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A Study on Catastrophic Retrial Feedback Queueing Network Model with Three Nodes in Steady State

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Abstract

Queueing networks, which are effectively used for performance analysis of various systems, including computer, communications, transportation networks, and manufacturing, are discussed in this article. Under the catastrophic condition, we evaluate a queueing network with feedback and retrial for single server. In the third node the customer can decide whether feedback or leave the system with probabilities t, u and 1-t-u. Also using Little's formula, we are determining the likelihood of having idle customers and determining n number of customers in the system. We are deriving L_q, L_s, W_q *and* W_s for the above model. The purpose of the numerical example gives the richness of our result and the feasibility of this concept.

Keywords: Queueing Network, Feedback, Retrial, Catastrophes.

1. Introduction

Queues form when there is a current demand for a certain service that exceeds supply. If the customer's requested service is not immediately available, a queue will emerge. Our everyday lives involve waiting in line for services; examples include waiting at hotels, at train reservation counters, and at banks. Other scenarios involve waiting, such as tasks that require a machine to complete, aircraft that must circle in the air before landing and vehicles that must stop at traffic signals, among others. Erlang [4], a Danish mathematician and engineer, initially contributed in queueing theory, which formed the foundation of the system. When the behaviour of the system depends on time, it is said to be in a transient state. When the operational characteristics do not depent on time, they are said to be in a steady state.

1.1 Queueing Network

A collection of nodes, with each one representing a different service facility, forms a queuing network. In 1957, Jackson [7] first proposed the application of queueing networks. For the queueing network, Jackson's network [8] has contributed the initial model which is still considering till date. Queuing network models have been utilized in a wide range of businesses and environments, including airport terminals, healthcare systems, flexible manufacturing and production systems, computer networks, and networks for communication. Queuing networks fall into three categories: mixed, closed, and open networks. An arrival enters the system get service and leaves the system in the open network. In closed networks, the number of users is fixed, and they never leave the system, they just switch between queues. Mixed networks allow certain kinds of users in but prohibit others.

1.2 Retrial Queue

Retrial queueing models are predicated on the idea that arriving customers do not queue up or move the system instantly when the server is crowded. Instead, they visit a virtual space called orbit and return later to try their luck. Many computer systems and communication networks may be mathematically modeled using retrial queues, which are employed with great effectiveness in these applications. Kosten [9] initially presents the Retrial queue in 1947. Falin [5] examined the key results and methods of retrial queues theory in a short discussion. Shanmugasundaram et al. [12] examined a feedback retrial queueing network consisting of three nodes under catastrophic circumstances.

1.3 Feedback Queue

In modern service systems, where specific activities may necessitate repeated services, the feedback queue is crucial. Feedback queues are queuing systems that allow customers to return for extra services or because they are dissatisfied with the service they received. Numerous real-world scenarios, including the redesign of manufacturing systems, communication networks, and supermarkets, demonstrate the feedback phenomenon. Takacs introduced the feedback mechanism in 1963 [15]. Working vacations, server breakdowns, and single-server queues with instant feedback have all been taken into consideration by Varalakshmi et al. [17].

1.4 Catastrophe

The evolution of networks and communication systems has resulted in increasing attention to queueing systems with disasters. The subject of disaster queues has been extensively researched by numerous researchers over the past 20 years. Every customer (including the one in service) must exit the system in the case of a complete disaster. There are numerous applications for these kinds of queueing systems. For instance, every customer are required to vacate the premises when an ATM defects in a bank. Moreover, viruses have the ability to erase all stored data from computer networks during a clearing process.

Gelenbe [6] introduced the concept of catastrophes, Furthermore, Chao [2] is motivated to assess the migration process in the context of disasters and virus-infected computer networks. Krishna Kumar et al. [10] considered the transient solution of the system size in the queuing model with the possibility of disasters in a service station. Shanmugasundaram et al. [13] focused on a time-dependent solution of an arrival of customer with retrial customers and providing feedback to non-retrial customers with disasters.

