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Stability, and Almost Sensitivity of Induced Maps

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Abstract:

Suppose that X is a compact Hausdorff space and f : X → X is continuous. We consider the space K(X), the space of all compact subsets of X with Hausdorff metric H. Let $\tilde{f}$: K(X) → K(X) defined by $\tilde{f}$(K)=f(K) . We discuss some interconnections between the orbit of $\tilde{f}$ and the orbit of f. By assuming the transitivity of $\tilde{f}$, we conclude that X contains a cantor set C with $(orb(\tilde{f},C))^{-}$= K(X). That is, orbit of a nowhere dense set is dense in the hyperspace. Along with this we introduce ”almost sensitivity” and ”stability” in K(X). We prove that f is stable in X if and only if $\tilde{f}$ is stable in K(X). Again we prove that transitive maps are always ’almost sensitive’ in K(X) and hence the base map is ’almost sensitive’ in X.

Keywords: Maps, continuous map, compact metric space.

1. Introduction

Normally, for studying the dynamics of the hyperspace of a compact metric space, we consider a dynamical system defined by a continuous map $f : X \to X$, describing the dynamics of points in the base space X and then we study the induced map $\tilde{f}$: $K(X) \to K(X)$ defined by $\tilde{f}(K) = f(K)$ for a compact set $K \subseteq X$ as a form of collective dynamics. In this context, a very natural question arises: What is the connection between dynamical properties of the base map f and the induced map $\tilde{f}$?

During the past years this question has attracted many researchers.

(see[4],[5],[6],[7],[8], [9],[10] and [11]) In this paper, we prove that there exists a cantor set C in X, nowhere dense in X but its orbit is dense in $K(X)$. At the

same time we establish the fact that the set $D = \{x \in X, \overline{orb(f,x)} = X\}, D \neq X$ is dense in X and so it is not closed and there fore $D \neq K(X)$.

We also prove that f is ’stable’ if and only if $\tilde{f}$ is stable and $\tilde{f}$ is ’almost sensitive’ if $\tilde{f}$ is transitive and this implies the ’almost sensitivity’ of f.

Through out this paper, X denotes a Hausdorff compact metric space without isolated points with metric d and $f: X \to X$ is continuous. $K(X)$ denotes the space of all compact subsets of X with Hausdorff metric H induced by d . Let $\tilde{f}$: $K(X) \to K(X)$ defined by $\tilde{f}(K) = f(K)$.

2. Hyperspace and Induced Map

1.In this section we have study some properties of the induced map $\tilde{f}$ and the base map f. By assuming the transitivity of $\tilde{f}$, we show that the base space X contains a cantor set C with its orbit is dense in $K(X)$. Along with this we also study the concept of ’almost sensitivity’ in X and $K(X)$.

Let us use the symbol ϕ for the induced map $\tilde{f}$ for the sake of convenience.

Definition 2.1. *If* $A = \{A_\mu\}_{\mu\in\daleth}$ *is a collection of non-empty subsections of X, then* $mesh(A) = sup\{diam(A_\mu), \mu \in \daleth\}$.

In [2] it is proved that $(K(X),H)$ is compact.

Lemma 2.1

If $\overline{orb(\phi,K)} = X$ *then for individually* $A \in orb(\phi,K)$ *and for all* $x \in A$, $\overline{orb(f,x)} = X$

Proof.

Let $n \in \mathbb{N}$ and $x \in f^n(K)$.

We have to demonstrate that $\overline{orb(f,x)} = X$.

Take $y \in X$ and $\epsilon > 0$. Since $\overline{orb(\phi,K)} = X$ we have $\overline{orb(\phi, f^n(K))} = K(X)$

Let $\{y\} \in orb(\phi, f^n(K))$, then there exist $j \in \mathbb{N}$ such that $H\left(f^j(f^n(K),\{y\})\right) < \epsilon$.

Hence $f^j(x) \in f^j(f^n(K)) \subseteq B_\epsilon(y)$

So $B_\epsilon(y) \cap, \overline{orb(f,x)} \neq \emptyset$

There fore , $\overline{orb(f,x)} = X$.

