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On Slightly Delta Generalized Pre-Continuous Functions

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Abstract:

The purpose of this article is to introduce and investigate the properties of slightly δgp-continuous functions. Also, the relationships of slightly δgp-continuous functions and graphs are investigated.

Keywords- slight continuity, slight pre-continuity, slight gpr-continuity, slight δgp-continuity.

1.Introduction and Preliminaries:

In 1982,Mashhour et al. [9] introduced preopen sets and pre-continuity in topology. R.C.Jain [8] presented the idea of slight continuity and studied its fundamental characteristics. Balasubramanian et al. (2011) introduced the notion of slight gpr-continuity[2] as a generalization of slight pre-continuity[2]. J.B. Toranagatti [15,16] recently introduced the concepts of δgp-continuity and contra δgp-continuity in topological spaces.

In this paper, a new strong form of slight gpr-continuity,called slight δgp-continuity,is introduced. It is shown that slight δgp-continuity is strictly weaker than contra δgp-continuity and δgp-continuity. Relationships between slight δgp-continuity and graphs are investigated. Also,additional properties of these functions are investigated.

Throughout this paper, (X, τ) and (Y, σ) (or X and Y) represents a topological space on which no separation axioms are assumed, unless otherwise mentioned. The closure and interior of $M \subseteq X$ will be denoted by cl(M) and int(M) respectively.

Definition 1.1. A subset K of a topological space X is called pre-closed[10](resp.,regular closed[13]) if $cl(int(K)) \subseteq K$(resp.,$cl(int(K)=K$.

Definition 1.2 A subset K of a topological space X is called δ-closed[18] if $K = cl_\delta(K)$ where

$cl_\delta(K) = \{x \in X: int(cl(U)) \cap K = \phi, U \in \tau \text{ and } x \in U\}$.

Definition 1.3 A subset K of a topological space X is called δgp-closed[3](resp,gpr-closed[6] and gp-closed[9]) if $pcl(K) \subseteq U$ whenever $K \subseteq U$ and U is δ-open (resp, regular open and open) in X.

Definition 1.4 A function $f:X \to Y$ from a topological space X into a topological space Y is called,

(i) δ-irresolute [7] if $f^{-1}(M)$ is δ-closed in X for every δ-closed set M of Y.
(ii) slightly continuous[8] (resp, slightly gp-continuous and slightly gpr-continuous[2]) if $f^{-1}(M)$ is closed (resp.,gp-closed and gpr-closed) in X for every clopen set M of Y.

(iii) δgp-continuous[15](resp, contra δgp-continuous[16]) f^{-1}(M) is δgp-open](resp, δgp-closed) in X for every open set M of Y.

(iv) δgp-irresolute [15] if f^{-1}(M) is δgp-closed in X for every δgp-closed set M of Y.

(v) pre δgp-closed[16] if the image of every δgp-closed set of X is δgp-closed in Y.

Definition 1.5 A space X is called,

(i) locally discrete[11] if every open subset is closed .

(ii) Submaximal[14] if every pre-open set is open in X.

(iii) δgp-additive[16]if δGPC(X) is closed under arbitrary intersections.

(iv) δgp$T_{1/2}$-space[15] if every δgp-closed subset of X is pre-closed.

Theorem 1.6[16]

If M and N are δgp-open subsets of a submaximal space X, then M∩ N is δgp-open in X.

2. Slightly δgp-Continuous Functions

Definition 2.1.

A function f: X → Y is called slightly δ-generalized pre-continuous (briefly slightly δgp-continuous) if inverse image of every clopen subset of Y is δgp-open in X.The proof of the following Theorem is straightforward and hence omitted.

Theorem 2.2. For a function f: X → Y, the following statements are equivalent:

i).f is slightly δgp-continuous.

ii). inverse image of every clopen subset of Y is δgp-closed in X.

iii). inverse image of every clopen subset of Y is δgp-clopen in X.