Thangaraj et al [16] studied the analytically obtained transient solution using continuous fractions of the feedback queue under catastrophic condition. Shanmugasundaram et al. [14] have studied a steady-state single server retrial queueing network with catastrophes. Using the continuous fractions approach, Chandrasekaran et al. [3] have examined the transient and reliable nature of the M/M/1 feedback queue in the face of disasters, server outages, and repairs. The past few years, investigation have been done in disaster-prone queueing systems by Chowdhury et al. [1] and Kumar et al. [11]

2. Description of The Model

An investigation is conducted on a queueing network consisting of three nodes, incorporating retrial, feedback, and catastrophe. External customers access the system through a Poisson process with a rate of arrival λ. If the server is not busy, there is a high likelihood that incoming customers will enter the queue with probability s. Customers have a likelihood of $1 - s$ to join the orbit when the server is occupied. It is considered that the capacity is infinite, and that the only person allowed to get service on a first-come, first-served basis is the one at the front of the line.

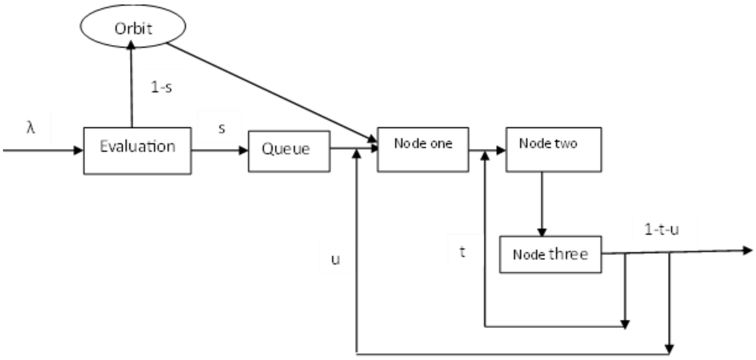
The customer returns to the orbit, if the server is busy. This procedure persists until the customer observes that the server on node one is idle. During the retrial, both requested and required services are provided. If the server is busy, customers are required to wait in an endless circle and repeat their request after an exponential rate. After receiving service at node one, they proceed to node two and then to node three. Here, the customer chooses whether or not to request feedback.

After receiving service in node three with probability $1 - t - u$, if the customer is happy with the service, they exit the system. If the customer is dissatisfied in node three, they will either travel to node two for instant feedback with a probability of t or to node one for delayed feedback with a probability of u. Every node runs on an M/M/1 basis. The service rate for all the three nodes are μ_1, μ_2 *and* μ_3 and are dispersed exponentially. The arrival and service processes, which are followed by the poisson process with rate γ, are the sources of catastrophes.

The server is deactivated, and all available customers are immediately destroyed in the event of a catastrophic system catastrophe. When someone arrives, the server is ready to serve.

In Fig. 1, this mechanism is displayed.

Fig. 1



The transient state probability of the system having n customers at time t is represented as

$$Q_n(t) = Q\{X(t) = n\}, n = 0,1,2,3 \ \ldots$$

Consider Q(x,t) as the probability generating function,

$$Q(x,t) = \sum_{n=0}^{\infty} Q_n(t) x^n$$

In most cases, when time t = 0 it is assumed that there are no customers in the system, denoted as $Q_0(0) = 1$

The probability of $Q_n(t)$ as per the system of differential – difference equations.

$$\begin{aligned} Q_0'(t) = & -\lambda Q_0(t) \\ & + \{[s\mu_1 + \mu_2 + (1-t-u)\mu_3] + [s\mu_1 + \mu_2 + \mu_3 + t\mu_2 + (1-t-u)\mu_3] \\ & + [s\mu_1 + \mu_2 + \mu_3 + u\mu_1 + \mu_2 + (1-t-u)\mu_3] + [(1-s)\mu_1 + \mu_2 + (1-t-u)\mu_3)] \\ & + [(1 - □)□_1 + □_2 + □_3 + □□_2 + (1 - □ - □)□_3] \\ & + [(1-s)\mu_1 + \mu_2 + \mu_3 + u\mu_1 + \mu_2 + (1-t-u)\mu_3]\}Q_1(t) + \gamma[1 - Q_0(t)] \\ = & -\lambda Q_0(t) + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]Q_1(t) + \gamma[1 - Q_0(t)] \qquad (1) \end{aligned}$$

