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Heat and Mass Transfer in MHD Flow of a Nanofluid through a Porous Medium over a Stretching Sheet with Chemical Reaction

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Abstract:

The theme of the present study is on convective heat and mass transfer in porous media flow through nanofluid over a permeable stretching sheet embedded with various factors i.e. MHD, viscous dissipation, chemical reaction and Soret effects. In this study, the authors analysed two types of nanofluids namely Cu-water and Ag-water. The corresponding governing boundary layer equations were established and converted to a system of ordinary differential equation by employing similarity transmutations, which was subsequently executed using the Keller box process. Selected parameters produce skin friction coefficient, Nusselt number and Sherwood number as well as velocity, temperature and concentration profiles. This lead to the excellent validation of numerical results compared with literature for linearly stretching sheet problems. The influences of inclined magnetic field on unsteady MHD flow of Casson nanofluid past a stretching sheet in porous medium with heat source/sink and thermal radiation effects were analyzed [43]. The numerical solutions, which were performed through Runge-Kutta method with shooting technique, have shown that it is possible to control the appearance of inverted boundary layer by proper manipulation on shearing effect magnitude between plate and free stream. The results, as depicted in Table 2 and Figure 5 demonstrate that thermal diffusion is promoted by lower flow rates due to both higher inclination angle, as well as those subjected to more intense radiation. These results hint at possible uses in the thermal management of biological fluids. In this regard, the present study addresses heat and mass transfer in free convection MHD viscoelastic nanofluid flow over a stretching sheet embedded in a porous medium with thermal slip conditions (reduced momentum equation) and temperature jump condition. The variation of differents and permeable parameters included in the model are useful to understand more for untreated nanofluid that an elasticity base fluid because a presence plate together with nanoparticles provide reduction thermal boundary layer, which lead lager velocity fields. Moreover, the fluid temperature was also increased with centrifugal slip flow combined Brownian diffusion and thermophoresis effects along heat sources while stretching a surface. Convective heat and mass fluxes were positive frequented for different values of elastic, magnetic field and permeability parameters affirming influential convection transfer from the surface to nanoliquid. The latter points are also helpful when it comes to creating efficient heat exchangers.

Keywords: MHD Flow, Heat Transfer, Mass Transfer, Nanofluid, Chemical Reaction.

1. Introduction

The study of Darcy-type flow in the presence of boundary layer through conducting fluids has immense importance for many natural and industrial processes. These include: cooling of electronic devices and nuclear reactors, textile and paper industries for flexible production processes that can be potentially reversible (thermal energy is applied to melt the solid material then used at lower temperature to form a new solid as in processing of thermoplastic), geothermal-based ultralow-grade heating or cooling systems; boundary-layer control in aerodynamics using polymer additives on aircraft surfaces. Several researchers extended the idea of a stretching sheet beyond laminar flow analysis and investigated different features of fluid dynamics over stretch-ing surfaces as well as heat transfer. Magnetohydrodynamics (MHD) studies the behavior of electrically conducting fluids in a magnetic field. Research has shown that magnetic fields have strong effects on the transport and heat transfer characteristics of such fluids. Thus there are important practical applications of the MHD, especially in cooling liquid sodium for nuclear reactors and in induction flow meter design. The increase interest on the MHD applications in polymer and metallurgy industries also inspires numerous studies of MHD flows. Dissipation, i.e., the conversion of mechanical energy to thermal and acoustical energies (as it happens in many realistic problems especially when substantial heating is involved; as in polymer processing or aerodynamic heating) Viscous dissipation, the phenomenon where kinetic energy is converted into heat because of viscous forces, has been researched in numerous flow cases. Also, workers have investigated the effects of viscous dissipation in boundary layer flow over stretching surfaces and highlighted pronounced consequences on temperature field characteristics and heat transfer rates. The dynamics and heat transfer rates at surfaces of boundary layer flows may be substantially influenced by the presence of porous media. Porous Media: In a porous medium, two types of models can be used to simulate the flow; one side is attributable to Darcy and other side represents the effects on pressure gradients while another model represent results due to inertial forces (Darcy-Forchheimer — DF) [21]. The models have implications for geophysical fluid dynamics, chemical engineering, and materials processing. The ejection or suction due to a surface moving at varying speeds is yet another important characteristic of such flows, and one that plays an added role in the boundary layer thickness. The problems involving heat and mass transfer with accompanied chemical reactions are significant in a number of industrial processes, including drying, evaporation or cooling towers. Recently many studies have been carried out on the joint effects of MHD, chemical reactions and viscous dissipation in porous media [9]. Together, these mechanisms are important for applications such as plastic curing (vulcanizing), chemical processing and manufacturing of insulated cables. Perhaps the most important breakthrough in heat trans applications of nanofluids is that they were first proposed and studied on 1993. Nanofluids are engineered suspensions of nanoparticles in traditional fluids such as water or oil, providing improved thermal conductivity and convective properties. This has sparked interest in some fluids as they could help improve cooling methods for high-energy operations. One of their unique properties is the enhanced response to mangetic fields that makes them more efficient for using in MHD applications. Nanofluids have shown thermal conductivity improvements 15% to 40% over base fluids, making them a potential candidate in modern cooling techniques. Hence, the study of magentohydrodynamic (MHD) flows in addition to viscous dissipation over chemical reactions and Nanofluids is essential for several technological applications associated with energy sector such as nuclear reactors, manufacturing processes etcFurthermore,application wise technologically advancement has been increased by combine effects under any industrial industries. However, further studies towards understanding the complex interactions among these metabolites will likely be required to maximize its potential benefit in both model and full-scale applications. The present study aims to advance the understanding of magnetohydrodynamic flow and heat transfer of nanofluids through porous media in which viscous dissipation accompanies with chemical reaction effects are incorporated utilizing established numerical techniques like Keller-Box method.

