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Some Results on Non-Isolated Resolving Number

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Abstract:
Connected graphs G. W = {w1, w2,..., wk} is a subset of V with a predetermined order. The line vector r(v|W) = (d(v, w1), d(v, w2),..., d(v, wk)) is the measurement depicting v with regards to W for each v ∈ V. If V's vertex utilize various metrics, W resolves G. Their fundamental magnitude, dim(G), is their lowest cardinality. A resolved set W is non-isolated if its influenced subsection ⟨W ⟩ has no single vertex. The simplest connection of a non-isolated resolved set of G is nr. An nr-set for G is a non-isolated resolution set of cardinality nr(G). In this study, we prove that the chart G has a unique nr-set. We also build a 2n-vertex graph G using nr-set. W which means nr(G) = n and r(vi|W) = (1,...2, 1), where 2 is in the ith location, represents every vertex not in W. Further we established the nr-value for the highly irregular graph Hn,n and for the Wheel Wn. Also we determined the nr-value for corona product of some graphs.

Keywords: Resolution set, metric dimensions, non-isolated resolution set, number.

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1 Introduction

We exclusively discuss limited, simple, uncontrolled, connected networks in this study. The vertex graph G are V (G) edge for E(G). See [6] for fundamental symbols and nomenclature.

The distance of a shortest pathway among the two points is d(u, v). The graph G1&G2 with a single instance of G1 and |V (G1)| duplicates of G2 is the corona of G1 and G2. It's built by connecting each G2 node to the ith G1 vertex.

A Wheel Wn network is generated from a single cycle Cn through adding a new vertex v and attaching it to all the cycle's corners. The ribs of the wheels are the newly added connections. The graph Hn,n is an irregular graph defined by V (Hn,n) = {v1, v2, ..., vn; u1, u2, ..., un} and E(Hn,n) = {viuj : 1 ≤ i ≤ n, n − i + 1 ≤ j ≤ n}. A graph component A spanning subsection with the same vertex collection as G. A k-factor is a spanned k-regular subsection. In particular, a 1-aspect matches perfectly.

Resolving sets have been mentioned in the literature. Slater established these concepts in [18] and [19] and utilized finding set for resolution set. Location number loc(G) is the relationship G's minimal resolved sets. Slater [10, 11, 12] describes a resolution set as an array W of vertices in a data structure G wherein distances to W precisely define every vertices. Harary and Melter [9] separately found location numbers but called them metric dimensions.

Let W = {w1, … wk} be a structured with G & V. G's resolved set is W if various edges of G have different interpretations in W. Bases for G are resolving sets with minimal cardinality, which is the metric depth of G (dim(G)). Resolved sets have uses for coin measuring, discovering drugs, robot navigating, network exploration, graph participates, and mass mind game techniques [16, 7, 8]. Chartrand and Zhang [7] survey measurement outcomes.

By setting requirements on the smaller graphs created by a resolved set, numerous designs have been examined. These factors are well-studied, including linked and independent resolved sets [15, 17].

A resolved set W of G exists independently if no two vertex are neighboring. A resolution set W of G has connections if its induced sub network ⟨W ⟩ is a non-trivial linked sub graph of G.

As in [13], a non-isolated resolve collection was developed. A resolution setting W of G with a minimum of 2 vertex is declared non-isolated if the resulting subgraph ⟨W ⟩ contains no isolation vertex. The smallest relationship of a non-isolating resolution setting graph G is termed nr(G). Unisolated resolving sets of relationship nr(G) are termed nr-sets of G.

In [13], the nr-value and cartesian products of various graph topologies were found. Furthermore, a line G of rank n within nr(G) = k created for any combination of k and n with $2 \leq k < n - 1$. In [1], the precise nr(G) networks and standard network subdivisions are given.

Additionally, [2] discusses the correlation among nr-value and factors like $\chi(G)$ & $\Delta(G)$. We show in this work that a graph has a unique nr-set. Furthermore, we built a 2n-vertex graph G with nr-set W. such that nr(G) = n and the representation of each vertex not in W is r(vi|W) = (1, 2, .. 1, ., 1) . Also we established the nr-value for the highly irregular graph Hn,n and for the Wheel Wn. And also the nr-value for corona product of some graphs are determined.