$n = 1,2,3, \ldots$

$$Q_n'(t) = \lambda Q_{n-1}(t) - \big[\lambda + \gamma + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]\big]Q_n(t) + [3\mu_1 + 6\mu_2 + 2(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3\mu_3]Q_{n+1}(t) \qquad (2)$$

In Steady state,

$$\lim_{n\to\infty} Q_n(t) = Q_n \text{ and } Q_n'(t) = 0 \ as t \to \infty$$

These are the steady-state equations that correspond with equations (1) and (2).

$$0 = -\lambda Q_0 + [3\mu_1 + 6\mu_2 + 2\mu_3]Q_1 + \gamma[1 - Q_0]$$

$$(\lambda+\gamma)Q_0 = \gamma + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]Q_1 \quad (3)$$

$$0 = \lambda Q_{n-1} - \big[\lambda + \gamma + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]\big]Q_n + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]Q_{n+1}$$

$$\big[\lambda + \gamma + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]\big]Q_n = \lambda Q_{n-1} + [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]Q_{n+1} \quad (4)$$

Equation (3) $\Longrightarrow$

$$\gamma = (\lambda+\gamma)Q_0 - [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]Q_1$$

$$= \left[(\lambda+\gamma) - [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]\frac{Q_1}{Q_0}\right]Q_0$$

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - [(3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3]\frac{Q_1}{Q_0}} \quad (5)$$

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{(\lambda+\gamma+(3+2u)\mu_1+(9+t)\mu_2+(10-6t-6u)\mu_3) - [(3+2u)\mu_1+(9+t)\mu_2+(10-6t-6u)\mu_3]\frac{Q_{n+1}}{Q_n}} \quad (6)$$

With the continuing fraction and the two equations mentioned previously,

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \frac{\eta\lambda}{(\lambda+\eta+\gamma) - \eta\,\frac{Q_2}{Q_1}}}$$

Where $\eta = (3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3$

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \frac{\eta\,\lambda}{(\lambda+\eta+\gamma) - \frac{\lambda\,\eta}{\lambda+\eta+\gamma} - \cdots}}.$$

$$Q_0 = \frac{\gamma}{(\lambda+\gamma)-\alpha},\ \alpha = \frac{\eta\,\lambda}{(\lambda+\eta+\gamma) - \frac{\lambda\eta}{\lambda+\eta+\gamma} - \cdots}. \quad (7)$$

The quadratic equation is satisfied by equation (7),

$$\alpha^2 - (\lambda+\eta+\gamma)\alpha + \lambda\eta = 0$$

Therefore, the roots are $\frac{\beta \pm \sqrt{\beta^2 - 4\lambda\delta}}{2}$, where $\beta = \lambda + \eta + \gamma$

Assuming α_1 and $\alpha_2$as the roots, we extract the only real root that falls inside [0, 1).

Substituting α_2 in equation (7), we get

$$Q_0 = \frac{\gamma}{(\lambda+\gamma) - \frac{\beta - \sqrt{\beta^2 - 4\lambda\eta}}{2}} \quad (8)$$

$$Q_0 = \frac{\gamma}{(\beta-\eta) - \frac{\beta - \sqrt{\beta^2 - 4\lambda\eta}}{2}}$$

Multiply and divide by $\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\lambda\eta}$

After some algebraic calculation, we get

$$Q_0 = \frac{\gamma\left(\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\lambda\eta}\right)}{1-\delta\left(\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\lambda\eta}\right)} \tag{9}$$

After expanding binomially,

$$Q_0 = \sum_{n=0}^{\infty}\eta^n\left(\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\lambda\eta}\right)^{n+1} + \gamma\sum_{n=0}^{\infty}\eta^n\left(\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\lambda\eta}\right)^{n+1} \tag{10}$$

Using equation (6), we can now calculate the remaining steady state probabilities in terms of Q_0,

$$\frac{Q_n}{Q_{n-1}} = \frac{\gamma}{(\lambda+\gamma+\eta)-\frac{\lambda\eta}{(\lambda+\eta+\gamma)-\frac{\lambda\eta}{\lambda+\eta+\gamma}-\cdots}}$$

