Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 2 (2025)

Generalized Pre-semi Homeomorphisms in Intuitionistic Fuzzy Topological Spaces

R. Revathy[1], P. Thirunavukarasu[2]

[1]Part Time Research Scholar, PG & Research, Department of Mathematics, Thanthai Periyar Government Arts & Science College, Thiruchirapalli, Tamil Nadu , India (Affiliated to Bharathidasan University)

e-mail:rrevathy085@gmail.com

[2]Assistant Professor, PG & Research Department of Mathematics, Thanthai Periyar Government Arts & Science College, Thiruchirapalli, Tamil Nadu , India. (Affiliated to Bharathidasan University)

email :ptavinash1967@gmail.com

Article History:

Received: *28-07-2024*

Revised: *09-09-2024*

Accepted: *17-09-2024*

Abstract:
This article presents a novel concept: generalized Intuitionistic Fuzzy pre-homeomorphisms. We delve into exploring several of their defining traits and characteristics.

Keywords: Intuitionistic Fuzzy Topology, Intuitionistic Fuzzy generalized pre-semi closed set, Intuitionistic Fuzzy generalized pre-semi homeomorphism.

1. Introduction

On encountering ambiguity, vagueness, and partial truth, Zadeh [14] proposed the fuzzy set. This quantifies the degree to which every element within universe of discourse

belongs towards particular subset within Fuzzy Set. Eventually, [3] Chang created fuzzy topology in 1967. [1] Atanassov formulated concept of Intuitionistic Fuzzy Sets (IF Sets), which offer a more detailed approach to uncertainty quantification. This framework allows for a more precise description of problems by utilizing existing information and observations. IFS adds the degree of non-membership to FS. There have since been a number of generalisations of the ideas behind fuzzy sets and fuzzy topology.

Many fuzzy notions have recently been used to Intuitionistic Fuzzy Sets. When utilizing idea of Intuitionistic Fuzzy Sets, Coker [3] established Intuitionistic Fuzzy Topological Spaces. Moreover, Dogan Coker and Selma Ozcag [16, 3] looked at connectedness in intuitionistic topological spaces. That feeble type of Intuitionistic Topological Spaces was later as research ideas by various scientists [10]. The focus of this paper is a generalised Intuitionistic Fuzzy pre-homeomorphisms. We look into some of their characteristics. We also show how different homeomorphisms relate to one another.

Preliminaries

Definition 1: [1]

If A is a non-empty static customary. Intuitionistic Fuzzy (IF) set U enclosed X is the item has method $U = \{\langle a, \mu U(a), \nu U(a)\rangle / a \in A\}$ somewhere the meanings $\mu U: A \to [0,1]$ and $\nu U : A \to [0,1]$ symbolize the continuum of connection also gradation involves non-membership of every section a ∈ A to the customary U, respectively, and $0 \leq \mu U(a) + \nu U(a) \leq 1$ for every a ∈ A. Symbolize by IF Set (A), the customary of all IF sets in A.

Definition 2: [1]

If U and V be IF Sets of the procedure U = {⟨a, μU(a), νU(a)⟩/ a ∈ A} and V = {⟨a, μV(a), νV(a)⟩/ a ∈ A}. Then

(i) U ⊆ V iff μU(a) ≤ μV(x) and νU(a) ≥ νV(a) for all a ∈ A.

(ii) U = V iff U ⊆ V and V ⊆ U.

(iii) U c = { ⟨a, νU(a), μU(a)⟩/ a ∈ A }.

(iv) U ∩ V = { ⟨a, μU(a) ∧ μV(a), νU(a) ∨ νV(a)⟩/ a ∈ A }.

(v) U ∪ V = { ⟨a, μU(a) ∨ μV(a), νU(a) ∧ νV(x)⟩/ a ∈ A }.

To keep things simple, we'll adopt the following notation: U = ⟨a, μU, νU⟩ in lieu of U = { ⟨a, μU(a), νU(a)⟩/ a ∈ A }. Furthermore, to maintain simplicity, we'll utilize the following notation: U = ⟨a, (μU, μV), (νU, νV)⟩ in preference to U = ⟨a, (U/μU, V/μV), (U/νU, V/νV)⟩. Intuitionistic Fuzzy Sets 0~ = { ⟨a, 0, 1⟩/ a ∈ A } also 1~ = { ⟨a, 1, 0⟩/ a ∈ A } These denote empty set also complete set of A.

