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Fuzzy Traveling Salesman Problem of Fuzzy Hexagonal Numbers using Ranking Method and Dijkstra's Algorithms

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Abstract

The aim of this article is to find the fuzzy shortest possible distance for a fuzzy traveling salesman problem of hexagonal fuzzy numbers. We used to formulate the numerical example using dijkstra's algorithm.

Introduction The traveling salesman problem (TSM) involves finding the shortest possible route to multiple destination and returning to the starting point. —_ there we several applications of TSP, such as vehicle routing, scheduling. The TSP in a serious challenge for the logistics and supply chain industry because of involves optimizing the delivery router for multiple destinations while considering various constraints such as, traffic, delivery windows and customer request with multiple vehicle, more cities and multiple sales professionals, TSP become a more challenging to solve. In this article we proposed shorter possible distance route from multiple destination and returning to the starting point using ranking method of hexagonal fuzzy numbers.

Conclusions: The ranking approach is used in this paper to transform hexagonal fuzzy numbers into expected time, or standard time, for each activity. Thus, using the DIJKSTRA algorithm, the fuzzy shortest path is found. It assists decision makers in selecting the optimal, shortest path in a fuzzy environment by applying the ranking algorithm.

Keywords: fuzzy number, Hexagonal fuzzy number, dijkstra's algorithm Ranking algorithm, sales man.

1. Introduction

The traveling salesman problem (TSM) involves finding the shortest possible route to multiple destination and returning to the starting point. —_ there we several applications of TSP, such as vehicle routing, scheduling. The TSP in a serious challenge for the logistics and supply chain industry because of involves optimizing the delivery router for multiple destinations while considering various constraints such as, traffic, delivery windows and customer request.With multiple vehicle, more cities and multiple sales professionals, TSP become a more challenging to solve. In this article we proposed shorter possible distance route from multiple destination and returning to the starting point using ranking method of hexagonal fuzzy numbers.

2 Preliminaries

2.1 Definition

Let the universe be U. The definition of the fuzzy setI on U is as follows,

$$\mathrm{I} = \{x, \chi_{\mathrm{A}}(x), x \in \mathrm{U}\}$$

In which $\chi_{\mathrm{A}}(x): \mathrm{U} \rightarrow [0,1]$ is called membership

2.2 Definition

If there is an element $x \in \mathrm{U}$ whose membership value is one, that is $\chi_{\mathrm{I}}(x) = 1$. Then a fuzzy set I is normal.

2.3 Definition

Fuzzy number is defined as fuzzy set I of real line R with membership function $\chi_{\mathrm{I}}(x): \mathrm{R} \rightarrow [0,1]$ if
(i) I is normal and convexity
(ii) I must be bounded
(iii) α_{I} must be closed interval for every $\alpha \in [0,1]$

2.4 Definition

A fuzzy number I_H is an HFN and is represented as $\mathrm{I}_H = (n_1, n_2, n_3, n_4, n_5, n_6)$. In which real numbers (n_1, n_2, n_3, n_4, n_5, and n_6) are found. The function its membership is listed below

$$\mathrm{f(x)} = \begin{cases} 0.5\left[\dfrac{x - n_1}{n_3 - n_1}\right] & \text{for } n_1 \leq x \leq n_2 \\ 0.5 + 0.5\left[\dfrac{x - n_2}{n_3 - n_2}\right] & \text{for } n_2 \leq x \leq n_3 \\ 1 & \text{for } n_3 \leq x \leq n_4 \\ 1 - 0.5\left[\dfrac{x - n_4}{n_5 - n_4}\right] & \text{for } n_4 \leq x \leq n_5 \\ 0.5\left[\dfrac{n_6 - x}{n_6 - n_5}\right] & \text{for } n_5 \leq x \leq n_6 \end{cases}$$

3 Operation of hexagonal fuzzy number

Assuming that $\mathrm{S}_H = (g_1, g_2, g_3, g_4, g_5, g_6)$ and $\mathrm{T}_H = (h_1, h_2, h_3, h_4, h_5, h_6)$. are two hexagonal fuzzy numbers, the three operations that can be carried out on them are as follows,

