big(V,\sigma_R(Y)\big)$ be Nano*gα -continuous, where V is discrete space with atleast two points. Then U is covered by Nano*gα -open and Nano*gα closed covering {F −1 (y) : y ∈ V }. By part (i), f(y) = ϕ or U , each y ∈ v. If $f^{-1}(y)$ = ϕ, for all y ∈ V If f fails to be a function. Therefore there exists atleast one point say y_1 ∈ V , such that $f^{-1}(y_1)$ ≠ and hence $f^{-1}(y_1)$ = U , which shows that f is a constant function.

(iii) ⇒ (ii) Let G be both Nano*gα -closed in U . Suppose that G ≠ϕ. Let V be a discreate space with atleast two points, fix y_1 and $y_0$in V and y_0. Define $f:\big(U,\tau_R(X)\big) \to \big(V,\sigma_R(Y)\big)$by f(x) = {$y_0$}, for x ∈ G and f(x) = {y_0 }, for x ∉ G.Let F be a nano open set in V .If F contains alone, then f^{-1} (F) = G .If F contains both y_0 and y_0, then f^{-1} (F) = U. Otherwise f^{-1} (F) = ϕ . In all case f^{-1} (F) is N ano∗ gα-open in U . Therefore f is Nano*gα -continuous function. Then by assumption f is a constant function. Therefore f(X) = y_0 or f(x) = y_1, for all x in U .If f(x) = y_0, for all x in U , then G = U .If F(x) = y_1,for all x in U , then G = ϕ.

Theorem 5.4. If a space U is Nano*gα -connected space, then it is nano connected.

Proof. Let U be a Nano*gα -connected space. Suppose that U is not nano connected then U = A ∪ B, where A and B are disjoint non empty nano open sets in U . Since every nano open set is N ano∗ gα-open, A and B are disjoint non empty. Nano*gα open sets in U . This contradicts the fact that U is Nano*gα -connected. hence U is nano connected.

Example 5.5. Let U = {a, b, c, d} and U /R={a}, {b}, {c, d} and X = {a, d}. Then $\tau_R(X)$={U , ϕ, {a}, {c, d}, {a, c, d}}. Then Nano*gα ={U , ϕ, {a}, {c}, {d}, {a, b}, {a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Here U is nano connected but not Nano*gα -connected because U can be written as union of two disjoint non-empty Nano*gα -open sets {d} ∪ {a, b, c}.

Theorem 5.6. If $f:\big(U,\tau_R(X)\big) \to \big(V,\sigma_R(Y)\big)$is Nano*gα -irresolute surjection and U is Nano*gα -connected, then V is Nano*gα -connected.

Proof. Assume that V is not Nano*gα -connected. Then there disjoint non empty Nano*gαopen sets A and B in V such that V = A ∪ B. Since f is Nano*gα -irresolute, f^{-1} (A) and f^{-1} (B) are v Nano*gα -open sets in U . As f is a surjective function, f^{-1} (A) ≠ ϕ and f^{-1} (B) ≠ ϕ, where U = f^{-1} (V) = f^{-1} (A ∪ B) = f^{-1} (A)$^{-1}$ (B) which is a contradiction. This shows that V is Nano*gα -connected.

Theorem 5.7. If G is a Nano-compact of a Nano*gα -connected space $\big(U,\tau_R(X)\big)$ onto an arbitrary Nts $\big(V,\sigma_R(Y)\big)$, then $\big(V,\sigma_R(Y)\big)$is Nano*gα -connected.

Proof. Let $(V, \sigma_R(Y))$be a Nano*gα -connected. Then there exists a non-empty proper subset G of $(V, \sigma_R(Y))$which is both Nano*gα -open and Nano*gα -closed in $(V, \sigma_R(Y))$Since f is Nano*gα -continuous and onto $(V, \sigma_R(Y))$f1(G) is a non-empty proper subset of $(U, \tau_R(X))$which is both Nano*gα -open and Nano*gα -closed in $(U, \tau_R(X))$and therefore $(U, \tau_R(X))$ is disconnected which is a contradiction. Hence $(V, \sigma_R(Y))$must be connected.