Definition 3: [3]

The set of axioms defining the IF topology on set A corresponds to a specific τ governing IF Sets within A.

(i) 0~, 1~ ∈ τ.

(ii) E1 ∩ E2 ∈ τ, per E1, E2 ∈ τ.

(iii) ∪ Ei ∈ τ per {Ei / i ∈ I} ⊆ τ.

In particular scenario, duo (A, τ) be termed an intuitionistic fuzzy topological space along with IF Set τ will be acknowledged as intuitionistic fuzzy open set with A. Counterpart U^c has IFO Set U be IFT Set (A, τ) termed as Intuitionistic Fuzzy closed set involves A.

Definition 4: [3]

Assume (A, τ) be IFTS and U = ⟨a, μU, νU⟩ be IFS in A. Next

(i) in(U) = ∪{ E / E is an IFOS in A and E ⊆ U }.

(ii) c(U) = ∩{ F / F is an IFCS in A and U ⊆ F }.

(iii) $c(U^c) = (in(U))^c$. (iv) $in(U^c) = (c(U))^c$.

Definition 5: [4]

Assume U is IF Set of A. Next

(i) pin(U) = ∪{E : E be IF P O Set has A also E ⊆ U}.

(ii) pc(U) = ∩{F : F be IF P C Set has A also U ⊆ F}.

Definition 6: [4]

An IF Set U has IFT Set (A, τ) has

(i) Intuitionistic Fuzzy semiclosed set provided that (c(U)) ⊆ U.

(ii) Intuitionistic Fuzzy semiopen defined that U ⊆ c(in(U)).

Definition 7: [8]

An IF Set U in an IFT Set (A, τ) fulfills certain conditions, it qualifies as Intuitionistic Fuzzy generalized pre-semi closed set when pc(U) ⊆ R at any time U ⊆ R also R is IFSO Set in (A, τ). IF Set U called as Intuitionistic Fuzzy generalized pre-semi open set has (A, τ) when complement U^c be IFGPSC Set with A.

Definition 8: [4]

If q: (A, τ) → (B, σ) is a plotting from an IFT Set (A, τ) into an IFT Set (B, σ). Next q is supposed chosen

(i) Intuitionistic Fuzzy continuous when q-1 (V) ∈ IF Open (A) PerV ∈ σ.

(ii) Intuitionistic Fuzzy α continuous when q-1 (V) ∈ If α Open(A) Per V ∈ σ.

Definition 9: [9]

A plotting q: (A, τ) → (B, σ) termed as intuitionistic fuzzy generalized pre-semi continuous functions if q-1 (S) is IFGPSC Set in (A, τ) in each case IFC Set S of (B, σ).

Definition 10: [11]

A plotting q: (A, τ) → (B, σ) labelled as Intuitionistic Fuzzy generalized semi-pre continuous mapping when q-1 (V) be IFGSPC Set in (A, τ) for each IFC Set V of (B, σ).

Definition 11: [5]

A plotting q: (A, τ) → (B, σ) is christened Intuitionistic Fuzzy generalized semi pre regular continuous mapping when q-1 (S) be IFGSPRC Set in (A, τ) every IFC Set V belongs to (B, σ).

Definition 12: [9]

A plotting q: (A, τ) → (B, σ) is Intuitionistic Fuzzy generalized pre-semi weak function when q-1 (S) termed as IFGPSC Set with (A, τ) Apiece IFGPSC Set S of (B, σ).

Definition 13: [14]

A map q: (A, τ) → (B, σ) baptised

(i) Intuitionistic Fuzzy closed mapping when q(U) is IFC Set involve B universally IFC Set U in A.

(ii) Intuitionistic Fuzzy α-open mapping when q(U) is If αO Set involve B universally IFO Set U in A.