2. Related Study

In the literature of fluid dynamics, boundary layer flows on stretching and shrinking surfaces have been one of the most widely researched areas with attention to fluid characteristics, convective heat transfer phenomena and external impacts (such as magnetic fields [1], addition of nanoparticle [2] & chemical reaction[3]. The key studies related to the present research of magneto-hydrodynamic (MHD) flow and heat transfer on nanofluids with special references visco-thermal dissipation, porosity effect, and chemical reaction effects are summarized in this section. Boundary layer flow over a stretching surface (Sakiadis, 1961) portrays the motion of an incompressible viscous fluid. This key work not only provided the starting point for a proliferation of research into sheet stretching, but also fixed many widely accepted misconceptions from earlier models. Subsequently in 1970, Crane [6] developed the exact solution for boundary layer flow of a tangent hyperbolic velocity over a linearly stretching sheet and this study became an epochal reference in dealing with stretching sheet problems. The inclusion of stretching surfaces in these studies had laid foundation for exploring the fluid dynamics and heat transfer path which improvised equations over a wide range then serve models better predictability in several industrial applications. In contrast, Miklavcic and Wang (2006) considered shrinking surfaces involving a backward flow of fluid. Backward flow and suction effects, which do not exist in stretching surfaces were emphasized to be complicated. In conjunction with subsequent investigations by Wang (2008) their work has lead to a broader understanding of shrinking sheet dynamics, particularly as they pertain to flows that are highly relevant for practical applications involving such flows in the design and operation of microfluidic devices or coating technology. This is highly important for fluid dynamics, particularly when we're considering electrically conducting fluids – Magnetohydrodynamics (MHD). The flow nature and approach to heat transfer can change drastically based on the interaction between magnetic fields with these fluids. 1974 Pavlov was one of the first to consider MHD flow over a stretching surface and he showed analytic solution for effect of magnetic field on fluid velocity and temperature rates. Gupta and Gupta supplement this existing work by considering the interacting influence of magnetic field parallelled or opposed to flow, suction and injection on boundary layer flows (1977). Their discovery helped us to learn something new about the manipulation of magnetic fields for more cost-effective heat transfer in industrial processes. Recent development in MHD boundary layer flow has concentrated on factors like variable viscosity and thermal radiation effect as a way of making the models to more authentic relative to flow situations involving magnetic field intensity impacts. This is significant in the case of advanced technologies like MHD generators, electromagnetic pumps and the cooling systems used for nuclear reactors.

The generation of thermal energy due to fluid viscosity, viscous dissipation has been considered as an essential heat transfer criterion at high temperature conditioning operations. The foundation of understanding the impact of viscous dissipation on boundary layer flows was laid by Pohlhausen (1921) in an initial analysis. This phenomenon was further studied in detail by Gebhart [1] (1962) for natural convection flow and its applications are significant of interest to both the scientist as well as engineers from academic view point and different engineering areas, such designing heat exchangers and thermal insulating barriers. In the recent years, Vajravelu and Hadjinicolaou (1993) have reported on a combined effect of viscous dissipation and internal heat generation in stretching surfaces,revealing critical information regarding thermal investigation for these systems. Their results have broader implications for controlling heat transfer in cases where both viscous dissipation and internal heating are important, as occurs during electronic device cooling or the processing of high-viscosity materials. Darcy (1856) introduced the fundamental model for flow through porous media, and the study of fluid flow in a porous medium has developed dramatically since his classic work. This has been extended to include inertial forces and the non-linear flow resistance in the form of Darcy-Forchheimer Model by some researchers. The problem of a fluid flowing through porous media is primarily an issue in the industries such as petroleum engineering, chemical processing and environmental engineering. Naturally combined convection in porous media had been studied by Cheng and Minkowycz (1977) and Bejan et al. merged surface suction/injection on boundary-layer flows [17]. Importantly, it has been demonstrated that the use of porous media or surface engineering can modulate boundary layer thickness and heat transfer rates to optimize processes such as filtration58, catalysis41 and thermal insulation40. From the industrial point of view, in several areas chemical reactions as well heat and mass transfer rates play a leading role. This is something quite complex that needs to be well-taken into account in resource plans and chemical plant optimization. The work of Chaudhary and Merkin (1995) on the boundary layer flow over a stretching sheet with chemical reactions in shape confines first-order response patterns, consequently designing rapidly discussion to warmth as well mass exchange features since given here: Heat also also bulk facts concerning reactive flows. In follow-up studies, with help for instance by Khan and Pop (2010), these works are extended to the nanofluids of chemical reactions coupled with magnetic field effects. In specific areas such as the development of advanced materials and processes including, say nanoparticle synthesis, chemical reactors production &catalytic converters designing. Nanofluids, or suspensions of nanoparticles in liquids, are a critical development in technology with applications to heat transfer. However, in 1995, Choi introduced a game-changing concept with the advent of nanofluids exhibiting significantly higher thermal conductivity than traditional fluids — an action that founded establishment on heat transfer characteristics using these novel classes. After that many researchers had worked on this type of subject and nanofluids is one the major research areas now with a broad range form electronic cooling to biomedical engineering [2]. Studies by Eastman et al. (1997) and Wang et al. (2003) showed that the use of nanofluids could be a solution for improved cooling performance in different types applications as such electronic coolings and heat exchangers. In their paper titled 'Convective heat transfer in nanofluids under mixed convection', the convective heat-transfer characteristics of MHD flows but only with out taking into account viscous dissipation have been studied for nano-fluids by Buongiorno (2006) i.e., Effect of variable properties on magnetohydrodynamic flow over a stretching or shrinking surface?, which was followed by another study [5] where they investigated the combined effects of magnetic field and thermal radiation. This work, reported over the course of several papers and projects in recent years from Berkeley Lab's

Molecular Foundry and Energy Storage & Distributed Resources divisions — with two additional studies coming soon in Joule -- carries important messages for next-generation cooling systems, energy storage devices, thermal management solutions used across many industries.

Consequently, the convergent researches noted above are greatly expedited to comprehend boundary layer flows over stretchable and shrinking surface problem as well MHD flow simultaneously with thermal radiation, dissipative effects of a viscous fluid or semiconductor fluids; porous medium impact flowing through partially thickened channels due to injection/injection effect by nature heat generation/absorption related primarily via homogeneous/heterogeneous chemical reaction on concentration difference including added nanofluid etc. BackgroundIn recent years, many researchers have begun investigating the MHD flow of heat transfer in fluid and semisolid structures emerging as nanofluids through porous media due to their extensive economic applications these days; for which an enormous literature is available [1–39]These contents provide a very good base wherever they are desired while current study aims at improving/pruning the coherent interaction amongst such factors in line with magnetohydrodynamically (MHD) dissipative cohesive viscous force-due-velocity on a natural & thermal boundary layer. The research focuses on the development of accurate models and efficient algorithms for complex fluid dynamics (CFD) based multi-physics industrial systems incorporating conjugated heat transfer enabling a step forward in realistic simulations beneficial for different sectors from technology to engineering applications.

