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Rayleigh Waves Propagation in a Micropolar Viscothermoelastic Half-Space with Impedance Boundary Conditions

Mamatha Kumari[1], E. Rama[2], Rajneesh Kumar[3], P Vikas Singh[4]

[1]Assistant Professor, St. Martin's Engineering College, Hyderabad. kumari4bu@gmail.com

[2]Associate Professor, Osmania University, Hyderabad. ramamathsou@gmail.com

[3]Department of Mathematics, Kurukshetra University, Kurukshetra 136119, India

rajneesh_kuk@rediffmail.com

[4]Department of Mathematics, Osmania University, pulsinghvikas@gmail.com

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Abstract

This paper deals with the propagation of Rayleigh waves in a micropolar viscothermoelastic half space with impedance boundary conditions. The boundary of the half space is thermally insulated/isothermal and it is assumed that normal traction, shear traction and shear couple traction at the surface, varies linearly with normal, tangential components of displacement and microrotation respectively. The secular equation for Rayleigh wave with impedance boundary conditions is obtained and this equation is in agreement with the classical secular equation for elastic solid with traction free boundary conditions when micropolar, thermal and impedance parameters are removed. The non-dimensional speed of Rayleigh wave is computed as a function of impedance parameters and presented graphically for a particular micropolar thermoelastic material.

Keywords: Micropolar thermoelasticity, Rayleigh waves, Impedance boundary conditions, Secular equation.

1.Introduction

Eringen's [1]micropolar theory of elasticity is now well known due to its possible utility in examining the deformation properties of materials such as cellular solids, polymers, composite fibrous, granular material, masonry, bones and many more with microstructures. This theory takes into account the intrinsic rotation along with linear displacement in the materials possess microstructure and the motion is governed by six degrees of freedom, three of microrotation and three of classical translation. The classical theory of elasticity, which ignores the microrotation degrees of freedoms, can explain the behaviour of common solid materials like coal, concrete etc. This theory is inadequate to explain the behaviour of materials with inner microstructure such as polycrystalline and materials with fibrous or coarse grain. Therefore, micropolar theory was developed to explain the microscopic motion and long-range interactions in solids.

As mechanical and thermal fields are associated in almost all practical engineering problems where, the application of mechanical forces can change the temperature of the system. Keeping this important interaction in view, Nowacki[2] and Eringen[3]extended the micropolar theory by including thermal effects and presented linear theory of micropolar thermoelasticity. Tauchert, Claus Jr and Ariman[4] developed the linear theory of micropolar thermoelasticity and formulate the constitutive equations. Various problems on micropolar thermoelasticity have been investigated extensively by researcher due its applications in various fields like earthquake, nuclear reactors, aeronautics, astronautics and modern sensor devices.

The generalized theory of thermoelasticity is a modified version of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of impossible phenomena of infinite velocity of thermal signals in the classical coupled theory of thermoelasticity. Lord and Shulman [5] and Green and Lindsay[6]includes the concept of thermal relaxation time and eliminate this paradox of infinite velocity of thermal signals. Based upon generalized theory of thermoelasticity given by Green and Lindsay, Boschi and Ieşan [7] proposed a generalized theory of linear micropolar thermoelasticity that admits the possibility of second sound effect. Ciarletta[8]established the finite speed of thermal waves by using theory of micropolar thermoelasticity without energy dissipation. Based upon Lord and Shulman theory[5]Sherief, Hamza and EI-Sayed [9],derived the generalized equation for the linear theory of micropolar thermoelasticity.