1. Addition: $\mathrm{S}_\alpha + \mathrm{T}_\alpha = (g_1 + h_1, g_2 + h_2, g_3 + h_3, g_4 + h_4, g_5 + h_5, g_6 + h_6)$
2. subtraction: $\mathrm{S}_\alpha - \mathrm{T}_\alpha = (g_1 - h_1, g_2 - h_2, g_3 - h_3, g_4 - h_4, g_5 - h_5, g_6 - h_6)$
3. multiplication : $\mathrm{S}_\alpha * \mathrm{T}_\alpha = (g_1 * h_1, g_2 * h_2, g_3 * h_3, g_4 * h_4, g_5 * h_5, g_6 * h_6)$

3.1 α-cut of Hexagonal Fuzzy Number

The α-cut of normal hexagonal fuzzy number $\mathrm{S}_H = (g_1, g_2, g_3, g_4, g_5, g_6)$ given by the definition (2.2). therefore $\chi_{\mathrm{I}}(x) = 1$ for all $\alpha \in [0,1]$

$$\mathrm{S}_H=\begin{cases}[2\alpha(g_3-g_1)+g_1,-2\alpha(g_6-g_5)+g_6] & \text{for } \alpha\in[0,0.5)\\ [2\alpha(g_3-g_2)-g_3+2g_2,-2\alpha(g_5-g_4)+2g_5-g_4] & \text{for } \alpha\in[0.5,1]\end{cases}$$

3.2 Preposition

1. Let us consider two $\mathrm{HFN}_H=(g_1,g_2,g_3,g_4,g_5,g_6)$ and $\mathrm{T}_H=(h_1,h_2,h_3,h_4,h_5,h_6)$. Then the addition of two hexagonal fuzzy number.

Proof

Let us add the alpha cut S_α and T_α of S_H and T_H using interval arithmetic.

$$\mathrm{S}_\alpha+\mathrm{T}_\alpha=\begin{cases}[2\alpha(g_3-g_1)+g_1,-2\alpha(g_6-g_5)+g_6]+\\ [2\alpha(h_3-h_1)+h_1,-2\alpha(h_6-h_5)+h_6], & \text{for } \alpha\in[0,0.5)\\ [2\alpha(g_3-g_2)-g_3+2g_2,-2\alpha(g_5-g_4)+2g_5-g_4]+\\ [2\alpha(h_3-h_2)-h_3+2h_2,-2\alpha(h_5-h_4)+2h_5-h_4], & \text{for } \alpha\in[0.5,1]\end{cases}$$

$\mathrm{S}_\alpha=[2\alpha+g_1,-2\alpha+g_6]$ for $\alpha\in[0,0.5)$
$\mathrm{T}_\alpha=[4\alpha+h_1,-4\alpha+h_6]$ for $\alpha\in[0.5,1]$
So that, $\mathrm{S}_\alpha+\mathrm{T}_\alpha=[6\alpha+(g_1+h_1),-6\alpha+(g_6+h_6)]$
Consider the example of HFN
$\mathrm{S}_H=(2,5,6,3,7,3)$ and $\mathrm{T}_H=(9,7,5,1,6,4)$
$\mathrm{S}_\alpha=[2\alpha+2,-2\alpha+3]$ for $\alpha\in[0,0.5)$
$\mathrm{T}_\alpha=[4\alpha+9,-4\alpha+4]$ for $\alpha\in[0.5,1]$
Since for both $\alpha\in[0,0.5)$ and $\alpha\in[0.5,1]$ arithmetic intervals are same.
Therefore $\mathrm{S}_\alpha+\mathrm{T}_\alpha=[6\alpha+11,-6\alpha+7]$ for all $\alpha\in[0,1]$

$$\alpha=0\implies \mathrm{S}_0+\mathrm{T}_0=[11,7]$$
$$\alpha=0.5\implies \mathrm{S}_{0.5}+\mathrm{T}_{0.5}=[13,4]$$
$$\alpha=1\implies \mathrm{S}_1+\mathrm{T}_1=[16,1]$$

Hence, $\mathrm{S}_\alpha+\mathrm{T}_\alpha=[11,13,16,1,4,7]$, Every point corresponds to part of the two HFN.