2 Graphs with Unique nr-set

Generally, graphs contain many nr-sets. This section demonstrates that a network with a distinctive nr-set of relationship n occurs for any integer $n \geq 2$.

Theor 2.1. Any linked graph G having order $n \geq 2$ has a unique nr-set of relationship n, resulting in an isotropic graph.

Proof. Let G be any connected graph on $n \geq 2$ vertices

and V (G) = {v0, … vn−1}.

Let

$$G' = K2n-1$$

with the vertex set

$$V(G') = \{u0, u1, u2, ..., u2n - 2\}.$$

A chart H with a complete vertex v is generated from G and G′ by adding edges connecting V (G) and V (G′): Give every number i ($1 < i \leq 2n - 2$) its base 2 (binary) form. Every such i may be described as a series of k co-ordinates, or k-vectors, in which the rightmost co-ordinate is an integer (either 0 or 1) at the 20 status, the value to its immediately left is the 21 status, etc.

For integers i, j with $0 \leq j \leq n - 1$ and $0 \leq i \leq 2n - 2$, we combine ui and vj if and only if i's (2j) th binary code value is 1.

First let us prove that nr(H) = n.

Let

$$W = V(G).\ \text{Else } r(v|W) = (1, 1, ..., 1)$$

and

$$r(ui|W) = (2 - bn-1, 2 - bn-2, ..., 2 - b0).$$

$$0 \leq i \leq 2n - 2.\ \text{Also } \langle W \rangle =\sim G.$$

W resolves H non-isolatedly. Therefore, nr(H) ≤ n. Just show that nr(H) > n.

Non-isolated H-resolved sets may be W.

If vj ∈/ W for some j, $0 \leq j \leq n - 1$, else r(vj|W) = r(ui|W) for some i, $0 \leq i \leq 2n - 2$, which is a contradiction.

Therefore,

W must contain all the vertices of G.

Hence

$$|W| \geq n.\quad \text{Thus } nr(H) = n.$$

Now we have to show that there is no other nr-set for H.

Let vj be the vertex which is in W but not in W′ for some j, $0 \leq j \leq n - 1$.

Else W′ must contain one vertex from each of {us, ut} with $t - s = 2j$ for all $0 \leq s < t \leq 2n - 2$.

Therefore $|W'| \geq 2n - 1$.

The following theorem constructs a graph G on 2n triangles with a nr-set W such that nr(G) = n and the value of every vertices is nr. W is r(vi|W) = (1, , 2,… 1, ..., 1).

Theorem 2.2.

There is a chart G on 2n vertex with a nr-set W,

$$nr(G) = n,$$

and

$$r(vi|W) = (1, 1,..2…1.., 1,..., 1)$$

for each significant integers $n \geq 2$.

Proof. Let

$$G = (Kn + Kn)/1 - \text{factor}$$

and

$$V(G) = V1 \cup V2$$

wherein

$$V1 \text{ and } V2 = \{v' \ldots .v'\}.$$

Let W = {v1, v2, ..., vn}.

Else r(v′|W) = (1, 2,.., 1)

Supposed that W ⊆ V1,

else if vi ∈/ W for any i, 1 ≤ i ≤ n,

else r(vi|W) = r(v′|W) = (1, …1),

a contradiction. W must contain all vi’s, 1 ≤ i ≤ n. Hence |W | ≥ n.

Similarly

if W ⊆ V2,

else |W | ≥ n.

Now assume that W ⊆ V1 ∪ V2.

If vi, v′ ∈/ W, 1 ≤ i ≤ n,

else r(vi|W) = r(v′|W) = (1,..., 1),

a contradiction. else W must contain either vi

or v′for all i, 1 ≤ i ≤ n. Hence |W | ≥ n.

Corollary 2.3 states that no graph G fulfilling the characteristics of the foregoing exists for positives integers n ≥ 2 and k ≥ 3.

Proof.

Let n ≥ 2, k ≥ 3.

Consider G on 2n vertex with a nr-set W

such that

nr(G) = n and r(vi|W) = (1, 1,..., 1, k, 1, 1,..., 1)

where k occurs in the ith location.

Let V (G) = {u1, ,…un; v1, V1 ..., vn}

and

W = {u1, u2, ..., un}.

Now by our assumption,

d(v1, u2) = 1.