A comprehensive study is available on the phenomenon of wave propagation in micropolar-generalized thermoelastic solid because of their practical applicability in the various fields of science and technology such as, seismology, acoustics, aerospace and submarine structures. Surface waves due to their destructive nature during earthquake are of particular importance in the study of seismology. Lord Rayleigh[10]was the first to study the wave propagating along the isotropic elastic solid and such waves after his name are known as Rayleigh waves. Several researchers have explored the concept of Rayleigh waves in different type of elastic materials. For example, Lockett [11]discussed the effects of thermal properties of an isotropic thermoelastic material on velocity of Rayleigh waves. Kumar and Singh[12] discussed about the existence Rayleigh wave in micropolar generalized thermoelastic half space with stretch. Rao and Reddy[13]studied the Rayleigh type wave propagation in a micropolar cylindrical surface. Kumar, Kaur and Rajvanshi[14]investigated the propagation of Lamb waves in micropolar-generalized thermoelastic solid with two temperatures bordered with layer of inviscid liquid. Kumar and Partap[15] studied propagation of Rayleigh Lamb waves in a micropolar elastic cylindrical plate. Sharma and Khator [22,24] examined some problems of power generation due to renewable sources. M.Marin [23,25,] explored some problems in bioheat thermoelastic, Cosserat thermoelastic media and non-local thermoelastic materials. Kaushal et al [26] investigated boundary value problem in frequency domain by considering modified Green- Lindsay thermoelastic medium. Kumar and Devi [27] analysed interaction due to hall current and rotation in modified couple stress elastic half-space subjected to ramp-type loading.

The boundary conditions in almost all the problems related to Rayleigh waves are considered as a traction free surface that is stresses vanishes on the surface. The possibilities of other type of boundary conditions are rarely consider in seismology or geophysics but there are other fields of physics like electromagnetism and acoustics, where it is common to use impedance boundary conditions. The impedance boundary conditions prescribed on the boundary is the linear combination of the unknown function and their derivatives. Tiersten [16]encountered these types of boundary conditions while studying the wave propagation in an isotropic elastic solid coated with thin film of different material. Malischewsky [17]modified the Tiersten's conditions in terms of stresses and displacement and obtained the secular equation for Rayleigh waves. Godoy, Duran and Nedelec [18]proved the existence of surface waves in an elastic half space with impedance boundary conditions and derived the secular equation with these conditions. Vinh and Hue[19] used impedance boundary conditions to investigate Rayleigh waves in an orthotropic and monoclinic half space. Recently Singh[20] studied about the Rayleigh wave in a thermoelastic solid half space subjected to impedance boundary conditions. Sharma and Khator [22,24] examined some problems of power generation due to renewable sources. M.Marin [23,25,27] explored some problems in bioheat thermoelastic, Cosserat thermoelastic media and non-local thermoelastic materials. Kaushal et al [31] investigated boundary value problem in frequency domain by considering modified Green- Lindsay thermoelastic medium. Kumar and Devi [26] analyzed interaction due to hall current and rotation in modified couple stress elastic half-space subjected to ramp-type loading.

Rayleigh waves are extremely useful for material characterization and to remove defects in the objects, as these are very sensitive to surface defects. Very few papers on Rayleigh waves with impedance boundary conditions are available but this concept has not been used in micropolar thermoelastic material. In this paper the propagation Rayleigh waves in a micropolar thermoelastic half space with impedance boundary condition has been investigated. Secular equation for thermally insulated and isothermal surface is obtained and this equation coincides with the secular equation of Rayleigh waves in thermoelastic solid when the micropolar effect is removed. On removing the micropolar effects, impedance parameters and thermal effects this equation reduces to famous secular equation of Rayleigh wave in isotropic elastic solid with traction free boundary conditions. Effect of micropolarity present in the medium on the phase velocity is highlighted through comparative study with respect impedance parameter.

2. Basic equations

Following Eringen[1], the governing equations for homogeneous, isotropic micropolar viscothermoelastic solid in absence of body forces and body couples are

$$(\mu^* + K^*)\nabla^2\vec{u} + (\lambda^* + \mu^*)\nabla(\nabla.\vec{u}) + k^*(\nabla \times \overrightarrow{\phi}) - \nu\nabla T = \rho\left(\frac{\partial^2\vec{u}}{\partial t^2}\right) \tag{1}$$

$$(\alpha^* + \beta^* + \gamma^*)\,\nabla\left(\nabla.\vec{\phi}\right) - \gamma^*\nabla \times \left(\nabla \times \vec{\phi}\right) + k^*(\nabla \times \vec{u}) - 2k^*\vec{\phi} = \rho j\left(\frac{\partial^2\vec{\phi}}{\partial t^2}\right) \tag{2}$$