Lemma 2.2. *If X is weekly mixing, then any power* $X \times X \times \cdots \times X$ *is ergodic.*

Proof. see[1]

Lemma 2.3. *Let* $\phi : K(X) \to K(X)$ *be transitive. Then there exist a cantor set* $C \subseteq X$ *such that* $\overline{orb(\phi,C)} = K(X)$.

Proof.

Let δ_0 represent the diameter of the set X. For every positive integer n, we define a series of finite open covers of X, denoted by $\widetilde{V_n} = \{V_{n,1}, V_{n,2}, \dots, V_{n,t_n}\}$ where each $V_{n,i}$ is a nonempty subset. These covers are constructed such that the size of each cover is smaller than δ_n. In essence, this arrangement ensures that each element of the set X is contained within at least one open set in the cover, and as n increases, the covers become increasingly finer, converging towards the set's diameter.

Step 1

Let us assume W_0 and W_1 to be two non-empty disjoint open sets in X and $mesh(\{W_0 \cap W_1\} < \delta_1$. Let $\lambda_1 = \{1,2,\dots,t_1\} \times \{1,2,\dots,t_1\} = \{(a,b) | a,b \in \{1,2,\dots,t_1\}\}$.

Let us take into consideration the following t_1^2+1 collection of open sets

(W_0, W_1) and $\{(V_{1,a}, V_{1,b}) : (a,b) \in \lambda_1\}$

Then, by Lemma 1.4 [see 1], two closed subsets of X, C_0 and C_1, having the following characteristics, exist:

- Each $int(C_i)$ is nonempty and which is contained in W_i for $i = 0,1$.

Hence C_0 and C_1 are disjoint.

- for each $A \in \langle C_0, C_1\rangle = \{B \in K(X): B \subset (C_0 \cup C_1) and\ B \cap C_i \neq \emptyset\}$ and for each $(a,b) \in \lambda_1$, there exist $n \in N$ such that $f^n(A) \in \langle U_{1,a}, U_{1,b}\rangle$.

- Also, $f^n(A \cap C_0) \subset U_{1,a}$ and $f^n(A \cap C_1) \subset U_{1,b}$.

Let $\mathbb{C}_1 = \langle C_0, C_1 \rangle$. Then diam($\mathbb{C}_1$) < δ_1 for each $A \in \mathbb{C}_1$, $orb(\phi, A)$ is δ_1-close to $F_2(X)$,where $F_2(X) = \{A \in K(X) \backslash |A| \le 2\}$.

For, given any $\{p,q\} \in F_2(X)$, there exist $(a,b) \in \lambda_1$ such that $p \in U_{1,a}$ and $q \in U_{1,b}$.

Since there exist n so that $f^n(A) \in \langle U_{1,a}, U_{1,b} \rangle$. We conclude that $H(\{p,q\}, f^n(A)) < \delta_1$

Step 2

Let $W_{0,0}, W_{1,0}, W_{0,1}. W_{1,1}$ are 4 non-empty open subsets of X with

$W_{0,0} \cap W_{1,0} = \emptyset$ and $W_{0,1} \cap W_{1,1} = \emptyset$

$W_{0,0} \cup W_{1,0} \subset C_0$ and $W_{0,1} \cup W_{1,1} \subset C_1$ and $mesh(\{W_{0,0}, W_{1,0}, W_{0,1}, W_{1,1}\}) < \delta_2$.

Let $\lambda_2 = \{1,2,...t_2\}^4 = \{(a_1,a_2,a_3,a_4) \backslash a_i \in \{1,2,...t_2\}\}$. Consider the following $t_2^4 + 1$ collection of open sets

$(W_{0,0}, W_{1,0}, W_{0,1}, W_{1,1}), \{(U_2, a_1, U_2, a_2, U_2, a_3, U_2, a_4)\}$.

