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Growth of Optimizational Principles in Portfolio Analysis through Discrete Entropic Models

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Abstract

There exists an extensive collection of parametric and non-parametric discrete and continuous information models but still inevitability arises to broaden supplementary parametric models to encourage flexibility in the system under study. Moreover, there appears well-built relation between information entropy and the theory of "Portfolio Analysis". Further, various approaches of measuring risk in portfolio analysis including entropy method, divergence technique and integrated methodology etc. are accessible in the existing literature of portfolio analysis. In the present communication, our intention is to provide advancement regarding certain well convinced optimizational principles by means of new discrete entropic models, and consequently to deliver their solicitations in portfolio analysis. Additionally, the well-established principle has been enlightened through the assistance of a numerical illustration.

Keywords: Portfolio analysis, Portfolio selection, Modern portfolio theory, Entropy, Variance, Covariance matrix, Uncertainty

1. Introduction

Portfolio analysis is a quantitative technique for choosing a best portfolio that can strike an equilibrium flanked by superlative return and slightest risk in innumerable ambiguous situations. For the selection of an optimal portfolio, "return of a portfolio" and "risk involved in the portfolio" are utmost significant matters. Portfolio analysis is an investigation of the constituents encompassed in a combination of products with the determination of creating decisions that are predictable to advance complete return. The term relates to the procedure that permits an administrator to identify enhanced methods to assign resources with the goal of cumulating profits. This is well accredited statement that portfolio choice is apprehensive to allocate one's wealth surrounded by dissimilar securities to accomplish the investment objective

In Portfolio Selection Analysis, we provide deliberations payable by the philosophy of Markowitz [9] who secondhand mean-variance methodology. However, this mean variance model customarily outcomes in outsized portfolio turnovers because the fundamental data-generating process is far away from normality, which limits its applicability to dynamic investment schemes. The mean variance analysis furthermore executes poorly in out-of-sample tests, which supports that asset returns may be asymmetrical or non-normal and a dissimilar measure of uncertainty is mandatory to characterize asset returns over time. It is well acknowledged evidence that however variance is solitary an average deviation measure of information, it is supposed to be a conjoint and advantageous risk measure in such philosophical models. To make available the solution of this problem, it is obligatory to acquaint with a diversification model. Encouraged by the impression, some researchers acquaint with Shannon's [17] entropy for investment proportions in the securities because of its operational instrumentation nature.

MacLean et al. [8] with the pretentious that the financial market at each point in time is demarcated by a hidden Markov model, which is characterized by the inclusive equity market returns and volatility, developed a dynamic portfolio selection model combining economic uncertainty for business cycles. The authors well-thought-out that the risk associated with investment decisions is measured by the exponential Rényi [16] entropy criterion summarizing the uncertainty in portfolio returns, developed an entropy-based selection model for their findings. MacLean et al. [8] further stressed that the key awareness for characterizing financial market dynamics by means of a hidden Markov model is to decide the concern of unobserved financial market strength. Financial instruments frequently exhibit dissimilar risk levels in innumerable market conditions. Hidden Markov models are proficient of capturing unexpected changes in a mechanism that generates the data. With a hidden Markov model, investors have a better understanding of the market condition by applying Bayesian analysis to modernize the probability distribution of regimes with dynamically experimental new information over time.

Bera and Park [2] delivered certain fundamental comments about Markowitz's [9] mean-variance efficient portfolio that this selection is one of the most commonly second-handed approaches in explaining portfolio diversification problem. However, contrary to the notion of diversification, this approach frequently provides indications about the excessive concentrations of the portfolios on limited assets. Furthermore, the authors pointed out that this method leads to deprived out-of-sample performances whereas entropy is a well-recognized measure of diversity and also has a shrinkage interpretation. In their research, the authors proposed to employ cross- entropy measure as the objective function with side conditions coming from the mean and variance-covariance matrix of the resampled asset returns. This approach can be observed as a shrinkage estimation of portfolio probabilities which are contracted towards the predetermined portfolio. Ou [12] developed an innovative philosophy of portfolio and risk based on incremental entropy and Markowitz's theory by replacing the mean return adopted by Markowitz [9] with geometric mean return as a criterion for evaluating a portfolio. The author called it incremental entropy which indicates that the incremental speed of capital is a supplementary unprejudiced and testable criterion. This new-fangled theory highlights that there is a factually optimal portfolio for prearranged probability of returns and additionally provides some formulas for optimizing portfolio allocations.