where $\vec{u}$ is the displacement vector, ρ is the density of the material, j is the microinertia, $\vec{\phi}$ is the microrotation vector, $\lambda, \mu, k, \alpha, \beta, \gamma, \lambda^*, \mu^*, k^*, \alpha^*, \beta^*, \gamma^*$ are material constants and

$$\lambda^* = \lambda + \lambda_\nu \frac{\partial}{\partial t}, \quad \mu^* = \mu + \mu_\nu \frac{\partial}{\partial t},$$

$$k^* = \kappa + \kappa_\nu \frac{\partial}{\partial t}, \alpha^* = \alpha + \alpha_\nu \frac{\partial}{\partial t},$$

$$\beta^* = \beta + \beta_\nu \frac{\partial}{\partial t}, \gamma^* = \gamma + \gamma_\nu \frac{\partial}{\partial t}$$

The constitutive relations are given by

$$\sigma_{ij} = \lambda^* u_{r,r}\delta_{ij} + \mu^*\left(u_{i,j} + u_{j,i}\right) + k^*\left(u_{j,i} - \epsilon_{ijr}\phi_r\right) - \nu T\delta_{ij} \tag{3}$$

$$m_{ij} = \alpha^*\phi_{r,r}\delta_{ij} + \beta^*\phi_{i,j} + \gamma^*\phi_{j,i} \tag{4}$$

where $(i, j, r = 1,2,3)$, σ_{ij} is the stress tensor,m_{ij} is the couple stress tensor and δ_{ij} is the kronecker delta.

Following Lord and Shulman (1967), the heat conduction equation is

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\vec{u} \tag{5}$$

where K^* is the coefficient of thermal conductivity, $\nu = (3\lambda + 2\,\mu + K)\alpha_t$, C^* is the specific heat at constant strain, α_t is the coefficient of thermal linear expansion, T is the change in temperature of the medium at any time, T_0 is the reference temperature of the body and$\tau_0$is the thermal relaxation time

3. Formulation of the problem

We consider a homogeneous and isotropic micropolar viscothermoelastic half space at uniform temperature T_0 in the undeformed state. Origin is placed at the plane surface and y-axis pointing vertically downward into the half space. The direction of propagation of the waves is considered along x-axis so that all particles vibrating on a line parallel to z-axis are equally displaced. Therefore, all the field quantities will be independent of z-coordinates. For the two-dimensional problem, we assume the components of the displacement $\vec{u}$ and microrotation vector $\vec{\phi}$ of the form

$$\vec{u} = (u\,, v\,, 0)\ ,\ \vec{\phi} = (0\,, 0\,, \phi) \tag{6}$$

Using (6), equation (1) and (2) can be written as

$$(\lambda^* + 2\mu^* + k^*)\frac{\partial^2 u}{\partial x^2} + (\mu^* + k^*)\frac{\partial^2 u}{\partial y^2} + (\lambda^* + \mu^*)\frac{\partial^2 v}{\partial x\partial y} + k^*\frac{\partial\phi}{\partial y} - \nu\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{7}$$

$$(\lambda^* + 2\mu^* + k^*)\frac{\partial^2 v}{\partial y^2} + (\mu^* + k^*)\frac{\partial^2 v}{\partial x^2} + (\lambda^* + \mu^*)\frac{\partial^2 u}{\partial x\partial y} - k^*\frac{\partial\phi}{\partial x} - \nu\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2} \tag{8}$$

$$\gamma^*\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + k^*\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2k^*\phi = \rho j\frac{\partial^2\phi}{\partial t^2} \tag{9}$$

Using Helmholtz's representation, the displacement components u and $v$can be written in terms of potential functions as

$$u = \frac{\partial\phi_1}{\partial x} + \frac{\partial\psi_1}{\partial y}\,,\ v = \frac{\partial\phi_1}{\partial y} - \frac{\partial\psi_1}{\partial x} \tag{10}$$