Remark 2.3. We have the following diagram for a function f : (X, τ)→(Y, σ) :

Slight precontinuity contra δgp-continuity

↓ ↓

slight gp-continuity → slight δgp-continuity → slight gpr-continuity

↑

δgp-continuity

None of the implications in above diagram is reversible.

Example 2.3. Let X =Y ={a, b, c,d}, τ = {X, ϕ, {a},{b},{a,b},{a,b,c}} and

σ={Y, ϕ, {a},{b, c}{a,b,c},{b,c,d}}.

(i)Define f : (X, τ)→(Y, σ) by f(a)=a=f(b),f(c)=b and f(d)=c,then f is slightly gpr-continuous but not slightly δgp-continuous since {a} is clopen in Y but f^{-1}({a})={a,b} is not δgp-closed in X .

(ii) Define g : (X, τ)→(Y, σ) by g(a)=a=g(c),g(b)=b and g(d)=c,then g is slightly δgp-continuous but not slightly gp-continuous since {a} is clopen in Y but g^{-1}({a})={a,c} is not gp-closed in X

(iii) Define h : (X, τ)→(Y, σ) by h(a)=a=h(d),h(b)=d and h(c)=b,then h is slightly δgp-continuous but not δgp-continuous since for closed set {d}, h^{-1}({d})={b} is not δgp-closed in X .

Example 2.4. Let R and Q be the real numbers and rational numbers, respectively.

Let M={x ∈ R:x is rational and 0<x<1}.We define two topologies on τ ={R, ϕ, M,R\M} and

σ = {R,ϕ, {0}}. Define f : (R, τ)→(R, σ) by f(x)=1 if x∈Q and f(x)=0 if x∉Q.

Then f is slightly δgp-continuous but not contra δgp-continuous since for closed set R\{0}, f^{-1}(R\{0})=Q is not δgp-closed in (R, τ) .

Theorem 2.5. If X is locally discrete, then the following are equivalent:

(i) f : X → Y is slightly δgp-continuous;
(ii) f : X → Y is δgp-continuous;
(iii) f : X → Y is contra δgp-continuous.

Theorem 2.6. If X is δgp$T_{1/2}$-space, then the following are equivalent:

(i) f : X → Y is slightly gpr-continuous;
(ii) f : X → Y is slightly δgp-continuous;
(iii) f : X → Y is slightly gp-continuous;
(iv) f : X → Y is slightly pre-continuous.

Theorem 2.7 Let f : (X, τ) → (Y, σ) be a slightly δgp-continuous function, then for each p ∈ X and each clopen set N containing f(p), there exists δgp-open set M containing such that f(M) ⊆ N.

Proof. Let p ∈ X and N be a clopen set such that f(N) ⊆ Y. Since f is slightly δgp-continuous f^{-1} (N) is δgp-open in X. If we put M = f^{-1} (N), we have p ∈ M and f(M) ⊆ N.

Let (X, τ) be a topological space. The quasi-topology on X is the topology having as base all clopen subsets of (X, τ). The open (resp. closed) subsets of the quasi- topology are said to be quasi-open (resp. quasi-closed). A point x of a space X is said to be quasi closure of a subset A of X, denoted by cl_qA, if A ∩ U ≠ ϕ for every clopen set U containing x. A subset A is said to be quasi closed if and only if A = cl_q [8]

Theorem 2.8

The following are equivalent for a function f:X⟶Y with X is δgp-additive.

(i) f is slightly δgp-continuous;

(ii) for each p ∈ X and each clopen set N of Y containing f(p), there exists an δgp-open set M in X containing p such that f(M) ⊆ N;

(iii) f(δgpcl(A)) ⊆ cl_q (f(A) for every subset A of X;

(iv) δgpcl(f^{-1}(B)) ⊆ f^{-1}(cl_q (B)) for every subset B of Y.

Proof: (i)→(ii)It follows from Theorem 2.7

(ii)→(i) Let M ⊆ Y be a clopen set such that f(p) ∈ M,then p ∈ f^{-1} (M). From (ii), there exists a δgp-open set Up containing p such that f(Up) ⊆ M, then p∈ Up⊆ f^{-1} (M).