Therefore, r(v1|W) = (2, 1, 1, ..., 1), a contradiction.

3 nr-values for the graphs Hn,n and Wn

The non-isolated resolution factor for extremely erratic graph Hn,n and for the Wheel Wn.

Theorem 3.1.

For any positive integer $n \geq 4$, $nr(Hn,n) = n - 1$.

Proof. Let

$$V (Hn,n) = \{v1, v2, ..., vn; u1, u2, ..., un\}$$

and

$$E(Hn,n) = \{viuj : 1 \leq i \leq n, n - i + 1 \leq j \leq n\}.$$

Let

$$W = \{un-1, un-2; v3, v4, ..., vn-1\}.$$

Else

$$r(v1|W) = (3, 2), r(v2|W) = (1, 3, 2, ...2),$$

$$r(un|W) = (2, 2, 1, 1, ..., 1),$$

$$r(u1|W) = (2, 2, 3, 3, ..., 3)$$

and

$$r(uj|W) = (2, 2, 3, 3, ..., 3,\text{‹} ,$$

$$x\ [n-(j+2)]\text{times } 1, 1, ..., 1)$$

where $2 \leq j \leq n - 3$.

Also ⟨W is isomorphic to K −.Hence $nr(Hn,n) \leq n - 1$.

It's sufficient to show that

$$nr(H_{n,n}) > n - 1.$$

Use any non-isolated resolution set for $H_{n,n}$.

If W contains only v_i's or u_i's, else ⟨W ⟩ is a null graph, a contradiction.

Suppose that $v_n \in W$.

If u_1, u_n belongs to W ,

else removal of them from W

will result again a non-isolated set.

Thus u1 and un need not be in W.

If $v_i \notin W$ for all i,

$1 \leq i \leq n - 1$,

else $r(u1|W) = r(u_n|W)$.

Hence at least one vi, $1 \leq i \leq n - 1$ must be in W.

Now if $u_j \notin W$ for some j,

$2 \leq j \leq n - 1$,

else for all $2 \leq j \leq n - i$,

$r(u_j|W) = r(u_1|W)$

and

for all $n - i - 1 \leq j \leq n - 1$,

$r(uj|W) = r(un|W)$, a contradiction.

Therefore,

all uj’s, where $2 \leq j \leq n - 1$ must be in W.

Hence $|W| \geq 1 + 1 + n - 3 = n - 1$.

Thus $|W| \geq n - 1$.

Similarly,

if un ∈ W,

else $|W| \geq n - 1$.

Now let

$v_n \in/ W$.

Suppose $v_1 \in W$,else un must be in W,since $N(v_1) = \{u_n\}$.

Hence as discussed above $|W| \geq n - 1$. Similarly,

if u1 ∈ W ,

else $|W| \geq n - 1$.

Now let v_n, v_1, u_1, u_n are all not in W.

If $v_2, u_{n-1} \notin W$,

else $r(v_1|W) = r(v_2|W)$

and

$r(u_n|W) = r(u_{n-1}|W$ be contract.

If $v_2 \in W$,

else $r(u_n|W) = r(u_{n-1}|W)$,

representation of the vertices v_1, u_2 and u_n.

Thus $u_{n-1} \in W$.

Now

$r(v_2|W) = r(v_3|W)$ and $r(u_n|W) = r(u_{n-2}|W)$

and u_{n-2}. and u_n, u_{n-2}.

Hence

$$u_{n-2} \in W. \text{ If } v_i \notin W \text{ for some } i,\ 3 \leq i \leq n-1,$$

else

$$r(v_i|W) = r(v_{i+1}|W) \text{ unless } u_{n-i} \in W.$$

Thus either v_i or u_{n-i} belongs to W for all i, $3 \leq i \leq n-1$.

Therefore,

$$|W| \geq 2+n-3 = n-1. \text{ Hence } |W| \geq n-1.$$

Next we find the nr- value for the Wheel W_n. It can be easily verify that,

$$nr(W_3) = 3,\ nr(W_4) = 2,\ nr(W_5) = 2 \text{ and } nr(W_9) = 4.$$

For $n \geq 6$ and$n \neq 9$, we define a formula for the wheel W_n in the following theorem.

Theorem 3.2.

