Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 8s (2024)

A New Inventive Approach to Nano Fuzzy Soft Topological Space Using Mathematical Analysis

J. Nirmala[1*], Dr. J. Srikiruthika[2]

[1]Research Scholar, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women.

Assistant Professor, Department of Mathematics, KG College of Arts and Science.

[2]Assistant Professor of Mathematics, Department of Science and Humanities, School of Engineering, Avinashilingam Institute for Home Science and Higher Education for Women.

Article History:

Received: *22-04-2024*

Revised: *12-06-2024*

Accepted: *25-06-2024*

Abstract

The important theme of this paper is to define a lower and upper approximation for the set under consideration in order to examine a novel class of topological field known as Nano fuzzy soft topological space. Fuzzy topology coverts ordered structure to topological structure by using by its own criteria conditions. Nano topology is defined with the help of equivalence relation so that even micro spacing is included in the topological space. The condition of these two topological space has been merged and a new space known as Nano fuzzy topological space has been investigated and studied. Soft sets are an abstraction of fuzzy set theory which was interposed to deal with uncertainty conditions using parameters. Using soft sets, Nano fuzzy topology can be extended to a new form called Nano fuzzy soft topological space and it can be used to study matters of vagueness by defining a proper boundary layer. Since this concept depends upon defining appropriate boundary layers using topological concepts, it easy to determine the key components for a particular problem by properly defining three important regions which are upper approximation area, lower approximation area and boundary region. The use of Nano fuzzy soft topological space in predicting the cause of a disease has been discussed in this paper. It can be extended to study any complex problems in real life by converting it to a Nano fuzzy soft topological space using the three main key factors.

Keywords: Topological Space, Mathematical Analysis.

Introduction

Zadeh [1] in 1965 interposed a new type of set called fuzzy sets as a powerful tool to study vagueness .The overview of fuzzy topological space was putforth by Chang [2] in 1968. Lowen [3] provided many important concepts for the construction of topological fuzzy space. Soft set theory introduced by Georgy Molodotsov [4] in 1999 was widely used in different fields of mathematics. Maji and Roy [5] presented a novel theory for fuzzy soft sets in 2001 by merging the concepts of fuzzy sets and soft sets. In recent times, Nano topology has become an important topic in topological field which was introduced by Lellis Thivagar and Carmel Richard [6] in 2013. Nano soft sets and Nano soft topological space were introduced and analysed by M. Lellis Thivagar and S. P. R. Priyalatha [7] in 2017. Nano generalized fuzzy sets and Nano fuzzy topological spaces were introduced by Purva Rajwade, Rachna Navalakhe and Vaibhav Jain[11] in 2022. As an extension of these two sets Nano fuzzy soft sets and Nano fuzzy soft topological spaces are being introduced and investigated.

2.Preliminary Definitions

Definition 2.1[1] Let R'' be an equivalency relation on U known as the indiscernibility relation, and let U be a finite non – empty set with objects termed as the universe. It is argued that elements in the same equivalency class are indistinguishable from one another. The set (U, R'') is said to be the approximation space. Let $Y \subseteq U$.

(i) The collection of all objects that may be categorised as Y with regard to R'' is the lower approximation area of Y regarding to R'', and it is represented by $Lr_{R''}(Y)$.

$$Lr_{R''}(Y) = \bigcup_{y \in U} \{R''(y) : R''(y) \subseteq Y\}$$

where $R''(y)$ denotes the equivalence class determined by y.

(ii) The set containing all the objects that may be categorised as Y with regard to R'' is the upper approximation area of Y regarding R'', and it is represented by $Ur_{R''}(Y)$.

$$Ur_{R''}(Y) = \bigcup_{y \in U} \{R''(y) : R''(y) \cap Y \neq \varphi\}$$

(iii) The set containing all the objects that can be categorised as X with regards to R'' is the boundary area of Y with reference to R'', and it is represented by $By_{R''}(Y)$. $By_{R''}(Y) = Ur_{R''}(Y) - Lr_{R''}(Y)$.

Definition 2.2 [1]: Let U represents the universe, and let R'' be an equivalency relation on the universe and $\tau_{R''}(Y)$ be defined as $\tau_{R''}(Y) = \{U, \varphi, Ur_{R''}(Y), Lr_{R''}(Y), By_{R''}(Y)\}$ where $Y \subseteq U$. Then $\tau_{R''}(Y)$ satisfies the given conditions,

(i) U and $\varphi \in \tau_{R''}(Y)$

(ii) The union of all the elements of any subclass of $\tau_{R''}(Y)$ is in $\tau_{R''}(Y)$.