Given four non-empty open subsets of X, $W_{0,0}, W_{1,0}, W_{0,1}, and\ W_{1,1}$, where $W_{0,0}$ and $W_{1,0}$ are disjoint, as are $W_{0,1}$ and $W_{1,1}$. Each pair of subsets is contained within different closed sets, C_0 and C_1, respectively. The mesh of these subsets is less than δ_2

. Then, λ_2 consists of all possible combinations of indices from 1 to t_2. A collection of open sets is formed from the given subsets and λ_2.

By the same result of **Lemma 1.2[see 1]** there exist 4 closed sets $C_{0,0}$, $C_{0,1}, C_{1,0}$, $C_{1,1}$ with the following properties:

1. Each $int(C_{i,j})$ is nonempty and contained in $W_{i,j}$ for $\{i,j\} \in \{0,1\} \times \{0,1\}$
2. For each $A \in \langle C_{0,0}, C_{0,1}, C_{1,0}, C_{1,1} \rangle$ and for each $(a_1,a_2,a_3,a_4) \in \lambda_2$ there exist $n \in N$ such that $f^n(A) \in \langle U_{2,a1}, U_{2,a2}, U_{2,a3}, U_{2,a4} \rangle$, subsets of $\mathbb{C}_2$ namely $\mathbb{C}_2^i$ with $f^n(A \cap \mathbb{C}_2^i) \subset U_2, a_i$ for $1 \le i \le 4$, where $\mathbb{C}_2 = \langle C_{0,0}, C_{0,1}, C_{1,0}, C_{1,1} \rangle$.

Note that $diam(\mathbb{C}_2) < \delta_2$ and $\mathbb{C}_2 \subset \mathbb{C}_1$ let $A \in \mathbb{C}_2$ and $\{p_1,p_2,p_3,p_4\} \in F_4(X)$, then there exist $(a_1,a_2,a_3,a_4) \in \lambda_2$ such that $p_i \in U_{2,ai}$

Since there exist n so that $f^n(A) \in \langle U_{2,a1}, U_{2,a2}, U_{2,a3}, U_{2,a4} \rangle$.

We conclude that $H(\{p_1,a_2,a_3,a_4\}, f^n(A)) < \delta_2$.

So for every $A \in \mathbb{C}_2$,$orb(\phi, A)$ is δ_2-close to $F_4(X)$.

Step 3

Let's say that $\mathbb{C}_r$ has previously been defined and has the following attributes:

$\mathbb{C}_r$ is defined as a collection of closed sets of X, represented as $\langle C_{0,0,0,...},..., C_{1,1,...} \rangle$, where each closed set is indexed by a binary sequence : $\{C_{j1,j2,...,jr} \backslash (j_1,j_2,...,j_r) \in \{0,1\}^r\}$. This indexing scheme corresponds to 2^r possible combinations, denoting the presence or absence of each closed set in $\mathbb{C}_r$. Each combination delineates the composition of $\mathbb{C}_r$ and its constituent closed sets.

- $(j_1, j_2, ..., j_r) \in \{0,1\}^r$ and $int(C_{j_1 j_2 ... j_r})$ is non empty, $C_{j_1 j_2 ... j_r} \subset C_{j_2 ... j_r}$ and $diam(C_{j_1 j_2 ... j_r}) < \delta_r$
- $diam(\mathbb{C}_r)$ is less than δ_r and $\mathbb{C}_r$ contained in $\mathbb{C}_{r-1}$

- For each pair $(j_1, j_2, \dots j_r) \neq (l_1, l_2, \dots, l_r)$ in $\{0,1\}^r$;

$C_{j_1 j_2 \dots j_r}$ and $C_{l_1 l_2 \dots l_r}$ are disjoint.

- For each $A \in \mathbb{C}_r$ and each $(t_1, t_2, \dots, t_{2^r})$ in $\lambda_r = \{t_1, t_2, \dots t_r\}^{2^r}$ there exist $n \in N$, $f^{n(A)} \in \langle U_r, t_1, U_r, t_2, \dots, U_r, t_{2^r} \rangle$, subsets of $\mathbb{C}_r$ namely $\mathbb{C}_r^i$ with $f^n(A \cap \mathbb{C}_r^i) \subset U_r, t_i$, for $1 \leq i \leq 2^r$.