Substituting (10) in equations (5) and (7)-(9), we obtained

$$(\lambda^* + 2\mu^* + k^*)\nabla^2\phi_1 - \nu T = \rho\frac{\partial^2\phi_1}{\partial t^2} \tag{11}$$

$$(\mu^* + k^*)\nabla^2\psi_1 + k^*\phi = \rho\frac{\partial^2\psi_1}{\partial t^2} \tag{12}$$

$$\gamma^*\nabla^2\phi - 2k^*\phi - k^*\nabla^2\psi_1 = \rho j\frac{\partial^2\phi}{\partial t^2} \tag{13}$$

$$K^*\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C^* T + \nu T_0\nabla^2\phi_1) \tag{14}$$

4. Solution of the Problem

The surface wave solutions of the equations (11)-(14) may be consider as

$$\{\phi_1, \psi_1, T, \phi\} = \{\overline{\phi_1}(y), \overline{\psi_1}(y), \bar{T}(y), \bar{\phi}(y)\}e^{iK(x-ct)} \tag{15}$$

where$c$ is the phase velocity, K is the wave number, $\omega = Kc$ is the circular frequency. It is assumed that the Rayleigh surface waves possibly damped in time, propagating along x-axis with wave speed $Re(c) = V > 0$and $Im(c) \leq 0$.

Using (15) in the equations (11) – (14), we have

$$[D^4 - AD^2 + B](\overline{\phi_1}(y), \bar{T}(y)) = 0 \tag{16}$$

$$[D^4 - A'D^2 + B'](\overline{\psi_1}(y), \bar{\phi}(y)) = 0 \tag{17}$$

Here

$$D = \frac{d}{dy}, \; A = K^2\left[2 - \frac{c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right)}{A_1}\right], \; B = K^4\left[\frac{A_1 - c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right) + \frac{c^4}{c_1^2}}{A_1}\right]$$

$$A' = K^2\left(1 - \frac{c^2}{c_2^2}\right) + K^2 - \frac{K^2 c^2 \rho j}{\gamma} + \frac{2k^*}{\gamma} - \frac{k^{*2}}{\gamma(\mu + K)} \tag{18}$$

$$B' = K^2\left(K^2 - \frac{K^2 c^2 \rho j}{\gamma} + \frac{2k^*}{\gamma}\right)\left(1 - \frac{c^2}{c_2^2}\right) - \frac{K^2 k^{*2}}{\gamma(\mu + K)}$$

$$c_1^2 = \frac{\lambda^* + 2\mu^* + k^*}{\rho}, \; c_2^2 = \frac{\mu^* + k^*}{\rho}, \tau^* = \tau_0 + \frac{i}{\omega}, \; A_1 = \frac{K^*}{\rho C^* \tau^*}, \; A_2 = \frac{\nu^2 T_0}{\rho^2 c_1^2 C^*}$$

$$(\lambda^*, \mu^*, \alpha^*, \beta^*, \gamma^*, \kappa^*) = (\lambda, \mu, \alpha, \beta, \gamma, \kappa) * (1 - \iota Q_i) \quad (i = 1 - 6)$$

$$Q_1 = \omega\left(\frac{\lambda_\nu}{\lambda}\right), Q_2 = \omega\left(\frac{\mu_\nu}{\mu}\right),$$

$$Q_3 = \omega\left(\frac{\alpha_\nu}{\alpha}\right), \; Q_4 = \omega\left(\frac{\beta_\nu}{\beta}\right),$$

$$Q_5 = \omega\left(\frac{\gamma_\nu}{\gamma}\right), Q_6 = \omega\left(\frac{\kappa_\nu}{\kappa}\right),$$

Using the radiation conditions $\overline{\phi_1}(y), \overline{\psi_1}(y), \bar{T}(y), \bar{\phi}(y) \to 0$ as $y \to \infty$ on the general solutions of the equations (16) and (17) and using (15), we obtained

$$\phi_1 = (B_1 e^{-Kb_1 y} + B_2 e^{-Kb_2 y}) e^{iK(x-ct)} \tag{19}$$

$$\psi_1 = (B_3 e^{-Kb_3 y} + B_4 e^{-Kb_4 y}) e^{iK(x-ct)} \tag{20}$$

$$T = (r_1 B_1 e^{-Kb_1 y} + r_2 B_2 e^{-Kb_2 y}) e^{iK(x-ct)} \tag{21}$$

$$\phi = (r_3 B_3 e^{-Kb_3 y} + r_4 B_4 e^{-Kb_4 y}) e^{iK(x-ct)} \tag{22}$$

where

$$b_1^2 + b_2^2 = \frac{A}{K^2},\ b_1^2 b_2^2 = \frac{B}{K^4},\ b_3^2 + b_4^2 = \frac{A'}{K^2},\ b_3^2 b_4^2 = \frac{B'}{K^4} \tag{23}$$