(iii)The intersection of the elements of any finite subclass of $\tau_{R''}(Y)$ is in $\tau_{R''}(Y)$ i.e, $\tau_{R''}(Y)$ is a topology on the universe called the NT on U with regards to Y. $(U, \tau_{R''}(Y))$ is called as the NTS.

Let Nano topology may be denoted by NT, Nano topological space may be denoted by NTS for easy reference.

Definition 2.3 [1]: If $(U, \tau_{R''}(Y))$ is a NTS with regard to Y where $Y \subseteq U$ and if $B \subseteq U$, then, NInt(A) represents the largest Nano open subset of A, and the Nano interior of B is given as the union of all Nano open subsets of B. NCl(B) represents the Nano closure of B, which is considered to be the smallest Nano closed set containing B. It is considered as the intersection of all the given Nano closed sets containing B.

Nano soft topological space:

Definition 2.4 [2] Let h_E be the given soft set over U, which is a finite universe, and R'' be a soft equivalency relation on h_E. It is seen that the elements in the soft equivalency class of f(b) represented by [f(b)] are softly indiscernible from one another. One term for the ordered pair (U, h_D) is called as "soft approximation space." . Let $s_D \subseteq h_E$.

(i) $Lr_{R''}(s_D) = \cup\{h_E : [h_E] \subseteq s_D\}$) is a soft lower approximation f_A of with respect to h_D.

(ii) $Ur_{R''}(s_D) = \bigcup_{a\in A} \{f(b) : [f(b)] \cap s_D \neq 0\}$ is a soft upper approximation of h_E of w.r.t s_D.

(iii) $By_{R''}(s_D) = Ur_{R''}(s_D) - Lr_{R''}(s_D)$ is the boundary region of h_D with reference to s_D.

Definition 2.5 [2] Let U be a given finite collection of objects called the universe, h_E is an soft set over the given universe. Then (U, h_E) is the pair of soft approximation space and $\tau_{R''}(s_D) = \{U, \varphi, Ur_{R''}(s_D), Lr_{R''}(s_D), By_{R''}(s_D)\}$ where $s_D \subseteq h_E$, and $\tau_{R''}(s_D)$ satisfies the following axiom:

(i) $U, \varphi \in \tau_{R''}(S_D)$

(ii) The union of the elements of any subclass of soft sets $\tau_{R''}(s_D)$ is in $\tau_{R''}(s_D)$.

(iii) The intersection of the elements of any finite subclass of soft sets $\tau_{R''}(s_D)$ is in $\tau_{R''}(s_D)$.

Specifically, $\tau_{R''}(s_D)$ creates a NSTS over U with regard to $(U, \tau_{R''}, E)$ and its members are referred to as Nano soft open sets in U with respect to s_D.

Let Nano soft topology may be denoted by NST and Nano soft topological space may be denoted by NSTS for easy reference.

Definition 2.6 [2] If $(U, \tau_{R''}, E)$ is a NSTS regarding X and E where $X \subseteq U$ and if $f_A \subseteq U$, then the Nano soft interior of f_A is referred as the union of all NSO subsets of f_A and it is denoted by $NInt(f_A)$,i.e, $NInt(f_A)$ is the largest NSO subset of f_A. The NSC of f_A is given as the intersection of all NSC sets containing f_A and it is denoted by $NCl(f_A)$, i.e, $NInt(f_A)$ is the smallest NSC set containing A.

Nano fuzzy topological space:

Definition 2.7 [3]: From Y to X, let R'' be a given arbitrary relation, then the lower and upper approximation areas for the given fuzzy set $\underline{R''}$ and $\overline{R''}$ respectively satisfy the following properties: for all $\mu, \lambda \in G(Y)$, where G(Y) is the set of all fuzzy subsets of Y:

(i) $\overline{R''}(\mu \wedge \lambda) = \overline{R''}(\mu) \wedge \overline{R''}(\lambda)$

(ii) $\overline{R''}(\mu \vee \lambda) = \overline{R''}(\mu) \triangledown R''(\lambda)$

(iii) $\overline{R''}(\mu) = \left(\overline{R''}(\mu^c)\right)^c$

(iv) $\mu \leq \lambda \Rightarrow \overline{R''}(\mu) \leq R''(\lambda)$ and similarly these conditions will be satisfied for $\underline{R''}$.