For any integer with positive $n \geq 6$ and $n \neq 9$:

$$nr(Wn) = \begin{cases} \frac{2n}{5} & \text{if } n \equiv 1,3 \text{ (modulo 5)} \\ \frac{2n}{5}+1 & \text{if } n \equiv 0,2,4 \text{ (modulo 5)} \end{cases}$$

Proof.

Let V (Wn) = {v, v1, v2, ..., vn}.

Case1.

Let n ≡ 0, 1(modulo 5).

Take W= {v, v1, v5, v5i+2, v5(i+1) : 1 ≤ i ≤ 6}.

Else for n≡ (modulo 5), |W| = $\frac{2n}{5}$+1

and for n≡ 1 (modulo 5),

|W| = $\frac{2n}{5}$ and r(v|W)= (1,1,2,2,…,2,1)

where 1 appears in the first, second, and $\frac{2n}{5}$ th places.

Case 2.

Let n≡2,4 (modulo 5)

Take W = {v, v1, v5, $v5_{i+2,}$ v5(i+1), vn : 1 ≤ i ≤ 6 .

Else |W | =$\frac{2n}{5}$+1 and $r(v_{n-1}|W)$ = (1,2,2,…,2,1,1)

where 1 appears in the first. $\frac{2n}{5} - th$, and $\frac{2n}{5} + 1 - th$ places.

Case 2.

Let $n \equiv 3$ (modulo 5)

$$\text{Take } W = \{v, v1, v5, v5_{i+2}, v5(i+1), vn : 1 \le i \le 6 .$$

$$\text{Else } |W| = \frac{2n}{5} \text{ and } r(v_{n-1}|W) = (1,2,2,\ldots,2,1,1)$$

where 1 appears in the first. $\frac{2n}{5} - th$, and $\frac{2n}{5} - 1 - th$ places, and $r(v_W)=(1,1,2,2,\ldots,2,1)$, where 1 appears in the first, second, and $\frac{2n}{5} - th$ places.

In all the above cases,

$$r(v_2|W) = (1, , ..., 2), r(v3|W) = (1, , ..., 2), r(v_4|W) = (1, 2,1, 2, 2, ..., 2), r(v_{5i+1}|W)$$

where 1 appears in the first and $(5i)^{th}$ and $(5i + 2)^{th}$ places,

$$r(v_{5i+3}|W) = (1, 2, 2, ..., 2, 1, 2, 2, ..., 2)$$

where 1 appears in the first and $(5i+2)^{th}$ places and $r(v_{5i+4}|W) = (1, 2, 2, ..., 2, 1, 2, 2, ..., 2)$ where 1 appears in the first and $(5i)^{th}$ places, $1 \le i \le 6$'$\underline{n}$'.

Also for

$$n \equiv 1, 3(\text{ modulo } 5), \langle W \rangle \text{ is isomorphic to } K1,\lfloor 2n \rfloor$$

and for

$$n \equiv 0, 2, 4(\text{ modulo } 5), \langle W \rangle \text{ is isomorphic to } K1,\lceil 2n \rceil.$$

Hence

$$n_r(Wn) \le 5 + 1, \text{ if } n \equiv 0, 2, 4(\text{ modulo }, 2n , 5)$$

and

$$nr(Wn) \le 5, \text{ if } n \equiv 1, 3(\text{ modulo } 5).$$

shows n = 6, 7, 8, and 14 examples.

Vertices in non-isolated resolution sets are expanded in the graph.

Consider the vertex vi,

$$1 \le i \le n. \text{ If } vi \in W ,$$

else one among

$$vi+2, vi+3, vi+4 \in W.$$

Since

$$vi+4 \in W , r(vi+5|W) = r(vi+3|W)$$

for any W unless

$$vi+6 \in W .$$

So we conclude that vi+6 ∈ W .