Consequently, the interval is covered by the addition of two α-cut.
2 If we assume two hexagonal fuzzy numbers $\mathrm{S}=(g_1,g_2,g_3,g_4,g_5,g_6)$ and $\mathrm{H}=(t_1,t_2,t_3,t_4,t_5,t_6)$. Then the subtraction of two numbers.

Proof

Let us add the alpha cut S_α and T_α of S_H and T_H using interval arithmetic.

$$\mathrm{S}_\alpha+\mathrm{T}_\alpha=\begin{cases}[2\alpha(g_3-g_1)+g_1,-2\alpha(g_6-g_5)+g_6]-\\ [2\alpha(h_3-h_1)+h_1,-2\alpha(h_6-h_5)+h_6], & \text{for } \alpha\in[0,0.5)\\ [2\alpha(g_3-g_2)-g_3+2g_2,-2\alpha(g_5-g_4)+2g_5-g_4]-\\ [2\alpha(h_3-h_2)-h_3+2h_2,-2\alpha(h_5-h_4)+2h_5-h_4], & \text{for } \alpha\in[0.5,1]\end{cases}.$$

$\mathrm{S}_\alpha=[2\alpha+g_1,-2\alpha+g_6]$ for $\alpha\in[0,0.5)$
$\mathrm{T}_\alpha=[4\alpha+h_1,-4\alpha+h_6]$ for $\alpha\in[0.5,1]$
since, $\mathrm{S}_\alpha-\mathrm{T}_\alpha=[-2\alpha+(g_1-h_1),2\alpha+(g_6-h_6)]$

Consider the example of HFN
$S_H = (1,3,5,7,9,2)$ and $T_H = (2,4,6,8,1,10)$
$S_\alpha = [2\alpha + 1, -2\alpha + 2]$ for $\alpha \in [0,0.5)$
$T_\alpha = [4\alpha + 2, -4\alpha + 10]$ for $\alpha \in [0.5,1]$

Since for both $\alpha \in [0,0.5)$ and $\alpha \in [0.5,1]$ arithmetic intervals are same.
Therefore $S_\alpha + T_\alpha = [-2\alpha - 1, 2\alpha - 8]$ for all $\alpha \in [0,1]$

$$\alpha = 0 \Rightarrow S_0 + T_0 = [-1, -8]$$
$$\alpha = 0.5 \Rightarrow S_{0.5} + T_{0.5} = [-2, -7]$$
$$\alpha = 1 \Rightarrow S_1 + T_1 = [-3, -6]$$

Hence, $S_\alpha - T_\alpha = [-1, -2, -3, -6, -7, -8]$, Every point corresponds to part of the two HFN.
Consequently, the interval is covered by the addition of two α-cut.
3 If we assume two hexagonal fuzzy numbers $S = (g_1, g_2, g_3, g_4, g_5, g_6)$ and $H = (t_1, t_2, t_3, t_4, t_5, t_6)$. Then the multiplication of two numbers.
proof
Let us add the alpha cut S_α and T_α of S_H and T_H using interval arithmetic.

$$S_\alpha + T_\alpha = \begin{cases} [2\alpha(g_3 - g_1) + g_1, -2\alpha(g_6 - g_5) + g_6] * \\ [2\alpha(h_3 - h_1) + h_1, -2\alpha(h_6 - h_5) + h_6], & \text{for } \alpha \in [0,0.5) \\ [2\alpha(g_3 - g_2) - g_3 + 2g_2, -2\alpha(g_5 - g_4) + 2g_5 - g_4] * \\ [2\alpha(h_3 - h_2) - h_3 + 2h_2, -2\alpha(h_5 - h_4) + 2h_5 - h_4], & \text{for } \alpha \in [0.5,1] \end{cases}.$$

$S_\alpha = [2\alpha + g_1, -2\alpha + g_6]$ for $\alpha \in [0,0.5)$
$T_\alpha = [4\alpha + h_1, -4\alpha + h_6]$ for $\alpha \in [0.5,1]$
since, $S_\alpha * T_\alpha = [(2\alpha + g_1)(4\alpha + h_1), (-2\alpha + g_6)(-4\alpha + h_6)]$
4 If we assume two hexagonal fuzzy numbers $S = (g_1, g_2, g_3, g_4, g_5, g_6)$ and $H = (t_1, t_2, t_3, t_4, t_5, t_6)$. Then the division of two numbers.