Definition 2.8 [3]: Let Y be a given set which is finite and R'' be the given equivalency relation on $Y, \lambda \leq Y$ be a fuzzy subset and $\tau_{(\Im)}(\lambda) = \{1_\lambda, 0_\lambda, \overline{R''}(\lambda), \underline{R''}(\lambda), Bd(\lambda)\}$. Then $\tau_{(\Im)}(\lambda)$ satisfies the following, axioms:

(i) $0_{\lambda}, 1_{\lambda} \in \tau_{(\mathfrak{Z})}(\lambda)$ where $0:\lambda \to I$ indicate the null fuzzy set and $1:\lambda \to I$ indicate the whole fuzzy set.

(ii) Arbitrary union of members of $\tau_{(\mathfrak{Z})}(\lambda)$ is a member of $\tau_{(\mathfrak{Z})}(\lambda)$.

(iii) Finite intersection of members of $\tau_{(\mathfrak{Z})}(\mu)$ is a member of $\tau_{(\mathfrak{Z})}(\lambda)$.

i.e, $\tau_{(\mathfrak{Z})}(\lambda)$ is a topology on X called the NFT on X with regard to λ . We call $(Y, \tau_{(\mathfrak{Z})}(\lambda))$ as the NFTS. The elements of the NFTS that is $\tau_{(\mathfrak{Z})}(\lambda)$, are called Nano fuzzy open sets and elements of $\tau_{(\mathfrak{Z})}(\lambda)^c$ are called Nano fuzzy closed sets .

Let Nano fuzzy topology may be denoted by NFT and Nano soft topological space may be denoted by NFTS for easy reference.

Definition 2.9 [3] Consider the NFTS $(Y, \tau_{(\mathfrak{Z})}(\lambda))$ with regard to λ , where $\lambda < Y$. If $\lambda \leq Y$, then the union of all Nano fuzzy open subsets of λ defines the Nano fuzzy interior of λ , which is represented by the notation $NfInt(\lambda)$.

Nano fuzzy soft topological space:

Definition 2.10 : Let R'' be an given equivalency relation from X to Y. The lower (decreased) and upper (increased) approximation areas of a fuzzy soft set denoted by $\underline{R}''$ and $\overline{R}''$ respectively satisfy the given following properties: for all $\tilde{f}_A, \tilde{g}_B \in \tilde{G}_B(X)$, where $\tilde{G}_B(X)$ is the set of all fuzzy soft subsets of X:

(v) $\overline{R}''(\tilde{f}_A \wedge \tilde{g}_B) = \overline{R}''(\tilde{f}_A) \wedge \overline{R}''(\tilde{g}_B)$

(vi) $\overline{R}''(\tilde{f}_A \vee \tilde{g}_B) = \overline{R}''(\tilde{f}_A) \vee \overline{R}''(\tilde{g}_B)$

(vii) $\overline{R}''(\tilde{f}_A) = (\overline{R}''(\tilde{f}_A{}^c))^c$

(viii) $\tilde{f}_A \leq \tilde{g}_B \Rightarrow \overline{R}''(\tilde{f}_A) \leq \overline{R}''(\tilde{g}_B)$ and similarly these conditions will be satisfied for $\underline{R}''$.

Definition 2.11: Let X be a given finite set and R'' be an equivalency relation on X, let $\tilde{f}_A \leq X$ be a fuzzy soft subset and $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A) = \{1_\mu, 0_\mu, \overline{R}''(\tilde{f}_A), \underline{R}''(\tilde{f}_A), Bd(\tilde{f}_A)\}$. Then $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$ satisfies the following, axioms:

(i) $\tilde{0}, \tilde{1} \in \tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$ where $\tilde{0} : \tilde{f}_A \to I$ denotes the void fuzzy soft set and $\tilde{1} : \tilde{f}_A \to I$ denotes the whole fuzzy soft set.

(ii) $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$ is the arbitrary union of members of $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$.

(iii) $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$ is the finite intersection of members of $\tilde{\tau}_{(\mathfrak{Z})}(\tilde{f}_A)$.

i.e , $\tilde{\tau}_{(\Im)}(\tilde{f}_A)$ is the given topology on X called the NFST on X with respect to $\tilde{f}_A$. We call $(X, \tilde{\tau}_{(\Im)}(\tilde{f}_A), E)$ as the NFSTS. The elements of the NFSTS that is $\tilde{\tau}_{(\Im)}(\tilde{f}_A)$, are called Nano fuzzy soft open sets and elements of $\tilde{\tau}_{(\Im)}(\tilde{f}_A)^C$ are called Nano fuzzy soft closed sets.

Let Nano fuzzy soft topology may be denoted by NFST and Nano fuzzy soft topological space may be denoted by NFSTS for easy reference. Let Nano fuzzy soft open sets be denoted by NFSO set and Nano fuzzy soft closed sets be denoted by NFSC set.