$$\begin{cases} r_i = K^2 \left[\dfrac{(b_i^2 - 1)(\lambda^* + 2\mu^* + k^*) + \rho c^2}{\nu} \right], & (i = 1,2) \\ r_j = \dfrac{K^2(\mu^* + k^*)}{k^*} \left[1 - \dfrac{c^2}{c_2^2} - b_j^2 \right], & (j = 3,4) \end{cases} \tag{24}$$

andB_1 , B_2 , B_3 and B_4 are arbitrary constants.

5. Boundary conditions and secular equation

The general form of impedance boundary conditions in two dimensions in terms of displacements and stresses given by Malischewsky (1988) can be written as $\sigma_{i2} + \epsilon_i u_i = 0$, for $y = 0$,where $\epsilon_i$are the impedance parameters and have the dimensions of stress/length. For elastic half space, Godoy, Duran and Nedelec (2012), expressed $\epsilon_i$as$\epsilon_i = \omega Z_i$. Here Z_i are impedance real valued parameters, has dimensions of stress/velocity and $\omega = kc$is the circular frequency. Here the impedance boundary conditions at the surface $y = 0$of a micropolar thermoelastic solid are consider as $\sigma_{2i} + \omega Z_i u_i = 0$,which can be written as

$$\sigma_{21} + \omega Z_1 u = 0\ ,\quad \sigma_{22} + \omega Z_2 v = 0\ ,\ m_{23} + \omega Z_3 \phi = 0\ ,\ \frac{\partial T}{\partial y} + hT = 0 \tag{25}$$

where $h \to 0$ corresponds to thermally insulated surface and $h \to \infty$ corresponds to isothermal surface.

Imposing boundary conditions (25) on the surface $y = 0$, we get a system of four homogeneous equations. For a non-trivial solution, the determinant of the coefficients B_1 , B_2 , B_3 and B_4 must vanishes which yields the following secular equation for the velocity of propagation of the Rayleigh waves

$$m_1[T_1(l_2 n_4 - n_2 l_4) - T_2(l_1 n_4 - n_1 l_4)] = m_2[T_1(l_2 n_3 - n_2 l_3) - T_2(l_1 n_3 - n_1 l_3)] \tag{26}$$

where

$$l_i = V_1 Z_1^* - b_i - \left(1 + \frac{k^*}{\mu}\right) b_i\ , \qquad (i = 1,2)$$

$$l_j = V_1 Z_1^* b_j - 1 - \left(1 + \frac{k^*}{\mu^*}\right)\left(1 - \frac{c^2}{c_2^2}\right)\ , \quad (j = 3,4)$$

$$n_i = 2 + \frac{k^*}{\mu^*} - V_1^2 - V_1 Z_2^* b_i \ , \qquad (i = 1,2)$$

$$n_j = \left(2 + \frac{k^*}{\mu^*}\right) b_j - V_1 Z_2^* \ , \qquad (j = 3,4)$$

$$m_1 = (\mu^* V_1 Z_3^* - \gamma^* b_3)\left(1 - \frac{c^2}{c_2^2} - b_3^2\right), m_2 = (\mu V_1 Z_3^* - \gamma^* b_4)\left(1 - \frac{c^2}{c_2^2} - b_4^2\right)$$

$$V_1 = \sqrt{\frac{\rho c^2}{\mu}} \ , Z_i^* = \frac{Z_i}{\sqrt{\rho\mu}} \ ,(i = 1,2,3)$$

For thermally insulated surface

$$T_i = b_i\left[\left(2 + \frac{\lambda^* + k^*}{\mu^*}\right)(b_i^2 - 1) + V_1^2\right] \ , (i = 1,2)$$