Hence f^{-1} (M) =∪{Up : Up⊂ f^{-1} (M)} is δgp-open in X .

(i)→(iii)Let A ⊆ X. Suppose y∉ cl_q (f(A)), then there exists a clopen set B ⊆ Y containing y such that f(A) ∩ B =ϕ. So,we have,A ∩ f^{-1} (B)=φ and

δgp-cl(A) ∩ f^{-1}(B)=φ which implies f(δgpcl(A)) ∩ B=ϕ and hence y∉f(δgpcl(A)).

Therefore f(δgpcl(A)) ⊆ cl_q (f(A)).

(iii)→(iv)Let B ⊆ Y, then f^{-1} (B) ⊆ X. By (iii), f (δgpcl(f^{-1}(B)) ⊆ cl_q (f (f^{-1} (B))) ⊆ cl_q (B).

Thus δgpcl(f^{-1} (B)) ⊆ f^{-1} (cl_q (B)).

(iv)→(i) Let M be any clopen subset of Y. Then by (iv),

δgpcl(f^{-1}(M) ⊆ f^{-1} (cl_q (M)) = f^{-1} (M) and δgpcl(f^{-1} (M)) = f^{-1} (M).

Therefore , f^{-1} (M) is δgp-closed set in X.

Remark 2.9 The composition of two slightly δgp-continuous functions need not be slightly δgp-continuous as seen from the following examples.

Example 2.10 Let X=Y={a,b,c,d},Z=={a,b,c}, τ = {X, ϕ, {a},{b},{a,b},{a,b,c}} and σ={Y, ϕ, {a},{b, c},{a,b,c},{b,c,d}} and η=={Z, ϕ, {a},{b, c}}. Define f : (X, τ)→(Y, σ) by f(a)=a=f(b),f(c)=b and f(d)=c and g : (Y, σ)→(Z, η) by g(a)=a,g(b)=b,g(c)=c=g(d). Then f and g are slightly δgp-continuous but g⋆f:X⟶Z is not slightly δgp-continuous, since for the clopen set {a} in Z , $(g\star f)^{-1}$ {a} = {a,b} is not δgp-closed in X.

Theorem 2.11

For any two functions f:X⟶Y and g:Y⟶Z,the following hold:

(i)g⋆f is slightlly δgp-continuous if f is δgp-irresolute and g is slightlly δgp-continuous.

(ii)g⋆f is slightlly δgp-continuous if f is δgp-irresolute and g is δgp-continuous.

(iii)g⋆f is slightlly δgp-continuous f is δgp-irresolute and g is slightlly continuous.

Proof:(i) Let N be any clopen set in Z. Then g^{-1}(V) is δgp-open in Y since g is slightlly δgp-continuous. Therefore , f^{-1} [g^{-1}(U)]=$(g\star f)^{-1}$(U) is δgp-closed in X because f is contra δgp-continuous.

Hence g⋆f is contra δgp-continuous.

The proofs of (ii) and (iii) are analogous to (i) with the obvious changes.

Theorem 2.12

Let f:X→Y be δgp-open surjection and g:Y→Z be a function such that g⋆f:X→Z is slightly δgp-continuous,then g is slightly δgp- continuous.

Proof: Let U be any clopen set in Z.Then $(g\star f)^{-1}(U)= f^{-1}(g^{-1}(U))$ is δgp-open in X. Since f is a δgp-open surjection, $f(f^{-1}(g^{-1}(U)))= g^{-1}(U)$ is δgp- open set in Y. Therefore, g is slightly δgp-continuous.

Theorem 2.13

Let f : X → Y be bijective, δ-irresolute and pre-closed . Then for every

δgp-closed set M of X, f(M) is δgp-closed in Y.