Proof

Let us add the alpha cut S_α and T_α of S_H and T_H using interval arithmetic.

$$S_\alpha / T_\alpha = \begin{cases} [2\alpha(g_3 - g_1) + g_1, -2\alpha(g_6 - g_5) + g_6] / \\ [2\alpha(h_3 - h_1) + h_6, -2\alpha(h_6 - h_5) + h_6], \\ [2\alpha(g_3 - g_2) - g_3 + 2g_2, -2\alpha(g_5 - g_4) + 2g_5 - g_4] / \\ [2\alpha(h_3 - h_2) - h_3 + 2h_2, -2\alpha(h_5 - h_4) + 2h_5 - h_4], & \text{for } \alpha \in [0,0.5) \end{cases}$$

$S_\alpha = \left[2\alpha + g_1, -2\alpha + g_6\right]$ for $\alpha \in [0.5,1]$
$T_\alpha = [2\alpha + h_1, -4\alpha + h_6]$ for $\alpha \in [0.5,1]$
since, $S_\alpha / T_\alpha = [(2\alpha + g_1)/(4\alpha + h_6), (-2\alpha + g_6)/(-4\alpha + h_6)]$

4 New Ranking Function

The classical set I_α called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $I_H = (n_1, n_2, n_3, n_4, n_5, n_6)$ is no less than, α it is defined as $I = \left\{x \in U / \chi_{I_{H(x)}} \geq \alpha\right\}$

$$= \begin{cases} [\mathrm{P}_1(\alpha), \mathrm{P}_2(\alpha)], & \text{for } \alpha \in [0,0.5) \\ [\mathrm{Q}_1(\alpha), \mathrm{Q}_2(\alpha)], & \text{for } \alpha \in [0.5,1] \end{cases}$$
$$= \begin{cases} [\mathrm{P}_1(\alpha), \mathrm{Q}_1(\alpha)], & \text{for } \alpha \in [0,0.5) \\ [\mathrm{P}_2(\alpha), \mathrm{Q}_2(\alpha)], & \text{for } \alpha \in [0.5,1] \end{cases}$$
$$= [n_1 + \alpha(n_3 - n_1) + n_6 + \alpha(n_4 - n_6)] \text{ for } \alpha \in [0,1]$$

It provides results, I is a fuzzy number then the ranking is defined by $\mathrm{R}(\mathrm{I}_H) = \int_0^1 2(0.5)(\mathrm{I}_h\alpha^L, \mathrm{I}_h\alpha^U)D\alpha$ where $(\mathrm{I}_h\alpha^L, \mathrm{I}_h\alpha^U)$ is the α level cut of the fuzzy number I_H $\mathrm{R}(\mathrm{I}_H) = \int_0^1 2(0.5)[n_1 + \alpha(n_3 - n_1) + n_6 + \alpha(n_4 - n_6)]D\alpha$

5 Description of the Model

Using the Ranking algorithm $\mathrm{R}[\mathrm{I}_H]$, hexagonal fuzzy numbers are transformed into anticipated time (Normal time) for every activity.
These numbers are interpreted as the typical travel time between the nodes and the provided algorithm is used to find the path, or least distance.

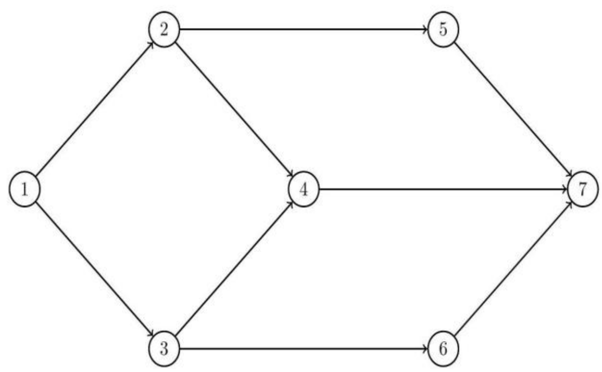
5.1 Algorithm

step 1:
Create the network diagram in accordance with the task assigned.
step 2:
Find the (normal) Expected time using the Ranking algorithm given the HFN.
step 3:
Find out how many paths, or possible routes, there are from the starting node to the finishing nodes.
step 4:
Finally, DIJKSTRA's algorithm will be used to find the shortest path or minimal traveling time.