Using the same reasoning as before, the equation above reduces to

$$\frac{Q_n}{Q_{n-1}} = \frac{\lambda}{\frac{\beta+\sqrt{\beta^2-4\lambda\eta}}{2}}, \quad n = 1, 2 \ldots\ldots \tag{11}$$

Equations (11) can be multiplied and divided by $\beta-\sqrt{\beta^2-4\lambda\eta}$

$$Q_n = \left(\frac{\beta-\sqrt{\beta^2-4\lambda\eta}}{2\eta}\right)^n Q_0 \quad , n = 1, 2, \ldots, \tag{12}$$

As we know $\beta = \lambda+\gamma+\eta$

Equations (10) and (11) gives the probabilities for Q_0 and Q_n under steady state condition for n = 1, 2 , . . . ,

2.1 Theorem

The average queue length asymptotic behavior in steady state, if $\gamma > 0$, is

$$L_q = \frac{\lambda-\eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma)-\left[(\lambda+\eta+\gamma)-\sqrt{(\lambda+\eta+\gamma)^2-4\lambda\eta}\right]}, \quad \eta = (3+2u)\mu_1 + (9+t)\mu_2 + (10-6t-6u)\mu_3$$

Proof:

Examine equations (1) and (2) in relation to the initial condition $Q_0(0) = 1$.

$$\frac{\partial Q(x,t)}{\partial t} = \left[\lambda x + \frac{\eta}{x} - (\lambda+\gamma+\eta)\right]Q(x,t) + \eta\left(1-\frac{1}{x}\right)Q_0(x) + \gamma \tag{13}$$

The mean size is $m(t) = \sum_{n=1}^{\infty} nQ_n(t) = \frac{\partial Q(x,t)}{\partial t}$ at x=1

By taking the derivative of equation (13) with respect to x and evaluating it at x = 1, we obtain

$$\frac{dm(t)}{dx} + \gamma m(t) = \lambda - (1-Q_0)\eta$$

Finding the solution for m(t) in the differential equation above, where

$$m(0) = \sum_{n=1}^{\infty} nQ_n(t) = 0$$

$$m(t) = \frac{\lambda}{\gamma}(1 - e^{-\gamma t}) - \frac{\eta}{\gamma}(1 - e^{-\gamma t}) + \eta \int_0^t Q_0(u)\, e^{-\gamma(t-u)} du \tag{14}$$

We get the below equation by applying laplace transform for Equation (1) and (2)

$$Q_0^*(x) = \frac{1 + \frac{\gamma}{x}}{(x+\lambda+\gamma) - \left[\frac{\beta - \sqrt{\beta^2 - 4\lambda\eta}}{2}\right]} \tag{15}$$

Taking Laplace Transforms for equation (14),

Let $m^*(x)$ be the laplace transform of $m(t)$

$$m^*(x) = \frac{\lambda-\eta}{x(x+\gamma)} + \frac{\eta}{(x+\gamma)} Q_0^*(x) \tag{16}$$

$$\lim_{t\to\infty} m(t) = \lim_{x\to 0} x\, m^*(x)$$

Using the Laplace Transform to solve Equations (1) and (2), we obtain

$$L_q = m(t) = \frac{\lambda - \eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma) - \left[(\lambda+\eta+\gamma) - \sqrt{(\lambda+\eta+\gamma)^2 - 4\lambda\eta}\right]}$$

3. Balance Equations

Using Little's formula, the following equations are obtained.

Average queue length of the system

$$L_q = \frac{\lambda-\eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma) - \left[(\lambda+\eta+\gamma) - \sqrt{(\lambda+\eta+\gamma)^2 - 4\lambda\eta}\right]},$$

$$\eta = (3 + 2u)\mu_1 + (9 + t)\mu_2 + (10 - 6t - 6u)$$

Customer wait time in each of the three lines

$$W_q = \frac{L_q}{\lambda} = \left(\frac{\lambda - \eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma) - \left[(\lambda+\eta+\gamma) - \sqrt{(\lambda+\eta+\gamma)^2 - 4\lambda\eta}\right]}\right)\frac{1}{\lambda}$$