For isothermal surface

$$T_i = \left[\left(2 + \frac{\lambda^* + k^*}{\mu^*}\right)(b_i^2 - 1) + V_1^2\right] \ , (i = 1,2)$$

6. Particular cases

1) In the absence of micropolar and viscous effects, the equation (26) reduces to the secular equation for the phase velocity of Rayleigh waves in a thermoelastic half space with impedance boundary conditions. Neglecting micropolar constants ($K = j = 0$) in the condition (18), we obtained

$$A' = k^2\left(1 - \frac{c^2}{c_2^2}\right) + k^2 \ , \ B' = k^4\left(1 - \frac{c^2}{c_2^2}\right)$$

Using equation (23), we get

$$b_3^2 = 1 - \frac{c^2}{c_2^2}, b_4^2 = 1 \ , m_1 = 0 \text{ and } m_2 \text{ be a non-zero value}$$

Consequently, the secular equation (26) reduces to

$$l_3(n_1T_2 - n_2T_1) - n_3(l_1T_2 - l_2T_1) = 0 \qquad (27)$$

The equation (27) coincides with the secular equation, obtained by author Singh[20] for Rayleigh waves in thermoelastic solid half space with impedance boundary conditions.

2) Further equation (27) reduces to secular equation for Rayleigh wave velocity with traction free boundary conditions when $Z_i^* = 0$, $(i = 1,2,3)$

3) If we neglect the impedance parameter, micropolarity, viscosity and thermal effects from the model i.e. $k^* = j = Z_1^* = Z_2^* = Z_3^* = \nu = 0$,the equation (26) reduces to

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1 - \frac{c^2}{c_1^2}}\sqrt{1 - \frac{c^2}{c_2^2}} \qquad (28)$$

where $c_1^2 = \frac{\lambda^* + 2\mu^*}{\rho}$, $c_2^2 = \frac{\mu^*}{\rho}$

Equation (28) is the well-known dispersion equation for the phase velocity of Rayleigh waves in classical elastic half space

7. Numerical results and discussions

To illustrate the theoretical results numerical computations have been carried out and non-dimensional Rayleigh wave speed has been calculated in a micropolar thermoelastic solid. The aluminium epoxy composite is taken as a micropolar thermoelastic solid and following Gauthier[21],the values of relevant physical constants of this material are

$\rho = 2.19 \times 10^3 kg/m^3$, $\lambda = 7.59 \times 10^{10} N/m^2$, $\mu = 1.89 \times 10^{10} N/m^2$,

$$K = 0.0149 \times 10^{10} N/m^2$$

$\alpha = 0.01 \times 10^6 N, \beta = 0.015 \times 10^6 N, \gamma = 0.268 \times 10^6 N$, $j = 0.196 \times 10^4 m^2$, $K^* = 0.492 \times 10^2 W/m\,K$, $C^* = 1.89 \times 10^{10} J/kg.K.$, $\tau_0 = 0.5 \times 10^{-10} s, T_0 = 298K, \alpha_t = 2.36 \times 10^{-6} K^{-1}$

secular equation (26) using the functional iteration method, assuming that c is a complex constant parameter with Re(c)=V ≥0. The graphic representations of the effects of viscosity, impedance parameters, and the Rayleigh wave speed's dependence on wave number are illustrated in Fig. 1 through Fig. 6.Figs. 1–3 illustrate the impact of viscosity on the non-dimensional Rayleigh wave speed V_1 in relation to impedance parameters Z. Figures 4 through 6 illustrate the variations in wave speed $V_1$with regard to the impedance parameter in a micropolar thermoelastic half space under thermally insulated and isothermal boundary conditions.

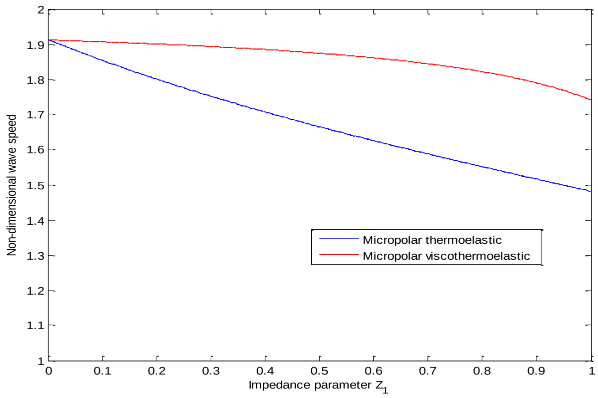

Fig. 1. Viscosity effects w.r.t. Impedance parameter Z_2 on non-dimensional speed $V_1$of Rayleigh wave.