6 Numerical example-1

Think of a project that has activities and nodes. HFN is a representation of the separation between them.

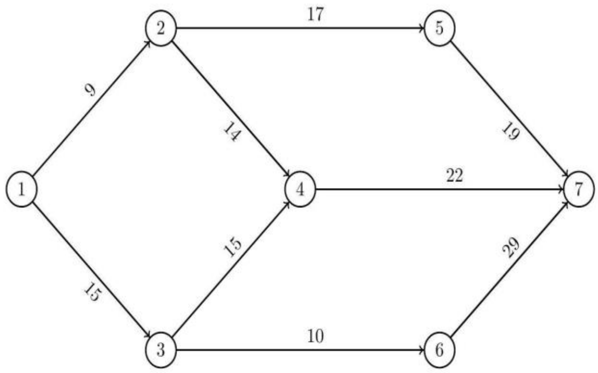
Step 1: Create the network diagram in accordance with the task assigned.



Activity	HFN
$1-2$	(4,3,2,7,6,5)
$1-3$	(6,5,1,14,3,9)
$3-4$	(9,1,3,12,10,6)
$2-4$	(1,3,10,9,7,18)
$3-6$	(2,4,1,9,6,8)
$2-6$	(8,4,3,14,1,9)
$5-7$	(2,9,8,11,10,17)
$4-7$	(4,8,10,12,13,18)
$6-7$	(16,11,12,17,10,13)

Table 1: Activities and hexagonal fuzzy number

step 2: Find the (normal) Expected time using the Ranking algorithm given the HFN.



Step 3 : Find out how many paths, or possible routes, there are from the starting node to the finishing nodes.

$1-2-4-7$	45
$1-2-5-7$	45
$1-3-4-7$	52
$1-3-6-7$	54

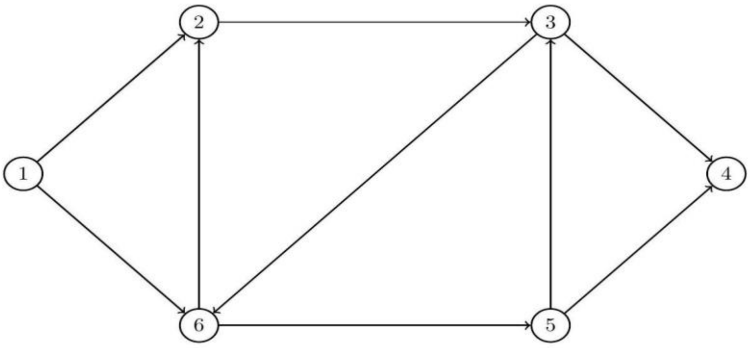
Table 2: path and time using shortest traveling path method

Step 4: Finally, DIJKSTRA's algorithm will be used to find the shortest path or minimal traveling time. Fuzzy shortest traveling path is $1-2-4-7=45$

7 Numerical example-2

one more numerical illustration of finding the shortest path. With a few activities and nodes in mind, the HFN represents the distance between them.

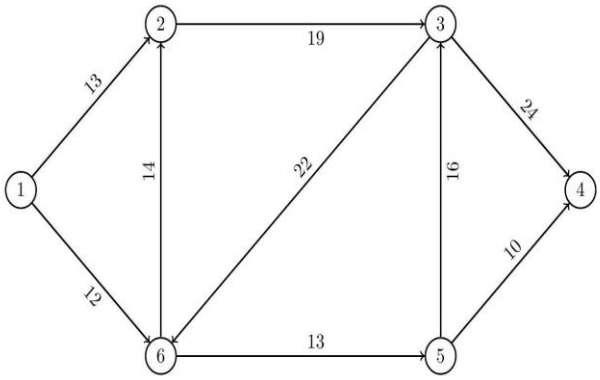
Step 1:Create the network diagram in accordance with the task assigned.