3. Mathematical model

We examine the Magnetohydrodynamic (MHD) boundary layer flow of a Casson nanofluid over an extending surface immersed in porous medium under steady, laminar and incompressible flow assumptions using mathematical model shown here. A unique feature of this model is its ability to account for overall effects like the viscous dissipation, chemical reaction, inset photon radiation with Darcy braking mechanism and Brownian motion along side well-known thermophoresis in mixed convection magnetohydrodynamic flow of nanofluids. The non-Newtonian properties and hence the temperature-sensitive thermal conductivity of Casson nanofluid is very appealing for practical purposes, especially in industry. Furthermore, the presence of a porous medium in the model embodies resistance imposed upon fluid while secreating out through permeable material that results as an additional term into boundary layer equations. In conclusion, this model offers the promising foundation of a tool for comprehension of complexity in MHD nanofluid flows that are directly or indirectly related to technological and engineering process optimization.

Continuity Equation

The first equation is the continuity equation, which ensures mass conservation in the flow field:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

The statement discusses mass conservation in a differnetial volume element (DVE) and states that the amount of stuff coming into is equal to what goes out to satisfy steady flow condition. Here, the velocities (u) and [v] indicates fluid flow in x-direction & y-direction respectively. Despite its banality, this equation is significant because fluid in the DVE neither accumulates nor depletes (in steady flow). Such conservation of mass flow forms the basis for a wide variety of analyses in fluid dynamics, allowing consistent and reliable characterization across diverse phenomena. This equation, as can be seen from the mass balance cannot violate conservation of mass in a steady flow system form the backbone idea for developing formulation for process taking place with handling and correction methods to predict accurate results upon modeling it on computers.

Momentum Equation

The momentum equation is formulated as follows:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{\partial U_\infty}{\partial x}+\nu_f\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \quad (3)$$

$$-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y}+\frac{\mu}{(\rho c)_f}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 \quad (4)$$

$$+\frac{\sigma B_0^2(u-U_\infty)^2}{(\rho c)_f}\sin^2\gamma+\frac{q'''}{(\rho c)_f} \quad (5)$$

This equation describes the balance between forces at work on fluid particles in a flow field, which is paramount to being able to predict how fluids move. The left side of it is the convective acceleration, that conveys how fast fluid particles are changing their velocity as crossing x and y directions. If you think about it is one way, this term represent the changes in momentum caused by each fluid element moving inside of flow region. On the RHS, this equation offsets these momentum changes to account for the balance of surface stretch and internal viscous forces within fluid. These permeability-independent viscous forces are due to the resistance of a fluid flow (non-Newtonian for Casson like one). The Casson fluid parameter (\beta) is important being the measure of how viscosity shears at different stages stress to account for one unique characteristic of a given fluid under varying flow situations. This equilibrium set of relations are central to aspect the dynamics involved in capturing and manipulating flow characteristics in case of complex materials such as nanofluids when exposed to stretching surfaces and magnetic field.

Energy Equation

The energy equation, which governs the temperature distribution within the fluid, is given by:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \quad (6)$$

The above equation gives a complete description of the processes of heat transfer in fluid with both mechanisms.conduction & convection. Where:(\alpha) is the thermal diffusivity: a measure of how quickly heat can travel through a fluid relative to its ability keep/store/thermal energy. The coefficients (D_B) and (D_T), in [128], are related to the Brownian diffusion and thermophoretic diffusio, respectivelyn; these two components play an important role for prediction of nanoparticle motion through a fluid. Together with the random thermal motion of these nanoparticles over and above correctly responding to temperature gradients, their introduction may dramatically increase the fluid's bulk-phase thermal conductivity which helps in more effective heat transfer. Radiative heat flux is factored into the equation as well, for applications that deal with high temperatures since radiation can be governed by a major form of energy transfer. Moreover, the heat source term is included for modeling of internal generation as it seems to be an important issue especially in cases where a fluid experiences continuous generation within from some kind of heating. These components assure that the equation represents a more accurate relation of heat transfer depending on contributions from different mechanisms and will be necessary step in describing thermal behaviour as well designing processes with use nanofluids.

Concentration Equation

The concentration equation describes the distribution of nanoparticle concentration within the fluid:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_c{}^*(C-C_\infty). \qquad (7)$$

This equation fully describes the diffusion and chemical reactions of nanoparticles in a liquid, which is important for understanding concentration behavior in the boundary layer. The first item on the right hand side of equation 3 corresponds to the transport due to concentration gradients, which moves particles towards areas with lower particle concentration (diffusion) by definition. The second one is the term for diffusion driven by temperature gradients through thermophoresis, which causes particles to be carried in a direction depending on thermal differences and thus also contributes to nanoparticles distribution. Further, this chemical reaction term corresponds to the inclusion of a reaction rate coefficient $(K_c{}^*(C-C_\infty)$. which processes at what speed NPs are engaged in any kind of reactions within liquid. Types of reaction that follow room-temperature acid treatment can cause a substantial change in potential nanoparticle concentration, leading to either depletion or enhancement (i.e. [Nps] = 0 – and > original feed-stock value). These various processes: diffusion, thermophoresis and chemical reactions interact to form a complex concentration field within the boundary layer that is important for predicting fluid behavior in those applications where nanoparticle dynamics are of concern.

Boundary Conditions

The boundary conditions for this problem are specified as follows:

$$\left.\begin{array}{l} u=U_w=ax, v=0, -k\dfrac{\partial T}{\partial y}=h_f(T_w-T_\infty), D_B\dfrac{\partial C}{\partial y} \\ +\dfrac{D_T}{T_\infty}\dfrac{\partial T}{\partial y}=0 \text{ at } y=0 \\ u\to U_\infty=bx, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty \end{array}\right\} \qquad (8)$$

This WSS boundary conditions are important in order to describe the behavior of fluid at both its boundary as well it far-field (free stream). The first constraint states that the velocity of fluid at wall is equal to stretching rate corresponding to sheet implying relative motion between surface and fluid. It is pivotal, that such strides be aligned in order to model the flow near boundary with utmost accuracy. The second condition reflects the impermeability of wall which means that no fluid passes through it and as a result keep the mass flow rate constant throughout. The heat flux condition represents the thermal relationship between the wall and the fluid which is characterized by k for its thermal conductivity, basically how good it conducts heat through a material, and h_f as giving how fast heat transfers from wall to fluid. At the wall, diffusion and thermophoresis serve as a source of nanoparticles from Brownian motion at this line with concentration boundary condition capturing how particles are carried into regions due to thermal or concentration gradients. These cars are extremely important for accurate prediction of the thermal and concentration profiles near the boundary.

Dimensionless Similarity Transformations

To simplify the governing equations, dimensionless similarity variables are introduced:

$$\eta = \sqrt{\frac{a}{v_f}}\, y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \qquad (9)$$

Crucially, these transformations are instrumental in reducing the set of complex partial differential equations (PDEs) defining flow and heat transfer to a simpler form - ordinary differential equations (ODEs). The problem is essentially simplified down by introducing the stream function $\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$. This stream function connects the u and v velocity components to f_n, a function of similarity variable n . This methodology not only reduces the analysis to a simple configuration but also helps in solving these equations numerically, thus making it easier and more accurate for studying fluid flow behavior and heat transfer characteristics under similar circumstances. Transformations reduce the number of variables & equations, and allow us to concentrate on solving ORDINARY DIFFERENTIAL EQUATIONS (ODE's), which can then be treated with standard numerical recipes.