The effect of viscosity parameter in the non-dimensional speed V_1 is significant pertinent and is noticed evidently from the figs (1)- (3)

In fig (1) the non-dimensional wave speed V_1 has been depicted against the non-dimensional impedance parameter Z_1 at constant frequency $\omega = 10\ rad/s$ and retaining the boundary free of normal and tangential couple traction $Z_2 = 0,\ Z_3 = 0$. It is evident that due to viscosity V_1 contains higher magnitude.

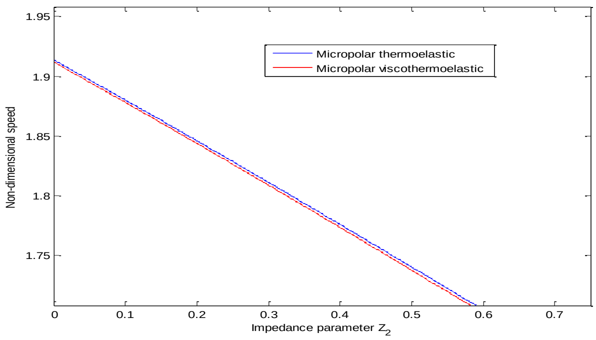

Fig. 2. Viscosity effects w.r.t. impedance parameter Z_2 on non-dimensional speed V_1 of Rayleigh wave

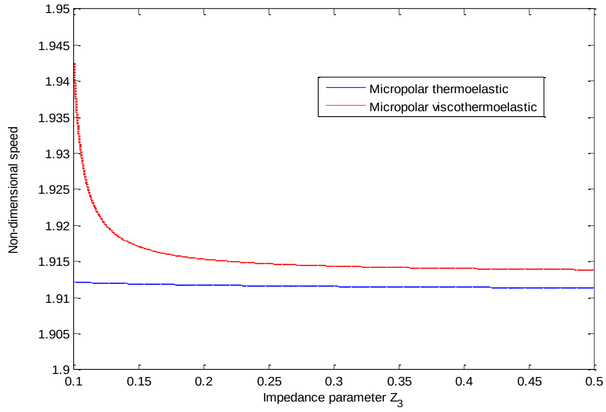

Fig. 3. Viscosity effects w.r.t. Impedance parameter Z_3 on non-dimensional speed V_1Rayleigh wave

Fig (2) and Fig (3) reveals the variations of wave speed V_1 with respect to non-dimensional impedance parameters $Z_2 and\ Z_3$ respectively keeping the other two impedance parameters fixed at zero value. From both the figs it is evidently visible that viscosity decreases and increases the wave speed respectively.

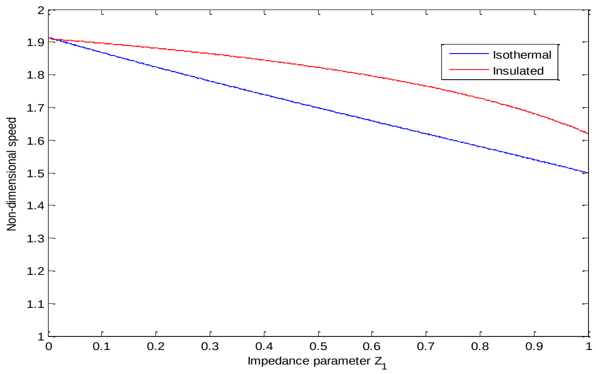

Fig4. Variation of non-dimensional wave speed V_1w.r.t. impedance parameter Z_1 in a micropolar viscothermally insulated and isothermal half space.