The mean quantity of customers within the system

$$L_s = L_q + \frac{\lambda}{\mu} = \left(\frac{\lambda - \eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma) - \left[(\lambda+\eta+\gamma) - \sqrt{(\lambda+\eta+\gamma)^2 - 4\lambda\eta}\right]}\right) + \frac{\lambda}{\mu}$$

Customer's wait time within the system

$$W_s = \frac{L_s}{\lambda} = \left[\frac{\lambda - \eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma) - \left[(\lambda+\eta+\gamma) - \sqrt{(\lambda+\eta+\gamma)^2 - 4\lambda\eta}\right]} + \frac{\lambda}{\mu}\right]\frac{1}{\lambda}$$

For Node one, the average queue length is

$$L_{q_1} = \frac{\lambda-\eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma)-\left[(\lambda+\eta+\gamma)-\sqrt{(\lambda+\eta+\gamma)^2-4\lambda\eta}\right]},$$

$$\eta = \mu_1 + 2\mu_2 + 2u\mu_3$$

For Node two, the average queue length is

$$L_{q_2} = \frac{\lambda-\eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma)-\left[(\lambda+\eta+\gamma)-\sqrt{(\lambda+\eta+\gamma)^2-4\lambda\eta}\right]},$$

$$\eta = \mu_1 + 2\mu_2 + 2t\mu_3$$

For Node three, the average queue length is

$$L_{q_3} = \frac{\lambda-\eta}{\gamma} + \frac{2\eta}{2(\lambda+\gamma)-\left[(\lambda+\eta+\gamma)-\sqrt{(\lambda+\eta+\gamma)^2-4\lambda\eta}\right]},$$

$$\eta = 2(1+u)\mu_1 + 8\mu_2 + 2t\mu_3$$

Using Little's formula the remaining parameters$W_{q_1}, W_{q_2}, W_{q_3}, L_{s_1}, L_{s_2}, L_{s_3}, W_{s_1}, W_{s_2}, W_{s_3}$ are calculated.

4. PARTICULAR CASE:

When $\mu_1 = \mu$ and $\mu_2 = \mu_3 = 0$, $t = s = 0$, there will only be a single node without feedback. When s = 0, there are no retrials leading to $\eta=\mu$.

Asymptotic behavior of the system L_q when $\gamma > 0$ is

$$L_q = \frac{\lambda-\mu}{\gamma} + \frac{2\mu}{2(\lambda+\gamma)-\left[(\lambda+\mu+\gamma)-\sqrt{(\lambda+\mu+\gamma)^2-4\lambda\mu}\right]} \quad (17)$$

Case 1:

When $\gamma > 0$, the average queue length of the system L_q behaves asymptotically

$$L_q = \frac{\lambda-\mu}{\gamma} + \frac{2\mu}{2(\lambda+\gamma)-\left[(\lambda+\mu+\gamma)-\sqrt{(\lambda+\mu+\gamma)^2-\alpha^2}\right]} \text{ as } t \to \infty, \alpha = 2\sqrt{\lambda\mu}$$

When $\gamma > 0$, equation (17) agrees with Krishnakumar et al. [8].

Case 2:

The mean queue length *m(t)* behaves asymptotically changes when γ is greater than 0.

$$L_q = \frac{\lambda-\mu q}{\gamma} + \frac{2\mu q}{2(\lambda+\gamma)-\left[(\lambda+\mu q+\gamma)-\sqrt{(\lambda+\mu q+\gamma)^2-4\lambda\mu q}\right]} \text{ as } t \to \infty$$

By assuming q = 1, the equation (17) agrees with Thangaraj et al. [13].

Case 3:

The mean queue length *m(t)* exhibits asymptotic behavior when $\gamma>0$.

$$L_q = \frac{\lambda-\beta}{\gamma} + \frac{2\beta}{2(\lambda+\gamma)-\left[(\lambda+\beta+\gamma)-\sqrt{(\lambda+\beta+\gamma)^2-4\lambda\beta}\right]} \text{ as } t \to \infty$$

Where $\beta = (5+2p)\mu_1 + 4\mu_2 + 2\mu_3$

Equation (17) agrees with Shanmugasundaram et al. [11] when p = 0, $\mu_1 = 0, \mu_2 = 0, and\ \beta = \mu$ are taken.