Proof. Let M be any δgp-closed set of X and N a δ-open set of Y containing f(M). Since

$f^{-1}(N)$ is a δ-open set of X containing M, pcl(M) ⊆ f^{-1} (N) and hence f(pcl(M)) ⊆ N. Since f is

pre-closed and pcl(M) is pre-closed in X, f (pcl(M)) is pre-closed in Y. Since

pcl (f(M)) ⊆ pcl(f(pcl(M))) ⊆ N, pcl(f(M)) ⊆ N. Therefore f(M) is δgp-closed in Y.

Theorem 2.14 Let g:X→X×Y be the graph function of f:X→Y,defined by g(x)=(x,f(x))

for each x∈X. Then f is slightly δgp-continuous,if g is slightly δgp-continuous.

Proof: Let V be any clopen set in Y, then X×V is a clopen set in X×Y.

It follows that $f^{-1}(U)= g^{-1}(X\times U)$ is δgp-closed in X since g is slightly δgp-continuous.

Hence f is slightly δgp-continuous.

Definition 2.15 A space X is called,

(i) co-T2 [5] (resp, δgp-Hausdorff[16])if for any pair of distinct points x and y, there exist disjoint clopen(δgp-open)sets G and H such that x ∈ G and y ∈ H.
(ii) co –normal[5](resp, strongly δgp-normal) if each pair of disjoint clopen(resp, δgp –closed) sets can be separated by disjoint open sets.
(iii) co-regular[5](resp,strongly δgp-regular) if for each clopen(resp, δgp-closed) set B and each x ∉ B,there exist disjoint open sets M and N such that B ⊂ M and x∈ M.
(iv) mildly compact[12](δgp-compact[17]) if every clopen(resp, δgp- cover of X has a finite sucover
(v) δgp-connected[16] if X is not the union of two disjoint nonempty δgp-open sets.

Theorem 2.16

If a surjective function f:X→Y is slightly δgp-continuous and X is δgp-connected space, then Y is connected.

Proof: Suppose that Y is not a connected space. Then there exist disjoint open sets U and V in Y such that Y=U∪V. Therefore U and V are clopen sets in Y. Since f is slightly δgp-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are δgp-open sets in X . Also f is surjective , $f^{-1}(U)$ and $f^{-1}(V)$

are non empty disjoint and X = $f^{-1}(U) \cup f^{-1}(V)$ which contradicts the fact that X is δgp-connected space. Hence Y is connected.

Theorem 2.17 If f : X → Y is slightly δgp-continuous injection and Y is co-T_2, then X is δgp-Hausdorff

Proof. Let $x_1, x_2, \in X$ and $x_1 \neq x_2$. Then since f is injective and Y is co-T_2, $f(x_1) \neq f(x_2)$ and there exist clopen subsets V_1, V_2 of Y such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. Since f is slightly δgp-continuous, $x_i \in f^{-1}(V_i) \in$ δGPO (X) for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is δgp- Hausdorff.

Theorem 2.18

Assume that X is δgp-additive. If f:X→Y and g:X→Y are slightly δgp-continuous, X is submaximal and Y is co-T_2 . Then F={p ∈ X:f(p)=g(p)} is δgp-closed in X.

Proof: Let p ∉ F, then f(p) ≠ g(p). Since Y is co-T2, there exist clopen subsets M_1 and M_2 of Y such that $f(p) \in M_1$, $g(p) \in M_2$ and $M_1 \cap M_2 = \phi$. Since f and g are slightly δgp-continuous , $f^{-1}(M_1)$ and $g^{-1}(M_2)$ are δgp-open sets in X . Let M= $f^{-1}(M_1)$ and N = $g^{-1}(M_2)$, then M and N are δgp-open sets containing p. Set O=M∩N, then O is δgp-open set containing p. Hence $f(O)\cap g(O)=f(M\cap N)\cap g(M\cap N)\subset f(M)\cap g(N)= M_1\cap M_2 =\phi$ and so O∩F=ϕ.

Then p ∉ δgpcl(F) and hence, F is δgp-closed in X.