Definition 14: [10]

A plotting q: (A, τ) → (B, σ) baptized Intuitionistic Fuzzy generalized pre-semi closed plotting if q(U) be IFGPS Closed Set has B in every instance IFC Set U involves A.

Definition 15: [10]

A plotting q: (A, τ) → (B, σ) labelled as intuitionistic fuzzy generalized pre semi open plotting when q(U) be IFGPS Open Set with B universally IFO Set involves A.

Definition 16: [12]

A plotting q: (A, τ) → (B, σ) termed as intuitionistic fuzzy generalized semi pre closed plotting if q (U) be IFGSP Closed Set with B universally IFC Set U involves A.

Definition 17: [6]

A plotting q: (A, τ) → (B, σ) labelled as intuitionistic fuzzy generalized semi pre regular closed function if fq(U) be IFGSPR Closed Set within B for all IFC Set U involves A.

Definition 18: [8]

If each IFGP Set Closed Set in (A, τ) is IFPC Set consists (A, τ), next space described as intuitionistic fuzzy pre semi M1/2 space.

Definition 19: [8]

An IFT Set (A, τ) termed as intuitionistic fuzzy pre semi M* 1/2 space if all IFGPS Closed Set is IFC Set involves (A, τ).

Definition 20: [7]

Let q is bijection mapping from IFT Set (A, τ) into IFT Set (B, σ). Then q called as

(i) Intuitionistic Fuzzy homeomorphism if q also q^{-1} be IF continuous functions.

(ii) Intuitionistic Fuzzy α homeomorphism if q also q^{-1} be IF α continuous functions.

Definition 21: [6]

If q : (A, τ) → (B, σ) is a Bijective function. Formerly q called as Intuitionistic Fuzzy generalized semi-pre regular homeomorphism when q be together IFGSP Regular continuous function also IFGSP Regular closed function.

Definition 22: [13]

If q : (A, τ) → (B, σ) is a Bijective mapping. Formerly q called as Intuitionistic Fuzzy generalized semi-pre homeomorphism when q is together IFGSP continuous function also IFGSP closed function.

Pre-Semi Homeomorphisms Generalized Context of Intuitionistic Fuzzy Topological Spaces

Authors proposed intuitionistic fuzzy generalised pre-semi homeomorphisms in this research also looked into several features.

Definition 23:

If q : (A, τ) → (B, σ) is a Bijective mapping. Then q is called as Intuitionistic Fuzzy generalized pre semi homeomorphism when q is both an IFGP Set constant plotting and IFGP Set closed plotting.

For the sake of clarity, consider the details U= ⟨a, (μ, μ), (ν, ν)⟩ as a substitute associated U= ⟨a,(u/μu , v/μv), (u/νu , v/νv)⟩ has all samples used in this paper.

Correspondingly V= ⟨a, (μ, μ), (ν, ν)⟩ as a substitute has V= ⟨a,(a/μa, b/μb), (a/νa, b/νb)⟩ has subsequent samples.

Definition 24:

If U is IF Set in an IFT Set (A, τ). Formerly comprehensive pre-semi inside has U also generalized pre-semi conclusion has U be demarcated with

(i) gpsin (U) = ∪{ E / E be IFGPS Open Set has A also E ⊆U }.

(ii) gpsc (U) = ∩{ F / F be IFGPS Closed Set has A also U ⊆F }.

Note that for any IF Set U in (A, τ), we have $gpsc(U^c) = (gpsin(U))^c$ and $gpsin(U^c) = (gpsc(U))^c$.

Theorem 25:

Each intuitionistic fuzzy homeomorphism is an intuitionistic fuzzy generalised pre-semi homeomorphisms.

Proof:

If q : (A, τ) → (B, σ) is an IFHM. Formerly q be IF continuous also IF closed. Given that Each function that is continuous within the realm of Intuitionistic Fuzzy (IF) be IFGP Set continuous, also all IF closed mapping be IFGP Set closed plot, q be IFGP Set continuous also IFGP Set closed. Therefore q is Intuitionistic Fuzzy generalised pre-semi homeomorphisms.