Recently, Mercurio et al. [10] presented an amended technique of demonstrating entropy model as a risk convoluted in portfolio problem. The authors have acquainted with an innovative multiplicity of problems entitled return-entropy optimization problems. This procedure streamlines calculations by means of a combinatorial methodology which discourses five foremost concrete apprehensions with the mean-variance optimization. Li et al. [7] well-thought-out the problem of diversified portfolio selection and mentioned that this problem provides a significant concern in indeterminate economic situation. In their findings, the authors deliberated the problem surrounded by the structure of uncertainty theory and as a consequence anticipated an uncertain extension mean-variance diversification model by selecting the mean as objective function along with variance and entropy by means of risk and diversity restrictions. The two discrepancies have then been explored for the determinations of slightest risk and concentrated return.

Recently, Bisht and Kumar [3] mentioned that with the developing areas of economy, the miscellaneous sector-based investment portfolios are deliberated more momentous and offered an integrated methodology of portfolio construction grounded on sector analysis for investment in National Stock Exchange of India. The authors developed a model workable in four stages and the results achieved by the anticipated portfolio are found very noteworthy which confirms its effectiveness of the proposed model. From the applications point of understanding, the authors provided the experimental study over the existing models.

Some accompanying modernizers who contributed a percentage for the enrichment of the portfolio analysis include Burger and Uzsoki [4], Nocetti [11], Baiden [1], Gulko [5], Lassance [6], Phillippatos and Wilson [14], Rau-Bredow [15], Simonelli [18], Smimou [19], Soyer and Tanyeri [20], Stuart and Markowitz [21], Tu [22], Usta and Kantar [23], Whitelaw [24], Xu et al. [25], Yang and Qiu [26] etc.

In the literature of information models, there occur numerous well-acknowledged entropy models originated primarily from Shannon's [17] model. This entropic model with astonishingly congenial possessions is specified by subsequent manifestation:

$$S(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{1.1}$$

In the present communication, we make use of various entropic models for the development of optimizational principles but beforehand, we make available a momentary outline to the conception of mean-efficient viewpoint outstanding to Markowitz [9]. To explicate this uncomplicated commencement, we advance subsequently:

Let p_j designate the probability of jth security outcome and ρ_{ij} symbolize the security return on ith security, then the expected return on ith security is specified by the subsequent mathematical manifestation:

$$\bar{\rho}_i = \sum_{j=1}^{m} p_j \rho_{ij} \tag{1.2}$$

Furthermore, the variances and the covariance's of security returns are prearranged by the subsequent expressions:

$$\sigma_i^2 = \sum_{j=1}^{m} p_j \left(\rho_{ij} - \overline{\rho_i}\right)^2, \tag{1.3}$$

$$r_{ik}\sigma_i\sigma_k = \sum_{j=1}^{m} p_j \left(\rho_{ij} - \overline{\rho_i}\right)\left(\rho_{kj} - \overline{\rho_k}\right) \tag{1.4}$$

Let us suppose that a person decides to make his investment extents $y_1, y_2, \ldots, y_n$ of his entire resources in n securities.

$$\sum_{i=1}^{n} y_i = 1; \quad y_i \geq 0 \tag{1.5}$$

At that moment, expected return on securities and variance return are specified by

$$E = \sum_{i=1}^{n} y_i \overline{\rho_i} \tag{1.6}$$

and

$$V = \sum_{i=1}^{n} y_i^2 \sigma_i^2 + 2\sum_{k=1}^{n} \sum_{i<k} y_i y_k r_{ik} \sigma_i \sigma_k \ . \tag{1.7}$$

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Markowitz [9] advocated his portfolio theory by the principle that $y_1, y_2, \dots, y_n$ be selected to capitalize on the expected return E and to decrease the variance V, or, otherwise, to reduce V once E is reserved at a stationary value. Now

$$V = \sum_{j=1}^{m} p_j \left(\rho_j - \overline{\rho}\right)^2, \tag{1.8}$$

where

$$\rho_j = \sum_{i=1}^{n} y_i \rho_{ij} \quad \text{and} \quad \overline{\rho} = \sum_{i=1}^{n} y_i \overline{\rho_i} \tag{1.9}$$

The Markowitz's [9] criterion can be elucidated geometrically where for each vector $\left(y_1, y_2, \dots, y_n\right)$, one can discover the values E and V and then represent a point in the $E-V$ plane.

This interpretation payable to Markowitz's [9] principle indicates that uncertainty or entropy models can be employed magnificently in the portfolio analysis. Keeping in observation this knowledge, we make advancement for the development of optimizational principles by means of our own entropy models.