Theorem 2.19

If f:X→Y be slightly δgp-continuous injective closed function from a strongly δgp-regular space X onto a space Y,then Y is co-regular.

Proof: Let M be a clopen set in Y and y ∉ M.Take y=f(x).Since f is slightly δgp-continuous , $f^{-1}(M)$ is

δgp-closed. Take N= $f^{-1}(M)$ .We have x ∉ N. Since X is strongly δgp-regular , disjoint open sets U and V in Y such that N ⊂ U and x ∈ V. Then we obtain that M=f(N)⊂ f(U) and y=f(x) ∈ f(V) such that f(U) and f(V) are disjoint open sets in Y. This shows Y is co-regular-normal.

Theorem 2.20

If f:X→Y be slightly δgp-continuous injective open function from a strongly δgp-normal space X onto a space Y,then Y is co-normal.

Proof: Let E and F be disjoint clopen subsets of Y. Since f is slightly δgp-continuous, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint δgp-closed sets in X. Since X is strongly δgp-normal, there exist disjoint open sets U and V such that $f^{-1}(E)\subset U$ and $f^{-1}(F)\subset V$. This implies E⊂ f(U) and F⊂ f(V) and injectivity and openness of f implies f(U) and f(V) are disjoint open sets in Y. This shows Y is co-normal.

Theorem 2.21

If f : X → Y is slightly δgp-continuous surjection, and X is δgp-compact, then Y is mildly compact.

Proof. Let { $V_\alpha : V_\alpha \in$ CO (Y), $\alpha \in I$ } be a cover of Y by clopen sets. Since f is slightly δgp-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ be δgp-open cover of X so there is a finite subset I_0 of I such that

$X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, $Y = U\{V_\alpha : \alpha \in I_0\}$ as f is surjective.

Thus Y is mildly compact.

Theorem 2.22

If f : X → Y is slightly δgp-continuous surjection, and X is δgp-connected, then Y is connected.

Proof. Assume that Y is disconnected. Then exist disjoint non-empty clopen sets U and V for which $Y = U \cup V$. Therefore, $X = f^{-1}(U) \cup f^{-1}(V)$ is the union of two disjoint δgp-open nonempty sets and hence is not δgp-connected.

Definition 2.23 The graph G(f) of a function f:(X,τ)→(Y,σ) is said to be is δgp-co-closed if for each (x,y)∈(X×Y)-G(f) there exist δgp-open set U in X containing x and clopen set V in Y containing y such that (U×V)∩G(f)= ϕ.

Theorem 2.24

The graph G(f) of a function f:(X,τ)→(Y,σ) is δgp-co-closed in X×Y if and only for each (x,y)∈(X×Y)-G(f) there exist δgp-open set U in X containing x and clopen set V in Y containing y such that f(U)∩V=ϕ.

Theorem 2.25

If f:(X,τ)→(Y,σ) is slightly δgp-continuous and Y is co-Hausdorf, then G(f) is is δgp-co-closed in the product space X×Y.

Proof: Let (x,y)∈(X×Y)-G(f),then f(x) ≠y and there exists clopen sets U and V such that f(x)∈U,y∈V and U∩V= ϕ. Since f is slightly δgp-continuous, then there exists a δgp-open set G such that x∈G and f(G)⊂ U and hence we obtain f(G)∩V = ϕ.This shows that G(f) is is δgp-co-closed.

Theorem 2.26

If f:(X,τ)→(Y,σ) is slightly δgp-continuous and Y is co-T_1, then G(f) is is δgp-co-closed in the product space X×Y.

Proof:Let (x,y)∈(X×Y)-G(f),then f(x) ≠y and there exists a clopen sets M of Y such that f(x)∈ M and y∉ M and U∩V= ϕ. Since f is slightly δgp-continuous, then there exists a δgp-open set G such that x∈G and f(G)⊂ M and hence we obtain f(G)∩(Y\M) = ϕ and Y\M is clopen set containing y.This shows that G(f) is δgp-co-closed.

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