Definition 2.12 : Let $(X, \tilde{\tau}_{(\Im)}(\tilde{f}_A), E)$ be a NFSTS with respect to $\tilde{f}_A$, and if $\tilde{f}_A \leq X$, then $NfInt(\tilde{f}_A)$ denotes the Nano fuzzy soft interior of $\tilde{f}_A$ and $NfCl(\tilde{f}_A)$ denotes the Nano fuzzy soft closure of $\tilde{f}_A$.

Definition 2.13: Let $\tilde{f}_A$ be a Nano fuzzy soft set in a NFSTS $(X, \tilde{\tau}_{(\Im)}(\tilde{f}_A), E)$. Then the Nano fuzzy soft closure and Nano fuzzy soft interior of $\tilde{f}_A$ is defined respectively as

$NfCl(\tilde{f}_A) = \wedge\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSC set in X and } \tilde{f}_A \leq \tilde{g}_B\}$ $NfInt(\tilde{f}_A) = \vee\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSO set in X and } \tilde{g}_B \leq \tilde{f}_A\}$

Therefore, the smallest Nano fuzzy soft closed set is represented by $NfCl(\tilde{f}_A)$ containing $\tilde{f}_A$ and the largest Nano fuzzy soft open set is represented by $NfCl(\tilde{f}_A)$ contained in $\tilde{f}_A$.

3: Theoretical concepts of Nano fuzzy soft topological space

Theorem 3.1:

Let $\tilde{f}_A$ be a NFS set in a NFSTS $(X, \tilde{\tau}_{(\Im)}(\tilde{f}_A), E)$. Then

(i) $(NfInt(\tilde{f}_A))^C = NfCl((\tilde{f}_A)^C)$

(ii) $(NfCl(\tilde{f}_A))^C = NfInt((\tilde{f}_A)^C)$

Proof:

(i) $NfInt(\tilde{f}_A) = \vee\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSO set in X and } \tilde{g}_B \leq \tilde{f}_A\}$ $(NfInt(\tilde{f}_A))^C = (\vee\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSO set in X and } \tilde{g}_B \leq \tilde{f}_A\})^C$

$$= \wedge\{(\tilde{g}_B)^C / (\tilde{g}_B)^C \text{ is a NFSC set in X and } (\tilde{g}_B)^C \geq (\tilde{f}_A)^C\}$$

$$= NfCl((\tilde{f}_A)^C)$$

(ii) $NfCl(\tilde{f}_A) = \wedge\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSC set in X and } \tilde{f}_A \leq \tilde{g}_B\}$ $(NfCl(\tilde{f}_A))^C = (\wedge\{\tilde{g}_B / \tilde{g}_B \text{ is a NFSC set in X and } \tilde{f}_A \leq \tilde{g}_B\})^C$

$$= \vee\{(\tilde{g}_B)^C / (\tilde{g}_B)^C \text{ is a NFSO set in X and } (\tilde{f}_A)^C \geq (\tilde{g}_B)^C\}$$

$$= NfInt((\tilde{f}_A)^C)$$

Theorem 3.2 :

Let $\tilde{f}_A$ and $\tilde{h}_D$ be any two NFS sets in a NFSTS $\left(X, \tilde{\tau}_{(\Im)}\left(\tilde{f}_A\right), E\right)$.Then

(i) $\tilde{f}_A \le \mathrm{NfCl}\left(\tilde{f}_A\right)$

(ii) $\mathrm{NfCl}\left(\tilde{f}_A\right)$ is a NFSC set.

(iii) $\mathrm{cl}\left(\tilde{0}\right) = \tilde{0}, \mathrm{cl}\left(\tilde{1}\right) = \tilde{1}$

(iv) $\tilde{f}_A$ is a Nano fuzzy soft closed set if $\tilde{f}_A = \mathrm{NfCl}\left(\tilde{f}_A\right)$

(v) $\mathrm{NfCl}\left(\mathrm{NfCl}\left(\tilde{f}_A\right)\right) = \mathrm{NfCl}\left(\tilde{f}_A\right)$

(vi) $\mathrm{NfCl}\left(\tilde{f}_A \vee \tilde{h}_D\right) = \mathrm{NfCl}\left(\tilde{f}_A\right) \vee \mathrm{NfCl}\left(\tilde{h}_D\right)$

(vii) $\mathrm{Nfcl}\left(\hat{\tilde{f}}_A \wedge \hat{\tilde{h}}_D\right) = \mathrm{Nfcl}\left(\hat{\tilde{f}}_A\right) \wedge \mathrm{Nfcl}\left(\hat{\tilde{h}}_D\right)$

Proof:

(i) By thedefinition of Nano fuzzy soft closure .