5. NUMERICAL EXAMPLES

Calculation for the system

For $t=0.6$, $u=0.4$,$\mu_1 = 7$, $\mu_2 = 8$, $\mu_3 = 9$, $\lambda = 1,2,3,4,5,6,7,8,9,10$, $\gamma = 1,3,5,7,9$

Total number of customers in all the three queues:

Table I shows the number of customers in each of the three nodes for arrival rates λ, which ranges from 1 to 10 and for the disaster impact γ, varying from 1 to 9.

TABLE I

	1	3	5	7	9
1	0.007173	0.007071	0.006972	0.006875	0.006781
2	0.014449	0.014241	0.014038	0.013841	0.01365
3	0.021831	0.021511	0.021201	0.020899	0.020606
4	0.029319	0.028884	0.028461	0.028051	0.027653
5	0.036918	0.036361	0.035822	0.035299	0.034792
6	0.044628	0.043946	0.043285	0.042644	0.042023
7	0.052453	0.05164	0.050852	0.050089	0.04935
8	0.060395	0.059445	0.058525	0.057635	0.056773
9	0.068457	0.067364	0.066307	0.065285	0.064296
10	0.076642	0.0754	0.0742	0.07304	0.071918

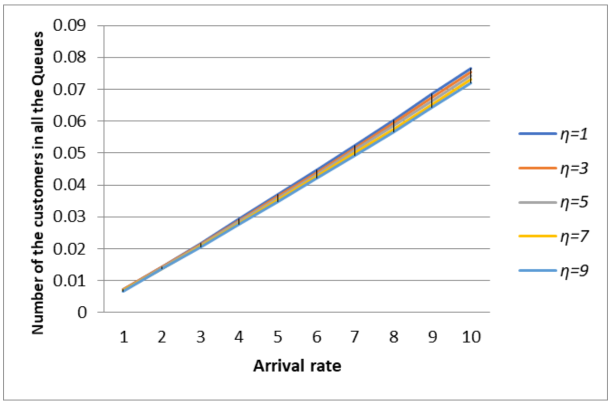


Fig. 2 The number of customers in the three queues varies depending on the arrival rate under calamity impact

It is evident from Fig. 2 that when the arrival rate rises, there are more customers in the three queues, and when γ value rises, there are less customers in three queues.

Customer's waiting time in the three queues:

In Table II, The customer's waiting time in the three queues is determined for arrival rates λ ranging from 1 to 10, and for γ (Catastrophe effect) ranging from 1 to 9.

TABLE II

	1	3	5	7	9
1	0.007173	0.007071	0.006972	0.006875	0.006781
2	0.007225	0.00712	0.007019	0.006921	0.006825
3	0.007277	0.00717	0.007067	0.006966	0.006869
4	0.00733	0.007221	0.007115	0.007013	0.006913

5	0.007384	0.007272	0.007164	0.00706	0.006958
6	0.007438	0.007324	0.007214	0.007107	0.007004
7	0.007493	0.007377	0.007265	0.007156	0.00705
8	0.007549	0.007431	0.007316	0.007204	0.007097
9	0.007606	0.007485	0.007367	0.007254	0.007144
10	0.007664	0.00754	0.00742	0.007304	0.007192

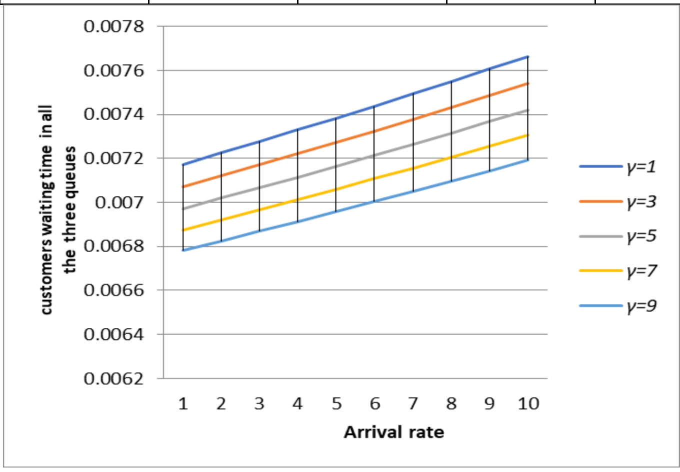


Fig 3. For different arrival rate the waiting time of a customer in all the three queues under catastrophe effect

It is evident from Fig. 3 that when the arrival rate rises, customer waiting times in all three queues increase, and when γ value rises, customers are waiting time in all three queues decrease.