Already

$$r(vi+5|W\ )\ \ /=\ r(vi+7|W\ )$$

Hence

$$vi+9 \in W$$

otherwise

$$r(vi+2|W\ ) = r(vi+8|W\ ).$$

Now, the vertex v must belong to W otherwise ⟨W ⟩ will contain the isolated vertices. Therefore, any non-isolated resolved sets it contain $\frac{2n}{5}$ vertices for n ≡ 1, 3 (modulo 5) and $\frac{2n}{5}$ + 1 vertices for n ≡ 0, 2, 4 (modulo 5). Hence

$$|W| \geq 0, 2, 4 \text{ (modulo 5).}$$

$$\frac{2n}{5}5\ ,$$

$$\text{if } n \equiv 1, 3 \text{ (modulo 5)}$$

and

$$|W| \geq \frac{2n}{5} + 1, \text{ if } n \equiv \frac{2n}{5}$$

Thus we conclude that

$$n_r(Wn\ , + 1, \text{ if } n \equiv 0, 2, 4 \text{ (modulo 5). }) = \frac{2n}{5}, \text{ if } n \equiv 1, 3 \text{ (modulo 5)}$$

and

nr(W) =nr-value of G◦Km, G◦Km and G◦K1,

m for m ≥ 2.

Theorem 3.3.

Consider G, a graph that is connected with order n ≥ 2.

Else,

for each significant integer m ≥ 3, nr(G ◦ Km) = nm.

Proof.

Let H = G ◦ Km and V (H) = {vi; ui1 , ui2 , ..., uim : 1 ≤ i ≤ n}

where V (G) = {vi : 1 ≤ i ≤ n} and ui , ui , ..., ui are the vertices in the ith copy of Km, 1 ≤ i ≤ n.

Let W = {vi, uij : 1 ≤ i ≤ n, 1 ≤ j ≤ m − 1}.

Else vi is the only vertices which is at a distance 1 from uim , for all 1 ≤ i ≤ n. Hence the representation of all ui , 1 ≤ i ≤ n differs at least in the ith place. nr(H) ≤ nm.

If u_{ij} , u_{ik} $\notin$ W for some i, j, k such that

$$1 \leq i \leq n \text{ and } 1 \leq j \neq k \leq m,$$

$$\text{else } r(u_{ij} |W) = r(u_{ik} |W)..$$

Also W must contain all v_i's, $1 \leq i \leq n$,

otherwise ⟨W ⟩ will contain u_{i1} as an isolated vertex.

$$\text{Thus } |W| \geq nm. \text{ Hence } nr(H) \geq nm.$$

Therefore,

$$nr(H) = nm.$$

Theorem 3.3.

If G is a connected graph with at least 2 vertices, and you create a new graph by combining G with multiple copies of a smaller graph K_m (where m is a positive integer greater than or equal to 3), else the minimum size of a non-isolated resolving set for this new graph is G × n×m.

Proof.

Consider the new graph obtained by adding to G copies of K m. In this graph, each vertex of G is adjacent to every vertex in one copy of K m.

We will exhibit a set of vertices that consists of one vertex of G and all but one vertex in each copy of K_m.

This set is large enough to distinguish all other vertices in the graph and therefore resolves the graph.

Let us now try to show that this set is the smallest possible.

Consider any other resolving set.

Suppose that the set does not contain enough vertices from each copy of K_m., some vertices of that copy would be indistinguishable, contradicting the defining property of a resolving set.

The set must therefore contain at least m×n vertices, so the minimum possible size is exactly that.

Theorem 3.4.

If G is a connected graph with at least 2 vertices, and you make a new graph by combining G with multiple copies of a smaller graph K (where, if m is a positive integer greater or equal than 3, else the minimum cardinality of a non-isolated resolving set for this new graph is nn×(m−1).

Proof.

In this case we will deal with another set of vertices, which contains all but one vertices from each copy of K.

This set still can distinguish all vertices in the graph and resolve it.

To see this set is minimal, consider any other resolving set.

If it contains too few vertices from each copy of K_m, else it won't be able to distinguish some pairs of vertices.

Hence the set must contain at least (m−1)nn×(m−1) vertices, which shows this is the minimum possible size.

Theorem 3.5.

If Let G be a connected graph with at least 2 vertices, and combine it with multiple copies of a star graph. A star graph stands for a special kind of small graph where one central vertex is connected to all others.

The minimum size of a non-isolated resolving set in this new graph would be equal to the total number of vertices of the combined graph.

Proof.

In such a graph, each vertex of G is connected to the center of a star graph.

The resolving set should contain enough vertices from each star so that all vertices are distinguishable.

If there aren't enough vertices else some vertices might not be distinguishable.

Thus the set will have at least the total number of vertices in the graph confirming that this is the minimum size.

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