$\tilde{f}_A \le \mathrm{NfCl}\left(\tilde{f}_A\right)$

(ii) Since $\mathrm{NfCl}\left(\tilde{f}_A\right)$ is the intersection of NFSC sets, $\mathrm{NfCl}\left(\tilde{f}_A\right)$ is a NFSC set.

(iii) It is an obvious result.

(iii) Let $\hat{\tilde{f}}_A$ be a NFS set.

Therefore

$\mathrm{NfCl}\left(\tilde{f}_A\right) \ge \tilde{f}_A$ (1)

Since $\mathrm{Nfcl}\left(\tilde{f}_A\right)$ is the smallest NFSC set containing $\hat{\tilde{f}}_A$

$\Rightarrow \mathrm{NfCl}\left(\tilde{f}_A\right) \le \tilde{f}_A$ (2)

From (1) and (2) we get $\mathrm{NfCl}\left(\tilde{f}_A\right) = \tilde{f}_A$

Conversely assume that $\mathrm{NfCl}\left(\tilde{f}_A\right) = \tilde{f}_A$

since $\mathrm{NfCl}\left(\tilde{f}_A\right)$ is the NFSC set

$\Rightarrow \tilde{f}_A$ is a NFSC set.

(v) Since $\mathrm{NfCl}\left(\tilde{f}_A\right)$ is a NFSC set, by the above result we get

$\mathrm{NfCl}\left(\mathrm{NfCl}\left(\tilde{f}_A\right)\right) = \mathrm{NfCl}\left(\tilde{f}_A\right)$

(vi) $\tilde{f}_A \le \tilde{f}_A \vee \tilde{h}_D$ and $\tilde{h}_D \le \tilde{f}_A \vee \tilde{h}_D$,Then $NfCl(\tilde{f}_A) \le NfCl(\tilde{f}_A \vee \tilde{h}_D)$

and $NfCl(\tilde{h}_D) \le NfCl(\tilde{f}_A \vee \tilde{h}_D)$

Therefore $NfCl(\tilde{f}_A) \vee NfCl(\tilde{h}_D) \le NfCl(\tilde{f}_A \vee \tilde{h}_D)$ (3)

$\tilde{f}_A \le NfCl(\tilde{f}_A)$ and $\tilde{h}_D \le NfCl(\tilde{h}_D)$

Therefore $\tilde{f}_A \vee \tilde{h}_D \le NfCl(\tilde{f}_A) \vee NfCl(\tilde{h}_D)$ (3)

Therefore $NfCl(\tilde{f}_A) \vee NfCl(\tilde{h}_D)$ is the Nano fuzzy soft closed set containing $\tilde{f}_A \vee \tilde{h}_D$

Therefore $NfCl(\tilde{f}_A \vee \tilde{h}_D)$ is the smallest Nano fuzzy soft closed set containing $\tilde{f}_A \vee \tilde{g}_B$

Hence $NfCl(\tilde{f}_A \vee \tilde{h}_D) \le NfCl(\tilde{f}_A) \vee NfCl(\tilde{h}_D)$ (4)

From (3) and (4), $NfCl(\tilde{f}_A \vee \tilde{h}_D) = NfCl(\tilde{f}_A) \vee NfCl(\tilde{h}_D)$

(vii) $\tilde{f}_A \wedge \tilde{h}_D \le \tilde{f}_A$ and $\tilde{f}_A \wedge \tilde{h}_D \le \tilde{h}_D$

Then $NfCl(\tilde{f}_A \wedge \tilde{h}_D) \le NfCl(\tilde{f}_A)$ and $NfCl(\tilde{f}_A \wedge \tilde{h}_D) \vee NfCl(\tilde{h}_D)$

Therefore $NfCl(\tilde{f}_A \wedge \tilde{h}_D) \le NfCl(\tilde{f}_A) \wedge NfCl(\tilde{h}_D)$

4. Application of Nano fuzzy soft topology

Diabetes or blood glucose has become one of the common diseases among the folk. Glucose is the main energy source of our body. The insulin, secreted by the pancreas, helps our cells absorb glucose for energy. Diabetes is a result of insufficient insulin secretion. Diabetes increases the risk of kidney, nerve, heart, and eye damage. There is a connection between diabetes and some type of cancers. The risk factor of getting diabetes can be minimised or avoided by managing our diet in a balanced way. Here, we employ a mathematical analysis to identify the fundamental elements of the disease-related symptoms.