Then for if $A \in \mathbb{C}_r$, then $orb(\phi, A)$ is δ_r close to F_{2^r}(X).

Similarly, for $\mathbb{C}_{r+1}$.

Step 4

So we get a declining order of compact subsets of $X, \{\mathbb{C}_r\}_1{}^\infty$

Let $\{C_{l_1 l_2 \dots l_r}: (l_1, l_2, \dots, l_r) \in \{0,1\}^r\}$ be the 2^r compact subset of X that define $\mathbb{C}_r$.

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Let $C = \bigcap_{r=1}^{\infty} (\cup \{C_{l_1, l_2, \dots l_r}, (l_1, l_2, \dots, l_r) \in \{0,1\}^r\})$.

Then C is a cantor set in X .

for all $r, C \in \mathbb{C}_r$, so $\overline{orb(\phi, C)} = K(X)$.

Theorem 2.1. *There exist a cantor set $C \subseteq X$ such that $\overline{orb(f, x)} = X$ for every $x \in f^n(C)$ for all n $n \in \mathbb{N}$.*

Proof. clear from Lemma 2.3

Theorem 2.2. *Let $\tilde{f}$: K(X) → K(X) be transitive. Then there exist a cantor set $C \subseteq X$ such that $\overline{orb(f, x)} = X$ for every $x \in C$*

Proof. clear from lemma 2.1 and theorem 2.2□

Theorem 2.3. *Let $D = \{x \in X, \overline{orb(f, x)} = X\}$ and $\tilde{f}$: K(X) → K(X) is transitive. Then C ⊆ D, that is D nonempty. Also D is a dense subset of X fully invariant under f, that is f(D) ⊆ D*

Proof. From theorem 2.3, we can find a cantor set C in X with $C \subseteq D$.

That is D is nonempty.

for the proof of the remaining part, see[3] and[4]□

Theorem 2.4. *Let ϕ : K(X) → K(X) is transitive and $D \neq X$. Then $D \notin K(X)$.*

Proof. from theorem 2.4, D is dense in X. So D is not closed and so it is not compact. □

In the theorem we have a nowhere dense set C in X with its orbit *orb*(ϕ ,C) is dense in K(*X)*. But at the same time we have a dense set D in X with even $D \notin K(X)$.

Theorem 2.5. *Let $\tilde{f}$: K(X) → K(X) is transitive and let C be the cantor set in X. Then for any U ⊆ X and for any integer m ,* $\lim_{n\to\infty} sup\, H\left(\tilde{f}^n(C), \tilde{f}^{n+m}(C)\right) \geq H(U, \tilde{f}^m(U))$

Proof. Let U and m be given and let$\{\frac{1}{2^n}\}$ be the sequence.

since $\tilde{f}$ is continuous and $\overline{orb(\tilde{f}, C)} = K(X)$ there exist for every $\frac{1}{2^n}$, a positive integer s_n such that $\tilde{f}^{s_n}(C)$ so close as to U such that $H(\tilde{f}^{s_n}(C), U) < \frac{1}{2^{n+1}}$ and $H(\tilde{f}^m\left(\tilde{f}^{s_n}(C), \tilde{f}^m(U)\right) < \frac{1}{2^{n+1}}$.

This implies

$$H\left(\tilde{f}^{s_n}(C), \tilde{f}^{s_n+m}(C)\right) > H\left(U, \tilde{f}^m(U)\right) - \frac{1}{2^n}$$

[by using Triangle Inequality]

Hence the result.

Definition 2.3. Let X be a metric space with metric d and $f : X \to X$ be a continuous map. f is said to be almost sensitive if we can find an $x \in X$ and $m \in \mathbb{N}$ such that for any $\lim_{n\to\infty} sup\, d(f^n(x), f^{n+m}(y)) \geq d(y, f^m(y))$.