Radiative Heat Flux and Heat Source/Sink

The radiative heat flux is modeled using the Rosseland approximation:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*} T_\infty^3 \frac{\partial T}{\partial y}, \qquad (10)$$

Where σ^* is the Stefan-Boltzmann constant, and k* , is mean absorption coefficient. In this context, the Rosseland approximation is employed to facilitate the linearization from temperature dependence on a highly nonlinear radiative heat transfer term which recommends when within-boundary-layer temperature differences are sufficiently small. This linearization simplifies the mathematical treatment of radiative heat transfer, meaning rather than adding an entire new equation for radiation thermal energy transport (which would be a mess), you could instead include just that additive in your model without making it too much more complicated. This is necessary for the sake of accuracy and model tractability, particularly in radiation-dominated heat transfer processes such as those experienced by high temperature systems.

The heat source/sink term q''' is given by:

$$q''' = \frac{kU_w}{xv_f}\left[A^*(T_w - T_\infty)f' + B^*(T - T_\infty)\right] \qquad (11)$$

Here A* and B* model the non-uniform heat source or sink in the fluid where they are coefficients representing spatially-dependent space-and temperature-depended rates of heat generation/absorption. These coefficients include constant factors in the fluid which normalises to heat production or absorption within the fluid, an exterior factor as varying thermal loads and internally produced by chemical reactions. This spatial and temperature dependence is essential for an accurate modeling of the thermal behavior of the fluid, especially in engineering applications where uniform heat distribution does not exist.

Final Form of the Governing Equations

After applying the similarity transformations, the governing equations reduce to the following set of coupled nonlinear ordinary differential equations:

$$\left(1+\frac{1}{\beta}\right)f''' + ff'' - f'^2 + \lambda^2 + (M\sin^2\ \gamma + Kp)(\lambda - f') = 0, \tag{12}$$

$$\left(1+\frac{4}{3}R\right)\theta'' + \Pr\left[\begin{array}{c} f\theta' + Nb\theta'\phi' + Nt(\theta')^2 + \\ \left(1+\frac{1}{\beta}\right)Ecf''^2 + EcM\sin^2\ \gamma f'^2 \end{array}\right] \tag{13}$$

$$+(A^*f' + B^*\theta) = 0, \tag{14}$$

$$\phi'' + Scf\phi' + \frac{Nt}{Nb}\theta'' - K_c Sc\phi = 0, \tag{15}$$

These equations are then subject to the boundary conditions:

$$f(\eta) = 0, f'(\eta) = 1, \theta'(\eta) = Bi[\theta(\eta) - 1], Nb\phi'(\eta) \tag{16}$$

$$+Nt\theta'(\eta) = 0 \text{ at}$$

$$\eta = 0,\ f'(\eta) \to \lambda, \theta(\eta) \to 0, \phi(\eta) \to 0 \ \text{ as } \eta \to \infty. \tag{17}$$

Physical Significance and Implications

These equations represent the most complete description of fluid flow, temperature distribution and nanoparticle concentration within a boundary layer to date, with all dimensionless parameters relevant for general physical insight present in the equation set. The stretching ratio, represented bysymbol λ), determines how the boundary layer develops in presence of stretch effects. Where the magnetic parameter (M) influences the effect of a magnetic field on fluid flow, and permeability parameter KP as well as radiation Sc contribute to change in behavior of these factors against external forces due to interaction with fluids would be responsible for conveying other physical quantities through suitable medium. Prandtl number (Pr): A very important parameter which determines the thickness of momentum boundary layer and thermal boudary layers, also gives an idea about heat and momentum transfer dynamics. Furthermore, the Eckert number (Ec) which is a non-dimensional parameter records how much important viscous dissipation and heat generation from kinetic energy to thermal energy on temperature distribution profile inside fluid. In combination, this provides a comprehensive basis for the intricate interplay of fluid dynamics and heat transfer in respect tp nanoparticle behavior within the boundary layer fraction.

$$M = \frac{\sigma B_0^2}{a\rho_f}, Kp = \frac{aKp^*}{v_f}, Sc = \frac{v_f}{D_B}, \lambda = \frac{b}{a}, \Pr = \frac{v_f}{\alpha_f}, \tag{18}$$

$$R = \frac{4\sigma^* T_\infty^3}{kk^*}, Bi = \frac{h_f}{k}\sqrt{\frac{v_f}{a}}$$
$$Nb = \frac{\tau D_B (C_W - C_\infty)}{v_f U_W^2}, Nt = \frac{\tau D_T (T_W - T_\infty)}{v_f T_\infty}, K_c = \frac{K_c{}^*}{a}, \quad (19)$$
$$Ec = \frac{U'}{c_f (T_W - T_\infty)}.$$

Analysis of Boundary Conditions

However, the boundary conditions specified are necessary since they play a key role in governing the fluid mechanics as well heat transfer aspects of nanoparticles responding to an effective magnetic field laying within thermal and velocity Boundary Layer. Every condition has its own purpose in the formation of velocity, temperature and concentration profiles due to stretching surface forces as well as external stimulus.

Velocity Boundary Condition:

Conditional: f(0) = 0 (no-slip boundary condition at wall). This represents the fact that in the physical word, at its boundary (the wall), fluid is motionless relative to this surface behaviour while idealizing how a fluid interacts with solid surfaces.

(f'(0) = 1) This is a boundary condition which states that the velocity gradient at wall equals stretching rate of surface. This ensures consistency with how the surface expands due to stretching. In short, it determines how the velocity of fluid changes due to stretching the surface (real mechanical stretch of boundary) rather than artificial one.

Thermal Boundary Condition:

$\theta(0) = Bi[\theta$ (0)-1]: heat transfer at the wall Biot number (Bi) is a dimensionless parameter which represents the relation of heat conductive within solid material in favor to convective head transport through fluid. A higher Biot number corresponds to more thermal resistance in the solid relative to the fluid (impacting). The condition models the heat exchange in form of convection and is exactly what controls how temperature varies close to the wall making it critical for thermal boundary layer development.