Activity	HFN
1-2	(2,1,5,8,3,11)
1-6	(3,11,7,5,2,9)
2-3	(9,3,12,7,14,10)
6-2	(5,1,10,4,8,9)
6-5	(7,15,9,4,10,6)
5-4	(1,5,3,7,6,9)
5-3	(9,13,5,7,12,3)
3-4	(12,10,14,9,7,11)
3-6	(12,9,16,6,2,10)

Table 3: Activities and hexagonal fuzzy number

Step 2:Find the (normal) Expected time using the Ranking algorithm given the HFN.



Step 3: Find out how many paths, or possible routes, there are from the starting node to the finishing nodes.

$1-2-3-4$	55
$1-6-5-4$	35
$1-6-2-3-4$	68

$1-6-5-3-4$	64
$1-2-3-6-5-4$	77

Table 4: path and time using shortest traveling path method

Step 4: Finally, "DIJKSTRA's algorithm" will be used to find the shortest path or minimal traveling time. Fuzzy shortest traveling path is $1-6-5-4=35$

8 Conclusion

The ranking approach is used in this paper to transform hexagonal fuzzy numbers into expected time, or standard time, for each activity. Thus, using the DIJKSTRA algorithm, the fuzzy shortest path is found. It assists decision makers in selecting the optimal, shortest path in a fuzzy environment by applying the ranking algorithm.

9 Declaration

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9.2 Conflicts of interests: The authors have no compelling interests to declare that are relevant to the content of this article.

Refrences

[1] Abbasi.F and Allahviranloo.T (2021) The Fuzzy Arithmetic Operations of Transmission Average on Pseudo-Hexagonal Fuzzy Numbers and Its Application in Fuzzy System Reliability Analysis, Fuzzy Information and Engineering ,Volume 13,58-78

[2] Ching-Hsue Cheng (1998) A new approach for ranking fuzzy numbers by distance method, A new approach for ranking fuzzy numbers by distance method,Volume 95, 307-317.

[3] Deng Feng, Jiang Xia Nan,Mao Jun Zhang, Venkataraman. Y and Balas.E.V (2010) A Ranking Method of Triangular Intuitionistic Fuzzy Numbers and Application to Decision Making, International Journal of Computational Intelligence Systems, Volume 3, 522-530.

[4] Kirtiwant P. and Pathade.A (2016) Optimal solution of balanced and unbalanced fuzzy transportation problem using hexagonal fuzzy numbers ,International Journal of Mathematical Research ,Volume 5(2), 131-137.

[5] Lakshmana Gomathi Nayagam.V, Jagadeeswari Murugan,and Suriyapriya.k (2020) Hexagonal fuzzy number inadvertences and its applications to MCDM and HFFLS based on complete ranking by score functions, Computational and Applied Mathematics, Volume 39,323.

[6] Najeeb Alam Khan, Oyoon Abdul Razzaq, Avishek Chakraborty, Sankar Parsad Mondal, and Shariful Alam (2020) Measures of Linear and Nonlinear Interval-Valued Hexagonal Fuzzy Number, International Journal of Fuzzy System Applications, Volume 9,40.

[7] Saeidifar.A (2011) Application of weighting functions to the ranking of fuzzy numbers, Computers and Mathematics with Applications, Volume 62, 2246 - 2258 .

[8] Sangeetha. K and Parimala. M (2021) On solving a fuzzy game problem using hexagonal fuzzy numbers, Materials Today: Proceedings, Volume $47{,}2102-2106$.

[9] Tharakeswari. V, Kameswari. M, Mariappan. P, Venkataraman. Y and Balas.E.V (2023) Role of Hexagonal Fuzzy Numbers While Applying the MaxMin Concept to a Transportation Problem, Fuzzy Optimization, Decisionmaking and Operations Research, 161-175.

[10] Vincent F. Yu, Luu Huu Van, Luu Quoc Dat, Ha Thi Xuan Chi, Shuo-Yan Chou and Truong Thi Thuy Duong (2016) Analyzing the Ranking Method forFuzzy Numbers in Fuzzy Decision Making Based on the Magnitude Concepts, International Journal of Fuzzy Systems, Volume 19, 1279-1289.