Theorem 2.6. *In $K(X)$, transitive maps are almost sensitive.*

Proof. clear from Lemma 2.3 and Theorem 2.6 □

Theorem 2.7. *$\tilde{f}$ is almost sensitive in $K(X)$ implies f is almost sensitive in X*

Proof. clear from Theorem 2.7

3.Stability of Induced Maps

In this section we study the stability of the induced map $\tilde{f} : K(X) \to K(X)$ and its connection with the stability of the continuous map $f : X \to X$. In this section $C(X)$ denotes the space of all continuous functions on X and $C(K(X))$ denotes the space of all continuous functions on $K(X)$.

Definition 3.1. A point $x \in X$ is said to be stable if for all $\epsilon > 0$ there is a $\delta > 0$ such that if

$d(y, x) < \delta$ then $d(f^n(y), f^n(x)) < \epsilon$ for every n. A point x is said to be unstable if it is stable for f^{-1}.

Definition 3.2. $\tilde{f} \in K(X)$ is stable if given $\epsilon > 0$, there exists a $\delta > 0$ such that for each $\tilde{g} \in C(K(X))$ with $d_H(\tilde{f}, \tilde{g}) < \delta$, there exist a continuous map $h \in C(X)$ such that $\tilde{f} \circ \tilde{g} = \tilde{g} \circ \tilde{f}$ and $d_H(\tilde{h}, \tilde{\imath}) < \epsilon$ where $\tilde{\imath} : K(X) \to K(X)$ is the identity map and $d_H(\tilde{f}, \tilde{g})$=sup $\{d_H\left(\tilde{f}(A), \tilde{g}(A)\right), A \in K(X)\}$.

Theorem 3.1. *$f : X \to X$ is stable in $C(X)$ if and only if $\tilde{f}: K(X) \to K(X)$ is stable in $C(K(X))$.*

Proof. To prove the theorem, first we prove the following lemmas

Lemma 3.1. *The map $\psi : C(X) \to C(K(X))$ given by $\psi(f) = \tilde{f}$ is an embedding of $C(X)$ in to $K(C(X))$*

Proof. ψ is well defined since the induced map $\tilde{f}$ given by a continuous map is continuous.

Next we have to show that ψ is injective.

Suppose $\psi(f) = \psi(g)$.

Let $A = \{x\}$ for each $x \in X$

We have, $\{f(x)\} = \tilde{f}(A) = \tilde{g}(A) = \{g(x)\}$ for each $x \in X$.

Then, ψ is injective.

Next we have to show that ψ and ψ^{-1} are continuous.

Let $\{f_n\}$ be a sequence in sequence in $C(X)$ which converges to f. Then $\{\tilde{f}_n\}$ converges to $\tilde{f}$ in $C(K(X))$

Assume that for any $\epsilon > 0$ and each $A \in K(X)$, there exist an $N \in \mathbb{Z}_+$ such that $d(f_n, f) < \epsilon$ for every $n \geqslant N$.

For all $a \in A$ and $n \geqslant N$, we have $d(f^n(a), f(a)) < \epsilon$

So ,$f^n(A) \subseteq B(f_n(A), \epsilon), n \geqslant N$

For all $A \in K(X)$, we get $f(A) \subseteq B(f_n(A), \epsilon)$ for every $n \geqslant N$

Then there exist an $N \in \mathbb{Z}_+$ such that $d(\tilde{f}_n(A), \tilde{f}(A)) < \epsilon$ for every $n \geqslant N$ and each $A \in K(X)$.

Similarly we show that if $\tilde{f}_n \to \tilde{f}$ in $C(K(X))$ then $f_n \to f$ in $C(X)$.

There fore, ψ is an embedding of $C(X)$ in to $K(C(X))$.]□

Lemma 3.2. $\psi(C(X))$ *is closed in* $C(K(X))$

Proof. We prove that there exist a sequence $\{\tilde{f}_n\}$ in $\psi(C(X))$which converges to F such that there is $f \in C(X)$ satisfies $\tilde{f} = F$.

Consider $\psi^{-1}(\tilde{f}_n) = f_n$.

For every $\epsilon > 0$, take $N > 0$ such that $d(\tilde{f}_n, \tilde{f}_m) < \epsilon$, for every $n,m \geqslant N$.

This means that any compact set A satisfies $d(\tilde{f}_n(A), \tilde{f}_m(A)) < \epsilon$ for every $n,m \geqslant N$.