Example 26:

Let A = {u,v}, B = {a,b} and E1 = ⟨a, (0.5, 0.6), (0.3, 0.1)⟩, E2 = ⟨b, (0.4, 0.3), (0.4, 0.5)⟩. Next τ = {0~, E1, 1~} also σ = {0~, E2, 1~} be IFTs happening X & Y correspondingly. Describe bijective function q : (A, τ) → (B, σ) via q(a) = a also q(v) = b. Then q be Intuitionistic Fuzzy generalised pre-semi homeomorphisms except Intuitionistic Fuzzy homeomorphisms.

Theorem 27:

Every Intuitionistic Fuzzy α homeomorphisms is Intuitionistic Fuzzy generalised pre-semi homeomorphisms.

Proof:

If q : (A, τ) → (B, σ) is intuitionistic fuzzy α homeomorphisms. Then q be IFα continuous also IFα closed. Subsequently all IFα nonstop mapping is IFGP Set continuous also all IFα closed mapping is IFGP Set closed plotting, q be IFGP Set continuous and IFGPS closed. Therefore q is intuitionistic fuzzy generalised pre-semi homeomorphisms.

Example 28:

In the given instance bijective mapping q : (A, τ) → (B, σ) in q(u) = a also q(v) = b termed as Intuitionistic Fuzzy generalised pre-semi homeomorphisms. but not Intuitionistic Fuzzy simplified α homeomorphisms.

Theorem 29:

Every Intuitionistic Fuzzy generalised pre-semi homeomorphisms be Intuitionistic Fuzzy generalised semi-pre regular homeomorphisms.

Proof:

Assume q : (A, τ) → (B, σ) is Intuitionistic Fuzzy generalised pre-semi homeomorphisms. Then q is IFGP Set continuous and IFGP Set closed. Given that every IFGP Set continuous function be IFGSP Regular continuous also all IFGP Set closed mapping be IFGSP Regular closed mapping, q is IFGSP Regular continuous also IFGSP Regular closed. Thus q be Intuitionistic Fuzzy generalised semi-pre regular homeomorphisms.

Example 30:

Let A = {u,v}, B = {a,b} and E1 = ⟨x, (0.3, 0.2), (0.7, 0.8)⟩, E2 = ⟨y, (0.5, 0.6), (0.5, 0.4)⟩. Next τ = {0~, E1, 1~} also σ = {0~, E2, 1~} be IFTs on Both A, B in that order. Describe a Bijective mapping q : (A, τ) → (B, σ) via q(u) = a and q(v) = b. Afterwards q is Intuitionistic Fuzzy generalised semi-pre regular homeomorphisms but not Intuitionistic Fuzzy generalised pre-semi homeomorphisms.

Theorem 31:

Every Intuitionistic Fuzzy generalised pre-semi regular homeomorphisms is intuitionistic fuzzy generalised semi-pre regular homeomorphisms.

Proof:

Assume q : (A, τ) → (B, σ) be Intuitionistic Fuzzy generalised pre-semi regular homeomorphisms intuitionistic fuzzy generalised semi-pre regular homeomorphisms. Next q is IFGP Set continuous also IFGP Set closed. Because all IFGP Set continuous function is IFGSP continuous also all IFGP Set closed mapping be IFGSP closed function, q be IFGSP continuous also IFGSP closed. For this reason q is intuitionistic fuzzy generalised semi-pre regular homeomorphisms.

Example 32:

illustration of the bijective mapping q : (A, τ) → (B, σ) by means of q(u) = a also q(v) = b termed as intuitionistic fuzzy generalised semi-pre regular homeomorphisms but not intuitionistic fuzzy generalised pre-semi regular homeomorphisms.

Theorem 33:

Let q: (A, τ) → (B, σ) be intuitionistic fuzzy generalised pre-semi regular homeomorphisms, then q is IF homeomorphism if A as well as B are IFPSM * 1/2 space.

Proof:

If V is IFC Set in B. Next q-1 (B) be Intuitionistic Fuzzy generalised pre-semi regular homeomorphisms in A, based on assumption. Because A is an IFPSM * 1/2 space, q-1 (V) is IFC Set in A. Consequently q is IF continuous plotting. Based on assumptions q-1 : (B, σ) → (A, τ) termed as IFGPS continuous plotting. Assume U termed as IFC Set within A. Next (q-1)-1 (U) = q(U) is intuitionistic fuzzy generalised pre-semi regular homeomorphisms involves B, via assumption. Since B be IFPSM * 1/2 space, q(U) termed as IFC Set in B. Thus q-1 be IF continuous mapping. Then fnction q be IF Homeomorphism.