2. Development of Optimization Principles Through Entropy Measures

Markowitz [9] promoted his criterion by deliberating the choice from investment proportions $y_1, y_2, \dots, y_n$ in such an approach which minimizes the variance, that is, to make $\rho_1, \rho_2, \dots, \rho_m$ as identical as probable. Consequently, any disappearance of $\rho_1, \rho_2, \dots, \rho_m$ from egalitarianism was deliberated as risk. The equivalent persistence can be made proficient if we select $y_1, y_2, \dots, y_n$ so as to maximize the entropy function. Since the probabilities have not been involved in the above conversation but there is convinced requirement for their enclosure, we can instead consider the maximization of the subsequent entropy model:

$$H\left(P\right) = -\sum_{j=1}^{m} \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \log \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \tag{2.1}$$

The manifestation (2.1) characterizes Shannon's [17] entropy function and discovers incredible applications in a diversity of disciplines.

Subsequently, we have designed a novel discrete parametric entropic model in probability spaces and studied its crucial properties for authenticity. The necessity for the development of the model arises due to its applicability in the field of "Portfolio Analysis". This quantitative discrete parametric entropic model is specified by the subsequent manifestation:

$$H_\alpha\left(P\right) = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\alpha} \sum_{i=1}^{n} \log\left[1 + \alpha p_i\right] - \frac{\log(1+\alpha)}{\alpha}; \alpha \neq 0, \alpha > 0 \tag{2.2}$$

We observe that

$$\lim_{\alpha \to 0} H_\alpha(P) = -\sum_{i=1}^{n} p_i \log p_i$$

Hence, we examine that the appearance $H_\alpha\left(P\right)$ provided in (2.2) is a generalization of the well accepted Shannon's [17] entropy.

Next, to authenticate that the measure (2.2) is a convincing entropy model, we study its fundamental properties as follows:

(i) We have $H_\alpha\left(P\right)\geq 0$

For n degenerate distributions

$\Delta_1=\left(1,0,0,...,0\right),\Delta_2=\left(0,1,0,...,0\right),\ldots,\Delta_n=\left(0,0,0,...,1\right)$, we have $H_\alpha\left(P\right)=0$

Since, entropy gives minimum value for degenerate distributions and the minimum value is 0, we must have $H_\alpha\left(P\right)\geq 0$.

(ii) $H_\alpha\left(P\right)$ is permutationally symmetric function of $p_1,p_2,...,p_n$ as it does not change if $p_1,p_2,...,p_n$ are re-ordered among themselves.

(iii) $H_\alpha\left(P\right)$ is a continuous function of p_i for all p_i's.

(iv) Concavity: To demonstrate the concavity property, we carry on subsequently:

We have $\dfrac{\partial H_\alpha(P)}{\partial p_i}=-1-\log p_i+\dfrac{1}{1+\alpha p_i}$

Also

$$\frac{\partial^2 H_\alpha(P)}{\partial p_i^2}=-\frac{1}{p_i}-\frac{\alpha}{\left\{1+\alpha p_i\right\}^2}<0$$

Thus, $H_\alpha\left(P\right)$ is a concave function of $p_1,p_2,...,p_n$.

Moreover, with the help of numerical data shown in the following Table-2.1 for $n=2$ and $\alpha=2$, we have presented the measure $H_\alpha\left(P\right)$ against p as shown in Figure-2.1.

Table-2.1: $H_\alpha\left(P\right)$ against p for $n=2$ and $\alpha=2$

p	$H_\alpha\left(P\right)$
0	0.0000000
0.1	0.2756456
0.2	0.3884567
0.3	0.4545678
0.4	0.4893489
0.5	0.5000000
0.6	0.4895636
0.7	0.4546767
0.8	0.3887878
0.9	0.2714587