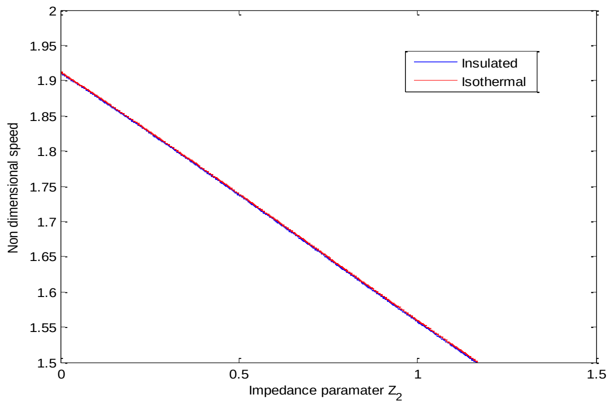

Fig.5. Variation of non-dimensional wave speed V_1w.r.t. impedance parameter Z_2 in a micropolar thermally insulated and isothermal half space

Fig 4. depicts the non-dimensional wave speed of Rayleigh wave as a function of impedance parameter Z_1 when the boundary is free from normal and couple traction $(Z_2 = 0, Z_3 = 0)$. Comparison of V_1 has been determined when the solid half space is due to thermally insulated and Isothermal boundary restrictions. It is evident that wave speed V_1 contracted in case of isothermal surface(curve-2) as contrast to thermally insulated surface(curve-1) for some value of impedance parameter Z_1 where $(Z_2 = Z_3 = 0, \omega = 10\ rad/\sec)$. It is noticed from the graph that wave speed contracted gradually with the increase in impedance parameters Z_1 for the region 0≤ $Z_1 \leq 1$ for both the conditions. The similar stencil of variant of wave speed is noticed with respect to impedance parameters $Z_2\ andZ_3$ as revealed in Fig5 and Fig6.The wave speed is more in case of thermally insulated state as contrast to isothermal condition.

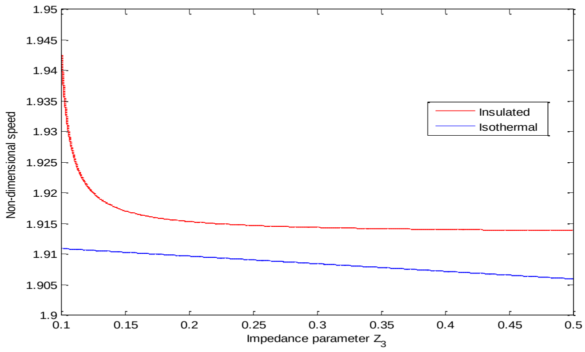

Fig.6 Variation of non-dimensional wave speed V_1w.r.t. impedance parameter Z_3 in a micropolar thermally insulated and isothermal half space

7. Conclusion

In this investigation, the Rayleigh waves in a micropolar viscothermoelastic half space with impedance boundary conditions for a thermally insulated and isothermal surface are examined. The secular equation for Rayleigh waves that satisfies impedance boundary conditions is obtained in the explicit form. The secular equation is consistent with the secular equation of Rayleigh waves for thermoelastic half space with impedance boundary conditions when the micropolar parameters are eliminated. Moreover, the classical equation for an elastic solid, as determined by Lord Rayleigh [10], is obtained by eliminating impedance and viscothermal effects from this equation. The numerical analysis yields the following conclusion:

• Rayleigh waves are present in a micropolar viscothermoelastic material with impedance boundary conditions.
• Rayleigh waves non-dimensional speed is affected by the viscosity of micropolar thermoelastic solids in relation to all impedance parameters, with both increases and decreases occurring.

The degree of dispersion of the non-dimensional Rayleigh wave speed is contingent upon the wave number and the impedance parameter range.

• The non-dimensional wave speed increases in the presence of a thermally insulated boundary when compared to isothermal boundary conditions when calculated as a function of impedance parameters. Rayleigh waves are particularly significant in the field of earthquake engineering due to their destructive character during earthquakes, and the investigation of waves in micropolar viscothermoelastic material is quite significant. Waves in certain rock behave like micropolar viscothermoelastic solids. As a consequence, the results of the present study, despite being a theoretical modal, are of paramount importance to researchers in the fields of geophysics, composite materials, geological materials, and seismology.

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