For $t = 0.6$, $u = 0.4$, $\mu_1 = 5,5.2,5.4,5.6,5.8,6.0,6.2,6.4,6.6,6.8$, $\mu_2 = 7$, $\mu_3 = 8$, $\lambda = 6$, $\gamma = 1,3,5,7,9$

Number of customers in the system:

Table III shows the total customers in the system with service rate μ_1 varying from 5 to 6.8 and catastrophe effect γ ranging from 1 to 9.

TABLE III

	1	3	5	7	9
5	0.952979	0.952013	0.951083	0.950188	0.949324
5.2	0.943715	0.942762	0.941845	0.940961	0.940108
5.4	0.93463	0.93369	0.932785	0.931912	0.931071
5.6	0.92572	0.924792	0.923899	0.923037	0.922206
5.8	0.916979	0.916064	0.915182	0.914331	0.91351
6	0.908402	0.907499	0.906628	0.905789	0.904978
6.2	0.899986	0.899094	0.898235	0.897405	0.896605
6.4	0.891724	0.890844	0.889996	0.889177	0.888386
6.6	0.883614	0.882745	0.881907	0.881099	0.880317
6.8	0.875651	0.874793	0.873966	0.873167	0.872395

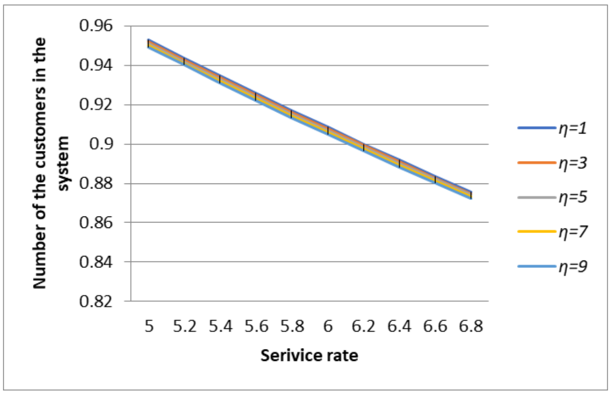


Fig 4. For different service rate number of customers in the system under catastrophe effect

It is evident from figure 4 that when the service rate rises, the number of customers in the system falls for different values of γ.

Customer's waiting time within the system:

Table IV determines the customer's waiting time based on two factors: the service rate (μ_1) which ranges from 5 to 6.8 and the catastrophe effect (γ) which ranges from 1 to 9.

TABLE IV

	1	3	5	7	9
5	0.15883	0.158669	0.158514	0.158365	0.158221
5.2	0.157286	0.157127	0.156974	0.156827	0.156685
5.4	0.155772	0.155615	0.155464	0.155319	0.155178
5.6	0.154287	0.154132	0.153983	0.15384	0.153701
5.8	0.15283	0.152677	0.15253	0.152389	0.152252
6	0.1514	0.15125	0.151105	0.150965	0.15083
6.2	0.149998	0.149849	0.149706	0.149568	0.149434
6.4	0.148621	0.148474	0.148333	0.148196	0.148064
6.6	0.147269	0.147124	0.146985	0.14685	0.14672
6.8	0.145942	0.145799	0.145661	0.145528	0.145399

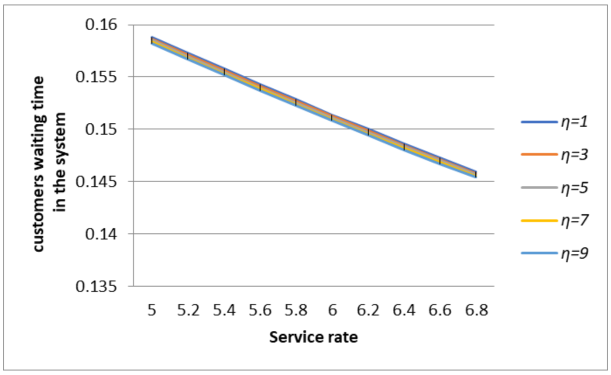


Fig 5. For different service rate the waiting time of the customer in the system under catastrophe effect

It is evident from Fig. 5 that when the service rate rises, customer waiting times in the system get shorter for different values of γ.