1	0.0000000

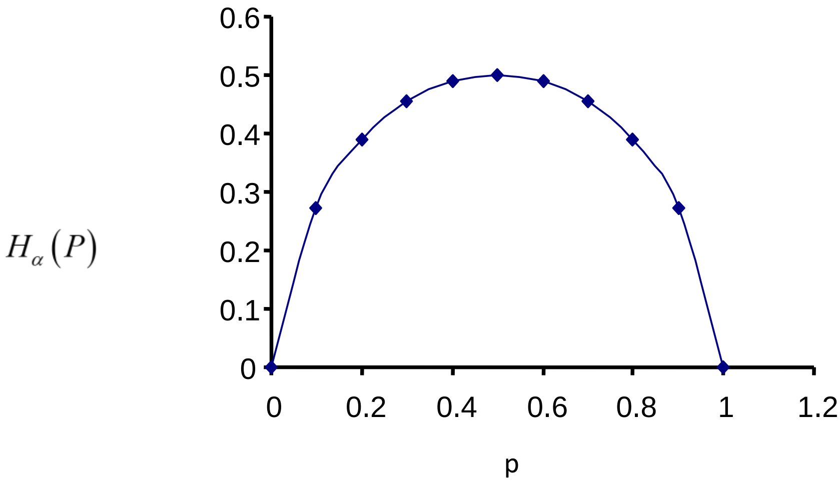

Figure-2.1: Concavity of $H_\alpha(P)$ with respect to P

The Figure-2.1 undoubtedly shows that the measure (2.2) is concave. Hence, under the above circumstances, the function $H_\alpha(P)$ is an acceptable entropy model.

(v) Maximization: We use Lagrange's method to maximize the entropy measure (2.2) subject to the natural constraint $\sum_{i=1}^{n} p_i = 1$. In this case, the corresponding Lagrange's function is

$$L \equiv H_\alpha(P) - \lambda\left(\sum_{i=1}^{n} p_i - 1\right)$$

Differentiating above equation with respect to $p_1, p_2, ..., p_n$ and equating the derivatives to zero, we acquire the subsequent appearance:

$$\frac{1}{1+\alpha p_1} - \log p_1 = \frac{1}{1+\alpha p_2} - \log p_2 = ... = \frac{1}{1+\alpha p_n} - \log p_n$$

which is possible only if $p_1 = p_2 = ... = p_n$. Further, using $\sum_{i=1}^{n} p_i = 1$, we get $p_i = \frac{1}{n}$, $\forall\, i = 1, 2, \ldots, n$.

Thus, we observe that the maximum value of $H_\alpha(P)$ arises for the uniform distribution $U = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$ and this result is mainly attractive one.

(vi) The maximum value $f(n)$ of the entropy is given by

$$f(n) = \log n + \frac{1}{\alpha}\left[n \log(n+\alpha) - n \log n\right] - \frac{\log(1+\alpha)}{\alpha}$$

Thus, we have $f'(n)=\dfrac{\alpha}{n(n+\alpha)}+\dfrac{1}{\alpha}\log\left\{1+\dfrac{\alpha}{n}\right\}>0$

This expression consequently entails $f(n)$ to be an increasing function of n, again an attractive consequence. With the study of these properties, we assertion that the model (2.2) is authentic entropy model. Thus, in the present circumstances, the model (2.2) takes the subsequent form:

$$H^{\alpha}(P)=Constant-\frac{1}{\bar{\rho}}\sum_{j=1}^{m}p_j\rho_j\log p_j\rho_j+\frac{1}{\alpha}\sum_{j=1}^{m}\log\left\{\bar{\rho}+\alpha p_j\rho_j\right\} \tag{2.3}$$

Accordingly, we articulate the subsequent entropy based principle as:

Optimizational Principle-I

Choose $y_1, y_2, \ldots, y_n$ so as to maximize

$$\begin{aligned}Z=-\sum_{j=1}^{m}p_j\left(y_1\rho_{1j}+y_2\rho_{2j}+\cdots+y_n\rho_{nj}\right)\log\left(y_1\rho_{1j}+y_2\rho_{2j}+\cdots+y_n\rho_{nj}\right)\\+\frac{1}{\alpha}\sum_{j=1}^{m}\log\left\{\bar{\rho}+\alpha\left(y_1\rho_{1j}+y_2\rho_{2j}+\cdots+y_n\rho_{nj}\right)\right\}\end{aligned} \tag{2.4}$$

subject to the subsequent constraints:

$$\sum_{j=1}^{m}p_j\left(y_1\rho_{1j}+y_2\rho_{2j}+\cdots+y_n\rho_{nj}\right)=Constant \tag{2.5}$$

and

$$y_1+y_2+\ldots+y_n=1 \tag{2.6}$$

$$y_1, y_2, \ldots, y_n \geq 0 \tag{2.7}$$

The principle has subsequently been enlightened:

Numerical Illustration: We replicate upon the case of two securities, each one with ten conceivable consequences displayed in the succeeding Table-2.2

Table-2.2

Probability	**Return-I**	**Return-II**
0.05	0.15	0.20
0.10	0.05	0.15
0.15	0.10	0.05
0.15	0.15	0.15
0.10	0.20	0.10
0.10	0.20	0.05
0.15	0.15	0.10
0.10	0.05	0.15
0.05	0.10	0.05
0.05	0.20	0.10