Theorem 34:

Assume q : (A, τ) → (B, σ) labelled as bijective plotting. When q is an IFGP Set continuous plotting, the next statements are equivalent: (i) q called as IFGPS open mapping. (ii) q be intuitionistic fuzzy generalised pre - semi regular homeomorphisms. (iii) q called as IFGPS closed mapping.

Lemma 35:

The composition of two intuitionistic fuzzy generalised pre semi regular homeomorphisms need not be an intuitionistic fuzzy generalised pre-semi regular homeomorphism in general.

Example 36:

Assume A = {u,v}, B = {x,y} and C = {r,s}. Let E1 = ⟨a, (0.2, 0.1), (0.6, 0.7)⟩, E2 = ⟨b, (0.2, 0.8), (0.8, 0.2)⟩, E3 = ⟨c, (0.5, 0.6), (0.5, 0.4)⟩. Then τ = {0~, E1, 1~}, σ = {0~, E2, 1~} also η = {0~, E3, 1~} are IFTs on A, B and C respectively.

Conclusion

In this study, generalised intuitionistic fuzzy pre-homeomorphisms are introduced. We take a look at some of their traits.

Reference

[1] Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 1986.
[2] Chang, Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, 1968.
[3] Coker, An introduction to Intuitionistic Fuzzy Topological Space, Fuzzy Sets and Systems, 88, 1997.
[4] Gurcay, Haydar and Coker, On fuzzy continuity in Intuitionistic Fuzzy Topological Spaces, The Journal of Mathematical Fuzzy, 1997.
[5] Ramesh, Thirumalaisamy, Kavunthi, Generalized semi pre regular continuous and irresolute mappings in Intuitionistic Fuzzy Topological Spaces, 2013.
[6] Ramesh , Thirumalaisamy, On GSPR Closed Mappings and GSPR Homeomorphisms in Intuitionistic Fuzzy Topological Spaces, International Journal of Mathematical Trends and Technology, 2013.
[7] Sakthivel, Alpha generalized homeomorphism in Intuitionistic Fuzzy Topological space, Notes on Intuitionistic Fuzzy Sets, 2011.
[8] Sampoornam, Gnanambal Ilango ,Ramesh, On Generalized Pre Semi Closed Sets in Intuitionistic Fuzzy Topological Spaces, International journal of Mathematical Trends and Technology, 2013.

[9] Sampoornam, Gnanambal Ilango, Ramesh, Generalized Pre Semi Continuous and Irresolute Mappings in Intuitionistic Fuzzy Topological Space, International Journal of Innovative Research in Science Engineering and Technology, Aug 2013, 4034- 4040.

[10] Sampoornam, Gnanambal Ilango, Arif mohammed, Generalized Pre semi Closed Mappings in Intuitionistic Fuzzy Topological Spaces, International Journal of Engineering Research and Technology, 2013.

[11] Santhi, Jayanthi, Intuitionistic Fuzzy Generalized Semi-Pre Continuous Mappings, International Journal of Contemporary Mathematical Sciences, 2010.

[12] Santhi , Jayanthi, Intuitionistic fuzzy generalized semi pre closed mappings, Journal of Informatics and Mathematical Sciences, 2010.

[13] Santhi, Jayanthi, Intuitionistic fuzzy generalized semi-pre homeomorphisms, Journal of Pure Mathematics, 2010.

[14] Seok Jong Lee, Eun Pyo Lee, The category of Intuitionistic Fuzzy Topological Spaces, Bulletin of the Korean Mathematical Society, 2000.

[15] Zadeh, Fuzzy Sets, Information and control, 1965.

[16] Selma Ozcag and Dogan Coker, On connectedness in intuitionistic fuzzy special topological spaces, International Journal of Mathematics and Mathematical Sciences, 1998.