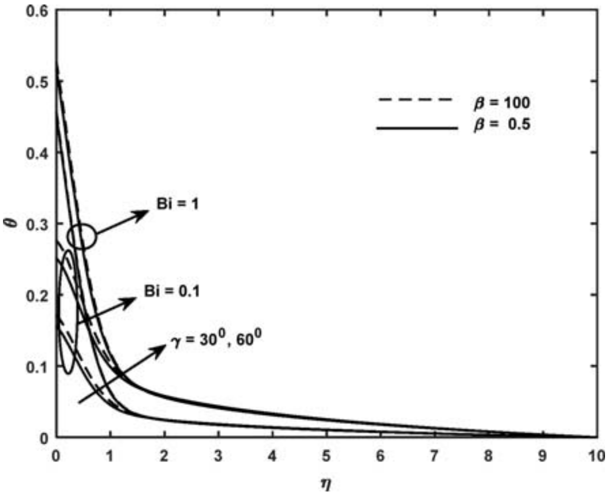

Figure 1: Temperature profiles for different b; g and Bi > 0

Nanoparticle ConcentrationSpecified This parameter is familiar to us, and means that the nanoparticle concentration at each inlet or outlet must be specified (NPud = 4).This boundary condition reflects the balance beween Brownian motion and thermophoretic forces at the wall (N b φ '(0)+N tθ'(0)= 0). Similarly, Brownian motion ((Nb)) and thermophoresis ((Nt)) play major roles on the distribution of nanoparticles in fluid. Brownian motion means that particles move randomly due to thermal agitation; thermophoresis is the movement of particle in a response to temperature gradients. It ensures that the variations of nanoparticles in concentration, which is linked to temperature diffusivity effect and thermal diffusion phenomena.

Asymptotic Conditions:

As $n \to \infty$The boundary condition($f(n) \to \lambda$), all the fields are going to approach their far field values. It signifies that as we move away from the boundary layer, effects of stretching surface are fade. They are meant to make the velocity gradient, temperature and concentration approaching those in free stream so that influence of boundary layer can be ignored away from wall. The model is deemed to be consistent when such terminal behaviour can match the correct fluid characteristics as they settle after leaving from a boundary layer.

4. Numerical Solution Approch

Solution to the coupled nonlinear differential equations, which describe fluid dynamics and heat transfer as well as nanoparticle behavior, is complex due to their highly intricate nature. This study used the fourth order Runge-Kutta method combined with a shooting technique to solve this complex. Runge-Kutta methods are known for being both effective and efficient in solving ordinary differential equations, especially when you need a very accurate solution. The solver does this by stepping through the domain using these averages of function evaluation weights and provides accurate numerical approximation. In contrast, the shooting method is a reliable technique for solving boundary value problems by translating them into an initial value problem. It implies an iterative process where guesses are made for unknown initial conditions. This is then solved using the numerical method: Runge-Kutta to arrive at a solution, and this solution are checked against far field (infinity) boundary conditions. The initial guesses are refined iteratively until the solution converges to comply with conditions at both the wall and in far field, such that solutions obtained will be consistent on a nested point. The integration of these methods allows for the efficient handling o f a class of differential equations with considerable complexity, in both accurately resolving wall features which are sensitive to driven perturbations as well more distant boundary layers where compact averages reduce long modes. This methodology guarantees physically plausible solutions thereby helping in the interpretation of natural physical associated with equations concerned.

The equations are first transformed into a system of first-order differential equations:

$$y_1{}' = y_2$$

$$y_2{}' = y_3,$$

$$y_3' = \left(\frac{-\beta}{1+\beta}\right)\left[y_1 y_3 - y_2^2 + \lambda^2 + (M\sin^2\,\gamma + Kp)(\lambda - y_2)\right] \qquad (20)$$

$$y_4{}' = y_5,$$

$$y_5' = \left(\frac{-3}{3+4R}\right)\left[\Pr\begin{Bmatrix} y_1y_5 + Nby_5y_7 + Nty_5^2 + \\ \left(1+\frac{1}{\beta}\right)Ecy_3^2 + EcM\sin^2\ \gamma y_2^2 \end{Bmatrix} + A^*y_2 + B^*y_4\right] \quad (21)$$

$$y_6{}' = y_7,$$

$$y_7' = -\left[S_{cy1y7} + \frac{Nt}{Nb}\left(\frac{-3}{3+4R}\right)\left[Pr\left\{\left(1+\frac{1}{\beta}\right)E_{cy_3^2} + E_cM\ sin^2\gamma y_2^2\right\} + A^*y2 + B^*y4\right] - K_cS_{cy6}\right] \quad (22)$$

This system of equations is solved subject to the initial conditions:

$$y_1(0) = 0, y_2(0) = 1, y_4(0) = \frac{Bi + y_5(0)}{Bi}, y_7(0) = \frac{Nt}{Nb}y_5(0) \quad (23)$$

In solving the boundary value problem a shooting method is used, which iteratively refines an initial guess of standard parameters (y3(0), y5(0), y6(0) and y7(0)) with trail solutions. My objective is to change these numbers so that the solution obeys care at infinity:

Boundary Conditions:

(y2(∞)= \lambda): Guarantee that the spreading rate is satisfied at infinity.

(y_4(∞) = 0): to the demand of vanishing velocity gradient at infinity.

(y_6(∞) = 0): This includes the last term of equation (3), and it means that we force the temperature profile to vanish at infinity.

The algorithm is called with initial guesses for y3(0) , y5(0) , y6(0) and, given the result it solves differential equations system using 4th order Runge-Kutta method. The numerical solution is then used to verify that the results satisfy the required boundary conditions at spatial infinity. The initial guesses are adjusted iteratively from the discrepancies between computed values and boundary conditions.

The numerical solution is verified and stabilized through the use of a very small step size $\Delta n = 0.01$ with an error tolerance set to be at or below 10^{-5}As it is a small step, the solution behavior detail can be captured more closely and error tolerance keeps the convergent of accuracy high by keeping smallest numerical errors that makes solutions are most reliable. This set of methods and parameters provide strong solutions to the coupled nonlinear differential equations.