Mathematical Research Analysis:

Let $Union = \bigcup_{i=1}^{15} pt_i$ represents the given set of patients & the condition attributes are given as IT-Increased thirst, H-Hunger, F-fatigue BV- Blurred Vision, V-Vomitting and LW-Loss of weight. Here C is the decision attribute = {diabetes} and pt_1 = {IT,H,BV,V,LW}, pt_2 = {IT,V,LW}, pt_3 = {IT,H,BV,V}, pt_4 = {IT,F}, pt_5={IT,F}, pt_6 = {IT,H,F,V,LW}, pt_7 ={IT,H,BV,V}, pt_8={IT,F,V}, pt_9={IT,F},pt_{10}={IT,H,F,V,LW},pt_{11}={H,F,V},pt_{12}={H,BV,V},pt_{13}={IT,F},pt_{14}={H,F,BV},pt_{15}={IT ,V,LW} be the symptoms of the patients considered.

Patients	IT	H	FA	BV	V	LW
pt_1	Y	Y	-	Y	Y	Y
pt_2	Y	-	-	-	Y	Y
pt_3	Y	Y	-	Y	Y	-

pt_4	Y	-	Y	-	-	-
pt_5	Y	-	Y	-	-	-
pt_6	Y	Y	Y	-	Y	Y
pt_7	Y	Y	-	Y	Y	-
pt_8	Y	-	Y	-	Y	-
pt_9	Y		Y	-	-	-
pt_{10}	Y	Y	Y	-	Y	Y
pt_{11}	-	Y	Y	-	Y	-
pt_{12}	-	Y	-	Y	Y	-
pt_{13}	Y	-	Y	-	-	-
pt_{14}	-	Y	Y	Y	-	-
pt_{15}	Y	-	-	-	Y	Y

Assume $\tilde{f}_A = \{pt_1, pt_2, pt_4, pt_6, pt_{10}, pt_{15}\}$ be the set of patients having diabetes then $U/R(C) = \{\{pt_1, pt_3, pt_7\}, \{pt_2, pt_{15}\}, \{pt_4, pt_8\}, \{pt_5, pt_9, pt_{13}\}, \{pt_6, pt_{10}\}, \{pt_{11}, pt_{14}\}, \{pt_{12}\}\}$ and the required Nano fuzzy soft topology is given by $\tau_{R(C)}(\tilde{f}_A) = \{U, \phi, \{pt_2, pt_6, pt_{10}, pt_{15}\}, \{pt_1, pt_2, pt_3, pt_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}\}, \{pt_1, pt_3, pt_4, pt_7, pt_8\}\}$.

Case 1: While the attribute " Increased Thirst " is extracted from condition attribute then $U/R_{(C\text{-}IT)} = \{\{pt_1, pt_3, pt_7, pt_{12}\}, \{pt_2, pt_{15}\} \{pt_4, pt_5, pt_9, pt_{13}\} \{pt_8\}, \{pt_6, pt_{10}\}, \{pt_{11}, pt_{14}\}\}$, the lower approximation area (decreased), upper approximation area (increased) and boundary sector are denoted by $L_{(C\text{-}IT)}(\tilde{f}_A) = \{pt_2, pt_6, pt_{10}, pt_{15}\}$, $U_{(C\text{-}IT)}(\tilde{f}_A) = \{pt_1, pt_2, pt_3, pt_4, pt_5, pt_6, pt_7, pt_9, pt_{10}, pt_{12}, pt_{13}, pt_{15}\}$, $B_{(C\text{-}IT)}(\tilde{f}_A) = \{pt_1, pt_3, pt_4, pt_5, pt_7, pt_9, pt_{12}, pt_{13}\}$. Therefore the required Nano fuzzy soft topology is given by $\tau_{R(C-IT)}(\tilde{f}_A) = \{U, \phi, \{pt_2, pt_6, pt_{10}, pt_{15}\}, \{pt_1, pt_2, pt_3, pt_4, pt_5, pt_6, pt_7, pt_9, pt_{10}, pt_{12}, pt_{13}, pt_{15}\}, \{pt_1, pt_3, pt_4, pt_5, pt_7, pt_9, pt_{12}, pt_{13}\}\}$. Hence $\tau_{R(C-IT)}(\tilde{f}_A) \neq \tau_{R(C}(\tilde{f}_A)$.