6. Conclusion

Here, we examine the feedback retrial queueing network's steady state probability in the event of a catastrophe and calculate$Q_0(t)$ or the number of customers in the system at time t, and $Q_n(t)$. We also determineL_q, W_q, L_s, *and* $W_s$for each of the three nodes. The numerical example displays the number of customers in each of the three system nodes, the number of customers in the system, and the amount of time a client has to wait in each of the three system queues. It shows the correctness of the results.

References

[1] Ankita Roy Chowdhury, Indra Rani, Steady-State Solution of Catastrophic Feedback Queue Subject to Balking Using Matrix-Geometric Approach, *Journal of Scientific Research*, 6 (65) (2021), 100 – 105.
[2] Chao X, A queuing network model with catastrophes and product form Solution, *Operation Research Letter*s, 18 (1995), 75-79.
[3] Chandrasekaran V.M, Saravanarajan M.C, Transient and Reliability Analysis of M/M/1 Feedback Queue Subject to Catastrophes, Server Failures and Repairs, *International Journal of Pure and Applied Mathematics*, 77 (5) (2012), 605-625.
[4] Erlang A.K, The theory of probabilities and telephone conversations, *Nyt J inds skriff Math*.B20, (1909), 33-39.
[5] Falin G.I, A survey of retrial queues, *Queueing system*, 7(1990), 127-167.
[6] Gelenbe E, Product form queueing networks with positive and negative customers, *Journal of Applied Probability*, 28(3) (1991), 656-663.
[7] Jackson J.R, Networks of Waiting Lines, *Operation Research*, 5(40) (1957), 518-521.
[8] Jackson J.R, Jobshop-like queueing systems, *Management Science*, 10 (1963), 131-142.
[9] Kosten L, On the influence of repeated calls in the theory of probabilities of blocking, *DeIngenieur*, (1947), 591-625.
[10] Krishna Kumar B, Arivudainambi D, Transient solution of an M/M/1queue with catastrophes, *Computers and Mathematics with Applications*, 40 (2000), 1233-1240.
[11] Kumar B.K, Anantha Lakshmi S. R, Anbarasu S, Pavai Madheswari S, Transient and steady-state analysis of queueing systems with catastrophes and impatient customers, *International Journal of Mathematics in Operational Research*, 6(5) (2014), 523-549.
[12] Shanmugasundaram S, Baby C, Study of M/M/1 feedback Retrial Queueing Network with three Nodes when Catastrophe Occurs, *Mathematicial Statistician and Engineering Applications*, 71(4) (2022), 3482 – 3494.
[13] Shanmugasundaram S, Chitra S, Time dependentsolution of M/M/1retrialqueue and feedback on non-retrial customers with catastrophes, *Global Journal of Pure and Applied Mathematics*, 11(1) (2015), 512-521.
[14] Shanmgasundaram S, Vanitha S, Steady state Analysis of M/M/1 Retrial Queueing Network with Catastrophes, *International Journal of Mathematics Trends and Technology*, 26(12) (2020), 170 – 181.
[15] Takacs L, A Single server queue with feedback, *The Bell System Technical Journal*, 42 (1963), 505-519.
[16] Thangaraj V, Vanitha S, On the analysis of M/M/1 feedback queue with Catastrophes using continued fractions, *International Journal of Pure and Applied Mathematics*, 53(1) (2009), 133-151.
[17] Varalakshmi, M, Chandrasekaran, V. M, Saravanarajan, M. C, A single server queue with immediate feedback, working vacation and server breakdown, *International Journal of Engineering & Technology*, 7 (4) (2018), 476-479.