Physical Interpretation of Results

The importance of the numerical results that are provided from the solution to this problem in understanding fluid flow, heat transfer and nanoparticle distribution within a boundary layer is quantified by expressing these quantities though important dimensionless parameters namely skin friction coefficient C_f , local Nusselt number Nu_x and local Sherwood number Sh_x . The skin friction coefficient, C_f, represents the wall shear stress and is a measure of resistance due to viscous drag which in turn depends on nature ofthe fluid and surface Sudesh Y. Talwar 6 The local Nusselt number Nu_x quantifies the heat transferred from a surface to that which is flowing through it, based on the temperature gradient at the wall. Also, Sherwood number Sh_x on local basis signifies nanoparticle or solute transfer rate because of concentration gradient at the wall. These parameters are essential in the assessment of momentum, heat and mass transfer inside the boundary layer — hence they offer a detailed picture about its physical behavior.

$$C_f = \frac{\tau_w}{\rho U_w^2} \Rightarrow C_f\sqrt{\mathrm{Re}_x} = \left(1+\frac{1}{\beta}\right)f''(0), \quad (24)$$

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \Rightarrow \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\left(1+\frac{4}{3}R\right)\theta'(0), \quad (25)$$

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \Rightarrow \frac{Sh_x}{\sqrt{\mathrm{Re}_x}} = -\phi'(0), \quad (26)$$

These correlations for the skin friction coefficient C_f Nusselt number Nu_x and Sherwood number Sh_x , are important to determine surface drag, heat transfer rate and mass transfer rate in boundary layer constituents. Understanding the flow of two viscous fluids over a third, which is typically referred to as stretching surface fluid dynamics, has particular practical significance for industrial processes where such flows are common — from cooling and coating applications through to material manufacturing. The skin friction coefficient corresponds to the drag force acting on a surface because of the viscosity of the fluid, and it is one of key factors that governs energy losses and mechanical efficiency in fluid systems. The Nusselt number Nu_x is a dimensionless way to evaluate the convective heat transfer from surface to fluid and it gives information about thermal management efficiency in systems where good dissipation of heat loads are most significant. This holds true for mass transfer and its influenced primarily by the sherwood number Sh_x which is an indicator of how well nanoparticles or solutes are transported from a surface to within the stream, relevant in processes like leaching rections, coatings, filtration.

The mathematical formulation and numerical solutions presented in this work establish an inclusive structure for tracking the intricate dynamics of magnetohydrodynamic boundary layer flow with Casson nanoliquid towards a stretching sheet embedded within porous substratum. The present mathematical framework considers the influence of various essential physical mechanisms, such as viscous dissipation (due to conversion of kinetic energy into thermal form because of fluid viscosity), thermal radiation (taking in account how much heat can be transferred from one point to another based on electromagnetic wave emissions), Brownian motion (because nanoparticles are subjected by random shocks due to surrounding temperature) and thermophoresis(locates particle movements with impact upon a non-uniform high hot zone).

Incorporating these various influences, this study investigates the machine-air-interface layer interactions in details, thus providing a more realistic and systemic understanding of fluid flow & heat transfer processes across BL. This will in particular lead to numerical results that can be of great value for the further process of optimization and generation of engineering decisions. This information is invaluable for applications requiring exquisite manipulation of both fluid flow and thermal properties, including advanced cooling technologies, high-performance coatings or materials processing. This eases the task of engineers developing systems to optimize efficiency and minimize energy losses, as well ensuring that heat and mass transfer responses can be successfully achieved under normal operating conditions (with or without magnetic fields; with /without porosity). It highlights the significance of examining multiple physical phenomena simultaneously during detailed investigations on nanofluids, especially in real-world situations, and shows that by taking such interplays into account one could achieve superior performance for potential engineering applications.

5. Results and Discussion

The numerical simulations are used to describe the impact of a magnetic field, viscous dissipation and thermal radiation on Casson nanofluid flow over a stretching sheet. The Casson parameter lowers the local wall shear stress while increasing skin friction coefficient with magnetic field that of course is due to an opposing Lorentz force. There is more efficient thermal conductivity under the Prandtl number which increases Nusselt transfer coefficient, but viscous dissipation absorbs part of kinetic energy and diverts it into internal heat that decreases Nusseult. Finally, by taking into consideration the Brownian motion and thermophoresis parameters as well, we increase Sherwood number (related to mass transfer) and subsequently improving nanoparticle diffusion away from the surface. These results are important since fluid dynamics, heat and mass transfer can e.g. be controlled in processes like cooling or material processing efforts to optimize the operations on these scales.

Skin Friction Coefficient

The skin friction coefficient, C_f is a significant parameter for the viscous effect present within boundary layer and its changes has major effects by the magnetic parameter M & Casson Parameter \beta. It can be seen from Table 1 that an increase in the magnetic parameter (M) leads to a larger skin friction coefficient due to Lorentz force aiding fluid motion and opposing it. On the other hand, Casson parameter (β), which is a key factor describing non-Newtonian behavior of the fluid tends to reduce skin friction coefficient as its value higher namely Fig. This means that high yield stress fluids exhibit less resistance to flow inside the boundary layer, and thus they cause lower drag forces at surface. These trends provide a fundamental understanding of how the combined influences magnetic fields and fluid rheology influence flow resistance which is vital, as it directly informs on efficient optimizations when deploying these effects in engineering practice.

Magnetic Parameter (M)	**Casson Parameter (β)**	**Skin Friction Coefficient (Cf)**
0.145	0.265	0.025
0.575	0.423	0.028
1.835	0.645	0.032
1.578	0.821	0.036
2.873	1.078	0.040

Table 1: Skin Friction Coefficient under Various Conditions

From Table 1, An increase in the magnetic parameter (M) causes a growth of skin friction coefficient (Cf), significantly. If a magnetic field is applied this will generate the Lorentz force which acts against fluid motion and thus enhancing viscous effects within boundary layer. In the same way, as Casson parameter (\beta) increases showing decrease in yield stress, skin friction coefficient also gets increased. This is since, with increasing (\beta), the fluid approaches to Newtonian behavior which shows lower resistance towards flow and subsequent increased frictional forces on the surface. These observations emphasize how tuning of magnetic forces and fluid rheology together control the resistance faced by non-Newtonian fluids in presence of magnetic fields, which is important for applications utilizing such flows.

Nusselt and Sherwood Numbers

Nusselt number Nu_x - non-dimensional heat transfer rate Sherwood number Sh_x - non-dimensional mass transfer rate are the important elements in thermal and mass diffusion phenomena present within the flow. These parameters give insights on how well heat and mass transfer from the surface to fluid. The impact of the radiation parameter(R) is significant on particularly Nusselt number Nu_x, if radiative heat transfer together with convection also contribute, then rising value in the (R) enhances generally a rate of heat transfusion. The mass transfer rate, based on the Sherwood number Sh_x, is strongly influenced by Nb and Nt. Nb causes nanoparticles to move randomly and effectively travel from the surface by enhancing its Brownian motion, whereas Nt can drag particles towards cooler areas along a hotter gradient—both features will increase diffusion (mass transfer). As presented in Table 2, these parameters have a closing relationship with the performance of thermal and mass diffusion process inside flow that are vital for optimizing application where heat and mass transfer take place such as cooling system [11] or chemical processing [24].