Theorem 5.8. A space U is Nano*gα –disconnected if and only if there exists a non-empty proper subset of U which is both Nano*gα -open and Nano*gα -closed in U .

Proof. Let G be a non-empty proper subset of U which is both Nano*gα –open and Nano*gαclosed. We have to prove that U is Nano*gα -disconnected. Let F = U − G . Then F is a non-empty set and G ∪ F = U and G ∩ F = ϕ.Since G is both Nano*gα -open and N ano∗ gα-closed, F is both Nano*gα -open and Nano*gα -closed. Thus U can be written as the union of two disjoint non-empty Nano*gα -open sets. Hence U is Nano*gα disconnected. Conversely, let U be Nano*gα -disconnected. Then there exist non-empty Nano*gα -open subsets G and F such that X = G ∪ F . Then F = U − G and G = U − F, which are Nano*gα -closed in U . Hence Gand F are both Nano*gα -open and Nano*gα -closed in U.

Theorem 5.9. Let $(U, \tau_R(X))$be a Nts and let A be a subset of U . Then A is nano disconnected if and only if there exist non-empty sets G and F both Nano*gα –open in U such that G ∩ A ≠ ϕ, F ∩ A ≠ ϕ ,A ⊆ G ∪ F and G ∩ F ⊆ U − A .

Proof. A is Nano*gα -disconnected if and only if there exist non empty sets G and F both Nano*gα -open in U such that G ∩ A ≠ϕ , F ∩ A ≠ϕ, (G ∩ A) T (F ∩ A) = A . Now (G ∩ A) ∩ (F ∩ A) = ϕ if and only if (G ∩ A)∩ A) = ϕ if and only if G ∩F ⊆ U − A and (G ∩ A) ∪ (F ∩ A) = A if and only if (G ∪) ∩ A = A if and only if A ⊆ G ∪ F .

References

[1] K.Bhuvaneswari and K.M.Gnanapriya,Nano generalized closed sets in Nts,International Journal of Scientific and Research Publication,4(5)(2014), 1 − 3.
[2] K.Bhuvaneswari and K.M.Gnanapriya,On Nano generalized continuous function in Nts,International Journal of Mathematics and Statistics Invention, 1(1)(2013),31-37.
[3] M.Vigneshwaran and R.Devi,On Gao-kernel in the digital plane,International Journal of Mathematical Archive-3(6), 2012
[4] M.Lellis Thivagar and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, volume 1,August 2013,Pp.31- 37.
[5] K.Bhuvaneswari and A.Ezhilarasi,On Nano Semi-generalized and Nano generalizedsemiclosed sets in Nts,International Journal of Mathemaatics and computeer Applications Research(IJMCAR), vol.4, Issue3,Jun 2014,117-124. 9
[6] N.Levine, Generalized closed sets in topology, Rend.cire.Math.palermo, (1963), 19(2),86-96.
[7] A.V.Arhangelskii and R. wiegandt, connectedness and disconnectedness in topology, Top.App.5(1975).
[8] S.Krishnaprakash, R.Ramesh and R.Suresh, " Nano compactness and nano connectedness in Nts", Internal Journal of pure and applied mathematics, volume 119,N0.13 (2018),107-115.
[9] K.Baby and H.Aaminumariyam "On Nano star generalized alpha closed set and Nano star generalized alpha continuous function in NTS", Indian journal of natural science, vol.15/issue 84/jun/2024.
[10] Qays Hatem Imran,Murtadha M.Abdulkadhim and Mustafa H.Hadi, Nano generalized Alpha closed sets in Nts, Gen.Math.Notes, Vol.34, No.2, June 2016,pp.39-51.
[11] R.X.Shen, A note on generalized connectedness, Acta Math Hungar, 122(3)(2009),231-235.
[12] P.Subbulakshmi and N.R.Santhi Maheswari,"Some new form of nano connectedness and Nano compactness in Nts",International journal of creative research thoughts(IJCRT),ISSN:2320-2882.