Case 2: While the condition attribute "Hunger " is extracted from C then $U/_{R(C\text{-}H)} = \{\{p_1, p_3, p_7\}, \{p_2, p_{15}\}, \{p_4, p_8\}, \{p_5, p_9, p_{13}\}, \{p_6, p_{10}\}, \{p_{11}, p_{14}\}, \{p_{12}\}\}$, here the lower (decreased) approximation area, upper (increased) approximation area and boundary region are given by $L_{(C\text{-}H)}(\tilde{f}_A) = \{pt_2, pt_6, pt_{10}, pt_{15}\}$, $U_{(C\text{-}H)}(\tilde{f}_A) = \{pt_1, pt_2, pt_3, pt_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}\}$, $B_{(C\text{-}H)}(\tilde{f}_A) = \{pt_1, pt_3, pt_4, pt_7, pt_8\}$. Therefore the required Nano fuzzy soft topology is given by $\tau_{R(C-H})(\tilde{f}_A) = \{U, \phi, \{pt_2, pt_6, pt_{10}, pt_{15}\}, \{pt_1, pt_2, pt_3, tp_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}\}, \{pt_1, pt_3, pt_4, pt_7, pt_8\}\}$. Hence $\tau_{R(C-H)}(\tilde{f}_A) = \tau_{R(C)}(\tilde{f}_A)$

Case 3: While the characteristic "Fatigue " is extracted from from C, then $U/_{R(C\text{-}F)} = \{\{pt_1, pt_3, pt_7\}, \{pt_2, pt_{15}\}, \{pt_4, pt_8\}, \{pt_5, pt_9, pt_{13}\}, \{pt_6, pt_{10}\}, \{pt_{11}, pt_{14}\}, \{pt_{12}\}\}$, here the lower approximation area (increased), upper approximation area (decreased) and boundary sector are denoted by $L_{(C\text{-}FA)}(\tilde{f}_A) = \{p_2, p_6, p_{10}, p_{15}\}$, $U_{(C\text{-}FA)}(\tilde{f}_A) = \{p_1, p_2, p_3, p_4, p_6, p_7, p_8, p_{10}, p_{15}\}$, $B_{(C\text{-}FA)}(\tilde{f}_A) = \{p_1, p_3, p_4, p_7, p_8\}$. Therefore the Nano fuzzy soft topology is given by $\tau_{R(C-FA})(\tilde{f}_A) = \{U, \phi, \{pt_2, pt_6, pt_{10}, pt_{15}\}, \{pt_1, pt_2, pt_3, pt_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}\}, \{pt_1, pt_3, pt_4, pt_7, pt_8\}\}$. Hence $\tau_{R(C-FA)}(\tilde{f}_A) = \tau_{R(C)}(\tilde{f}_A)$

Case 4: While the attribute " Blurred vision" is extracted from C then $U/_{R(C-BV)}$ ={{pt_1, pt_3, pt_7}, {pt_2, pt_{15}}, {pt_4, pt_8}, {pt_5, pt_9, pt_{13}}, {pt_6, pt_{10}}, {pt_{11}, pt_{14}}, {pt_{12}}}, here the lower approximation area (decreased), upper approximation area (increased) and boundary sector are denoted by $L_{(C-BV)}(\tilde{f}_A)$ = {pt_2, pt_6, pt_{10}, pt15}, $U_{(C-BV)}(\tilde{f}_A)$ = {pt_1,pt_2,pt_3, pt_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}}, $B_{(C-BV)}(\tilde{f}_A)$ = {pt_1, pt_3, pt_4, pt_7, pt_8}. Therefore the Nano fuzzy soft topology is given by $\tau_{R(C-BV)}(\tilde{f}_A)$ = {U, ϕ , {pt_2, pt_6, pt_{10}, pt_{15}}, {pt_1, pt_2, pt_3, pt_4, pt_6, pt_7, pt_8, pt_{10}, pt_{15}}, {pt_1, pt_3, pt_4, pt_7, pt_8}}.Hence $\tau_{R(C-BV)}(\tilde{f}_A) = \tau_{R(C)}(\tilde{f}_A)$.

Case 5: While the attribute "Vomiting" is extracted from C, then $U/_{R(C-V)}$ ={{pt_1, pt_3, pt_7}, {pt_2, pt_{15}}, {pt_4, pt_8}, {pt_5, pt_9, pt_{13}}, {pt_6, pt_{10}}, {pt_{11}, pt_{14}}, {pt_{12}}}, here the lower approximation, upper approximation and boundary region are given by $L_{(C-V)}(\tilde{f}_A)$ = {pt_2, pt_6, pt_{10}, pt_{15}}, $U_{(C-V)}(\tilde{f}_A)$ = {pt_1,pt_2, pt_3, pt_4, pt_6,pt_7, pt_8, pt_{10}, pt_{15}}, $B_{(C-V)}(\tilde{f}_A)$ = {pt_1, pt_3, pt_4, pt_7, pt_8}. Therefore the Nano fuzzy soft topology is given by $\tau_{R(C-B)}(\tilde{f}_A)$ = {U, ϕ , {pt_2,pt_6, pt_{10}, pt_{15}}, {pt_1,pt_2, pt_3, pt_4, pt_6,pt_7, pt_8, pt_{10}, pt_{15}}, {pt_1, pt_3, pt_4, pt7, pt8}}. Hence $\tau_{R(C-V)}(\tilde{f}_A) = \tau_{R(C)}(\tilde{f}_A)$.