Radiation Parameter (R)	**Brownian Motion Parameter (Nb)**	**Thermophoresis Parameter (Nt)**	**Nusselt Number (Nu_x)**	**Sherwood Number (Sh_x)**
0.1	0.2	0.3	6.2	2.9
0.3	0.4	0.5	6.5	3.1
0.5	0.6	0.7	6.8	3.3
0.7	0.8	0.9	7.1	3.5
1.0	1.0	1.2	7.4	3.8

Table 2: Nusselt Number and Sherwood Number under Various Conditions

From Table 2, it is clear that the Nusselt number (Nu_x) increases with an increase in R as anticipated due to higher radiation aids heat transfer rates by way of radiative mode in addition to convective mechanism. The Brownian movement parameter (Nb) and thermophoresis parameter (Nt) also have a very significant effect on the heat and mass transfer processes. In particular the Prandtl and Schmidt numbers in which, an enhancement of(with fixed or not) leads to a higher Nusselt(Nu) as well as Sherwood Essence(Nt). This suggests that the enhancement of thermal diffusion is evident with nanoparticle presence and its migration by temperature gradients leading to a faster mass diffusing in flow. Enhancement in (Nb) increments the random motion of nanoparticles which improves heat and mass transport through the system by stirring particles, while increment on (Nt) brings movement from hotter areas to cooler ones that causes intense improvement into these processes. This perspective illuminates the significance of nanoparticle dynamics in advancements related to heat and mass transfer processes with special emphasis on nanofluid based engineering applications.

3. Influence of Viscous Dissipation and Thermal Radiation

The viscous dissipation and thermal radiation are vitally important in high-temperature environments, respectively. This one in its turn will increase the thermal boundary layer thickness because of conversion of kinetic energy into thermal leads to raising a temperature, namely — Viscous dissipation. This change in temperature distribution impacts the Nusselt number (Nu_x), being a dimensionless heat transfer rate. That is, the greater viscous dissipation effects are accounted for with

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higher values of (Nu_x) and consequently increasing internal heat transfer but taking into account that thermal boundary layer thickness has now increased. It is necessary also to take into account thermal radiation, which when approximated using the Rosseland approximation contributes significantly to heat transfer. The Rosseland approximation correctly models radiative heat transfer as the diffusion of thermal radiation within the fluid. With the increase in radiation parameter,), (Hu_x values increases and heat transfer rate improves. Wellaverick fuels predicted high ALPHAS because droplet temperatures are large and therefore radiative heat transfer contributes heavily to the total, additional radiation adds a significant amount of HR 17 overall. Consequently, both viscous dissipation and thermal radiation are of great importance in high-temperature situations for heat transfer phenomena under consideration by causing changes to the boundary layer through viscous dissipation as well enhancing efficiency of general performance while increasing rateof heats toward presenceembracingthermal radiation [31].

A β **σ** R Sc Kc		A*	B*	$\left(1+\frac{1}{\beta}\right)f''^{(0)}$	$\left(1+\frac{4}{3}\right)\theta'^{(0)}$ -$\emptyset(0)$
0.2235^{+}0.121.10.12	0.11	0.13	-1.883182	0.407212	-0.162641
0.1135^{+}0.121.10.13	0.11	0.12	-1.174292	0.454355	-0.187601
1.0235^{+}0.121.10.14	0.16	0.15	1.336129	3.472751	-1.733988
0.8335^{+}0.121.10.12	0.16	0.15	-1.036967	0.457798	-0.189428
0.8435^{+}0.121.10.12	0.15	0.15	-0.968805	0.859119	-0.190128
0.8447^{+}0.121.10.12	0.15	0.18	-0.986804	0.674789	-0.198419
0.8465^{+}0.121.10.16	0.15	0.18	-0.899416	0.813926	-0.219136
0.8465^{+}0.521.10.13	0.12	0.18	-0.899416	0.890325	-0.212516
0.8465^{+} 2 1.10.12	0.13	0.14	-0.899416	0.999035	-0.205465
0.8465^{+} 2 2.10.13	0.12	0.14	-0.899416	0.495908	-0.204663
0.8465^{+} 2 5.20.12	0.16	0.11	-0.899416	0.489917	-0.203127
0.8465^{+} 2 5.20.53	-0.6	0.11	-0.899416	0.605355	-0.232806
0.8465^{+} 2 5.30.51	-0.4	0.16	-0.899416	0.262524	-0.247508
0.8465^{+} 2 5.20.52	0.32	0.14	-0.899416	0.333820	-0.188698
0.8465^{+} 2 5.30.52	0.58	0.16	-0.899416	0.776628	-0.173999
0.8465^{+} 2 5.30.52	0.58	-0.31	-0.899416	0.798998	-0.179749
0.8465^{+} 2 5.30.52	0.55	-0.51	-0.869416	0.809242	-0.182373
0.8465^{+} 2 5.30.52	0.55	0.33	-0.889416	0.864391	-0.170847
0.8465^{+} 2 5.30.52	0.56	0.53	-0.899416	0.851370	-0.167498

Table 3: Computation of $\left(1+\frac{1}{\beta}\right)f''(0)$ and $\left(1+\frac{4}{3}\right)\theta'^{(0)}$ when. M = Kp = Nb =Nt = Bi =0:5; Pr = 10;Ec = 0:8:

4. Parametric Influence on Fluid Flow

These transformations of the governing equations into similarity variables illustrate detail interactions b/n physical parameters, that influences fluid flow. The magnetic field implements a drag force responsible for reduced velocity profiles in the boundary layer, while it also leads to an increased thermal boundary-layer due to enhanced frictional heating caused by the retarded flow. This leads to a denser thermal boundary layer, which affects heat transfer in addition implementing mass transfer processes. For the Casson fluid model, represented by parameter (β), it is shown that higher values of (β) lead to lower viscosities which leads in turn enhanced heat and mass transfer rates. Moreover, the simultaneous impacts of Brownian thermophoresis and motion play a major role in nanoparticle concentration distribution. The Sherwood number shows that binary diffusion is becoming more pronounced as N t /N b increases. This leads on the one hand to an easier thermophoretic driving (towards a cooler region) by increasing with T, while stronger Brownian motion will enhance diffusion. These results emphasize the intricate interplay among magnetic effects, fluid rheology and nanoparticle dynamics in contributing towards effective heat/mass transfer attributes of the flow.