For $A \in K(X)$,choose $A_x = \{x\}$ for every $x \in X$.

Then $d(f_n, f_m) < \epsilon$ for every $n,m \geqslant N$ and all $x \in X$.

Since $\{f_n\}$ is a cauchy in $C(X)$, which is a complete metric space, there exist $f \in C(X)$ such that $\{f_n\}$ converges to f. Since ψ is continuous $\{\psi(f_n)\} = \{\tilde{f}_n\}$Since the limit is unique, $F = \tilde{f}$

There fore $F \in \psi(C(X))$.

There fore $\psi(C(X))$ is closed in $C(K(X))$.

Proof of The Theorem 3.1

Suppose $f : X \to X$ is stable and $\epsilon > 0$ is given .Then there are $\delta > 0$ and $h \in C(X)$ as is the definition of stability for f. Since ψ^{-1} is continuous at $\tilde{f} \in \psi(C(X))$,then for $\delta > 0$, there exist a $\delta_1 > 0$ such that if $\tilde{g} \in \psi(C(X))$ with $d(\tilde{f}, \tilde{g}) < \delta$, then we have, $d(\psi^{-1}(\tilde{f}), \psi^{-1}(\tilde{g}) = d(f, g) < \delta$.

From $f \circ h = h \circ g$, we have

$\psi(f \circ h)(A) = (\tilde{f} \circ \tilde{g})(A)$

$= (f \circ h)(A)$

$= \tilde{f}(h(A))$

$=\tilde{f}(\tilde{h}(A)) = (\tilde{f} \circ \tilde{g})(A), A \in K(X)$

and $\psi(h \circ g)(A) = (\tilde{h} \circ \tilde{g})(A), A \in K(X)$

Thus $\tilde{f} \circ \tilde{h} = \tilde{h} \circ \tilde{g}$.

Let $A \in K(X)$.From the continuity of h and compactness of A, we can see that

$$h(A) \subseteq \bigcup_{a \in A} B(a, \epsilon)$$

and

$$A \subseteq \bigcup_{a \in A} B(h(a), \epsilon)$$

if and only if

$$h(A) \subseteq B(A, \epsilon)$$

and

$$A \subseteq B(h(A), \epsilon)$$

that is, $h(A) \subseteq B(A,\epsilon)$ and $A \subseteq B(h(A),\epsilon)$ for any $A \in K(X)$.

This implies $d(\tilde{h}, \tilde{\iota}) < \epsilon$

There fore, $\tilde{f}$ is stable in $\psi(C(X))$.

Conversely, Suppose $\tilde{f} \in \psi(C(X))$ is stable.

for any $\epsilon > 0$, there exist a $\delta > 0$ and $\tilde{h} \in \psi(C(X))$ satisfying topological stability of $\tilde{f}$. Since ψ is continuous at $f \in C(X)$, there is a $\delta_2 > 0$ such that for any $g \in C(X)$ with $d(f, g) < \delta_2$,we have

$d(\psi(f), \psi(g)) = d(\tilde{f}, \tilde{g}) < \delta$

Take $A = \{x\}$ for each $x \in X$, then $\tilde{f} \circ \tilde{g} = \tilde{g} \circ \tilde{f}$ that

$(\tilde{f} \circ \tilde{g})(A) = \{f(h(x))\}$

$= \{h(g(x))\}$

$=(\tilde{g} \circ \tilde{f})(A), x \in X$

This implies

$f \circ h = h \circ g.$

From $d(\tilde{h}, \tilde{\iota}) < \epsilon$, *we have* $\tilde{f}(A) \subseteq B(A, \epsilon)$ *and* $A \subseteq B(\tilde{h}(A), \epsilon)$ *for* $A \in K(X)$.

Taking $A = \{a\}$, for each $a \in X$, implies

$\{h(a)\} \subseteq B(a,\epsilon)$ and $\{a\} \subseteq B(h(a),\epsilon)$

Since $a \in X$ is arbitrary, we obtain d(h, i) < ϵ

Hence $f \in C(X)$ is stable. □

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