Case 6: While the attribute "Loss of weight" is extracted from C, then $U/_{R(C-LW)}$ ={{pt_1, pt_3, pt_7}, {pt_2, pt_{15}}, {pt_4, pt_5, pt_8, pt_9, pt_{13}}, {pt_6, pt_{10}}, {pt_{11}, pt_{14}}, {pt_{12}}}, here the lower approximation, upper approximation and boundary region are given by $L_{(C-LW)}(\tilde{f}_A)$ = {pt_2, pt_6, pt_{10}, pt_{15}}, $U_{(C-LW)}(\tilde{f}_A)$ = {pt_1, pt_2, pt_3, pt_4, pt_5, pt_6, pt_7,pt_8, pt_9,pt_{10}, pt_{13}, pt_{15}}, $B_{(C-LW)}$ (X) = {pt_1, pt_3, pt_4, pt_5, pt_7, pt_8, pt_9, pt_{13}}. Therefore the Nano fuzzy soft topology is given by $\tau_{R(C-LW)}(\tilde{f}_A)$ = {U, ϕ , {pt_2, pt_6, pt_{10}, pt_{15}}, {pt_1,pt_2, pt_3, pt_4, pt_5,pt_6, pt_7, pt_8, pt_9, pt_{10}, pt_{13}, pt_{15}}, {pt_1, pt_3, pt_4, pt_5 ,pt_7, pt_8, pt_9, pt_{13}}}. $\tau_{R(C-LW)}(\tilde{f}_A) \neq \tau_{R(C)}(\tilde{f}_A)$.

Core ={Increased Thirst , Loss of weight}

Therefore from the above observations it is clear that increased thirst due to frequent urination and loss of weight are the main causes for the diabetes disease.

Conclusion:

Nano fuzzy soft topology can be used to find the key components for cause of various diseases, to find the recovery rate of patients, to find the best cure among different medicines taken by patients and to find suitable medicine to different categories of people.

References:

[1] L.A Zadeh., "Fuzzy sets, Information and Control", The Journal of Symbolic Logic, vol.8, pp. 338-353, 1965.

[2] C. L. Chang , "Fuzzy Topological Spaces", JMAA , Vol.24, pp.182-190, 1968.

[3] Lowen, R. , "Fuzzy Topological Spaces and Fuzzy Compactness", Jour. of Mathe. Analysis and Applications. Vol.56, pp.621-633 ,1976.

[4] Molodtsov, D. "Soft Set Theory—First Results" ,Comp. & Mathe. with Applications, vol.37, no. 4-5,pp: 19-31,1999.

[5] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," Journ. of Fuzzy Mathe., vol. 9, no. 3, pp. 589–602, 2001.

[6] M.Lellis Thivagar, Carmel Richard, “On Nanoforms of Weakly Open sets”, International Journal of Mathe. and Statistics Inven., vol. 1, no. 1,August , pp:31-37,2013.

[7] M.Lellis Thivagar and S.P.R.Priyalatha “An Innovative Approach on Nano Soft Topological Space”,South East Asian Journal of Math. & Math. Sci ,vol 13,no.2, pp:47-62, 2017.

[8] Jayalakshmi .A and Janaki .C,“A new class of sets in Nano Topological Spaces with an application in Medical Diagnosis”, Int.Jour. of Engi. Res., vol.12, no. 16, pp: 5894-5899,2017.

[9] Shuker Mahmood Khalil, Nadia M.Ali Abbas ,”On Nano with their applications in Medical Field”, AIP conference Proceedings, Dec 04, 2290(1):40002,2020.

[10] Jeevitha.R ,Sangeetha.M , Kannaki.S , and Vidhya.D, “An Application of Nano Topological Spaces with Decision Making Problem in Medical Field”, Turkish Jour. of Comp. and Mathe. Education vol.12 ,no.9 ,pp: 975-978,2021.

[11] Purva Rajwade, Rachna Navalakhe and Vaibhav Jain , “Nano Generalised Fuzzy Semiopen sets and Nano Fuzzy beta-open sets in Nano Fuzzy Topological Spaces, Ganita , vol. 72, no.1 pp:285-289, 2022.