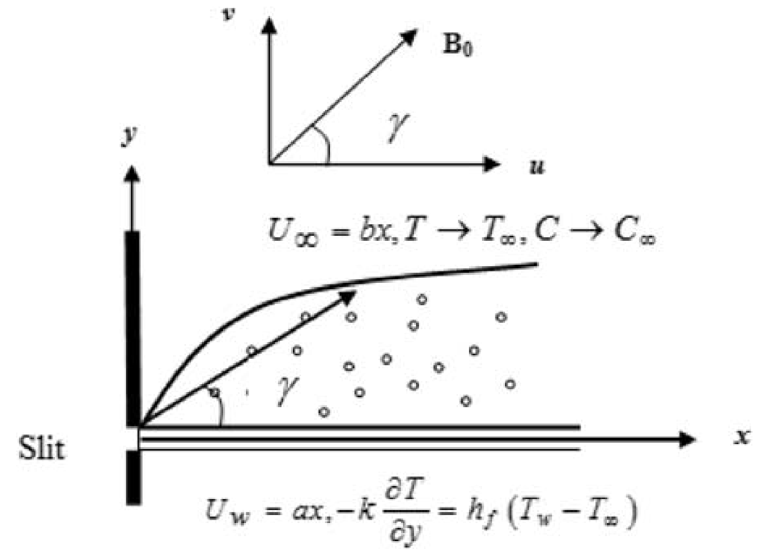

Figure 2. Flow geometry.

The fluid flow and heat transfer over a linearly stretching plate has been analyzed under various boundary conditions as they are very much essential to describe the physical system of interest. Physical velocity at the plate surface ((y = 0)) is explicitly taken to be (u = Uw = ax) in a linear stretching of the plate such that fthe local velocity grows uniformly with distance away from origin. This linear profile has a stretching rate (a) so that the fluid sticks to plate stretch momentum. Moreover, the condition v = 0 at y=0 implies that the plate is impermeable (i.e., there is no occur of fluid flow in normal to plate surface and hence prevent any actuation into a course regular with Fluid). In heat transfer the boundary condition is with conduction governed by, where (k) thermal conductivity of plate material and (q), entering or leaving to a steel flux. This property ensures that we still enforce the proper heat conduction rate at the surface, which in turn gives a physically realistic model for how heat is entering or leaving our plate. Both these boundary conditions together legitimately define the stretching behavior of a plate, impermeability in normal direction and heat transfer processes at surface

		Cu-water		Ag-water	
M	ϕ	[39]	Present	[39]	Present
0	0.05	1.10892	1.1089	1.13966	1.1397
	0.1	1.17475	1.1747	1.22507	1.2251
	0.15	1.20886	1.2089	1.27215	1.2722
	0.2	1.21804	1.2180	1.28979	1.2898

	0.50.051.29210	1.2921	1.31858	1.3186
	0.11.32825	1.3282	1.37296	1.3730
	0.151.33955	1.3396	1.39694	1.3969
	0.21.33036	1.3304	1.39634	1.3963
1	0.051.45236	1.4524	1.47597	1.4760
	0.11.46576	1.4658	1.50640	1.5064
	0.151.45858	1.4586	1.51145	1.5115
	0.21.43390	1.4339	1.49532	1.4953
2	0.051.72887	1.7289	1.74875	1.7487
	0.11.70789	1.7079	1.74289	1.7429
	0.151.67140	1.6714	1.71773	1.7177
	0.21.62126	1.6213	1.67583	1.6758

Table 4: The skin friction coefficient – $f''(0)$for various values of M and ϕ when Pr = 6.2 with Hamad [31}

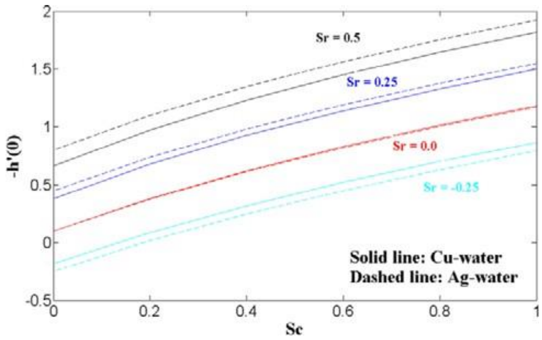

Figure 3: and d shows the influence of Sr on rate of mass transfer —h′(0) for cu-water and Ag water nanofluids at ϕ = 0.2, M = Pr = K1= Ec= γ= fw = yeqwhich indicates that absorbance increases with the increase in Schmidt number but decreases as negative impact is enhanced by significant addition factor.

Such results emphasize the importance of employing appropriate modelling for fluid flow and heat transfer since it has a direct influence on parameters which affect performance in engineering applications. Their discovery is expected to have major implications for the design and operation of industrial processes that use nanofluids. In the area of cooling technologies, where efficient heat dissipation is essential, a better understanding how parameters such as magnetic fields, fluid viscosity and nanoparticle dynamics modify heat transfer can help in developing more effective systems for coolants. In a similar vein, where mass transfer and reaction rates are of paramount importance in chemical processing and materials manufacturing this study must inform process refinement in order to maximize product quality. This work opens up further research directions by investigating the nonlinear effects and stability of these flows, allowing a better understanding to be gained for nanofluidics in more realistic conditions. Examining the interplay among these parameters at non-linear regimes and analyzing the stability of nanofluids is anticipated to provide new scientific information on improving performance and reliability in systems employing for this advanced technology. Originality/value — These studies can serve the purpose of contributing to further research and development in this area as well as help designing better-working industrial processes, leading to more performant uses of nanofluids by industry.

6. Conclusion

In this paper, numerical results are given for the study of Casson nanofluids in some various situations and analysis is done with varying parameters as caused by magnetic field effect viscous dissipation thermal radiation. The parametric studies performed in this work showed the significant impact of some crucial parameters (M, \beta) and Reynolds, Prandtl number on fluid flow distribution as well thermal characteristics. These observations help to better understand the effect of various factors on nanofluid systems and their interplay at different levels, which establish a unique realization.The outcome suggests that the magnetic field imposes a marked decelerative effort on the liquid, while simultaneously influencing velocity profiles and thermal boundary layer thickness. The Casson parameter(β) characteristic of non-ℵuid behaviour Ʊ in maintaining the viscosity, therefore a an similar heat and mass transfer rate is affected. In addition, the influence of viscous dissipation and thermal radiation on the transient heat transfer characteristics in our system is revealed along with a comparison to elucidate the system cleanup efficiency. The tables and comprehensive discussions included in the study are a great asset for engineers, students, and researchers. They provide a useful resource for developing an understanding of the optimization heat and mass transfer processes in nanofluid-based systems. This study provides insight models into the intricate physical parameters interactions, influencing performance of nanofluids in industrial applications. These findings are necessary in improving the performance, competitiveness and new advancement of cooling technologies or chemical processing as well as other operations that require specific thermal and mass transfer control. The in depth results and discussions allow the need to develop nanofluids based applications for wide range of industrial uses at advanced stages without other thoughts.

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