We need to discover the optimum values of y_1 and y_2, with mean return is 0.12. The mathematical optimization problem for $\alpha=2$ can be articulated subsequently:

Maximize

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$$Z=-\sum_{j=1}^{10}p_j\left(y_1\rho_{1j}+y_2\rho_{2j}\right)\log\left(y_1\rho_{1j}+y_2\rho_{2j}\right)+\frac{1}{2}\sum_{j=1}^{10}\log\left\{\overline{\rho}+2\left(y_1\overline{\rho}_{1j}+y_2\rho_{2j}\right)\right\} \tag{2.8}$$

subject to the set of subsequent constraints:

$$\sum_{j=1}^{10}p_j\left(y_1\rho_{1j}+y_2\rho_{2j}\right)=0.12 \tag{2.9}$$

$$y_1+y_2=1 \tag{2.10}$$

and

$y_1\geq 0, y_2\geq 0$ where

$p_1=0.05$, $p_2=0.10$, $p_3=0.15$, $p_4=0.15$, $p_5=0.10$, $p_6=0.10$, $p_7=0.15$, $p_8=0.10$, $p_9=0.05$, $p_{10}=0.05$

and

$\rho_{11}=0.15,\ \rho_{12}=0.05,\ \rho_{13}=0.10,\ \rho_{14}=0.15,\ \rho_{15}=0.20,\rho_{16}=0.20,\rho_{17}=0.15,\rho_{18}=0.05,$

$\rho_{19}=0.10,\ \rho_{110}=0.20,\ \rho_{21}=0.20,\rho_{22}=0.15,\ \rho_{23}=0.05,\ \rho_{24}=0.15,\ \rho_{25}=0.10,\ \rho_{26}=0.05,$

$\rho_{27}=0.10,\ \rho_{28}=0.15,\ \rho_{29}=0.05,\ \rho_{210}=0.10$

Upon differentiation the corresponding Lagrangian and associating to zero, we acquire the simultaneous set of equations with subsequent manifestations:

$$-\sum_{j=1}^{10}p_j\left\{1+\rho_{1j}\log\left(y_1\rho_{1j}+y_2\rho_{2j}\right)\right\}+\sum_{j=1}^{10}\frac{\rho_{1j}}{\overline{\rho}+2\left(y_1\rho_{1j}+y_2\rho_{2j}\right)}-\lambda\sum_{j=1}^{10}p_j\rho_{1j}-\mu=0 \tag{2.11}$$

and

$$-\sum_{j=1}^{10}p_j\left\{1+\rho_{2j}\log\left(y_1\rho_{1j}+y_2\rho_{2j}\right)\right\}+\sum_{j=1}^{10}\frac{\rho_{2j}}{\overline{\rho}+2\left(y_1\rho_{1j}+y_2\rho_{2j}\right)}-\lambda\sum_{j=1}^{10}p_j\rho_{2j}-\mu=0 \tag{2.12}$$

Now employing the relation $\overline{\rho}=\sum_{i=1}^{2}y_i\overline{\rho_i}$, the above data provides $\overline{\rho}=0.1325y_1+0.1075y_2$

Consequently, the simultaneous set of equations (2.11) and (2.12) becomes

$$-\sum_{j=1}^{10}p_j\left\{1+\rho_{1j}\log\left(y_1\rho_{1j}+y_2\rho_{2j}\right)\right\}+\sum_{j=1}^{10}\frac{\rho_{1j}}{\left\{0.1325y_1+0.1075y_2\right\}+2\left(y_1\rho_{1j}+y_2\rho_{2j}\right)} \tag{2.13}$$
$$-\lambda\sum_{j=1}^{10}p_j\rho_{1j}-\mu=0$$

and

$$-\sum_{j=1}^{10} p_j \left\{1+\rho_{2j}\log\left(y_1\rho_{1j}+y_2\rho_{2j}\right)\right\}+\sum_{j=1}^{10}\frac{\rho_{2j}}{\{0.1325y_1+0.1075y_2\}+2\left(y_1\rho_{1j}+y_2\rho_{2j}\right)} \tag{2.14}$$
$$-\lambda\sum_{j=1}^{10} p_j\rho_{2j}-\mu=0$$

By means of p_j's and ρ_{ij}'s from the given data and solving the equations (2.13) and (2.14) for random vales of $\lambda\ and\ \mu,$ we acquire $y_1=0.4333$ and $y_2=0.5667$ providing the optimum values of y_1 and y_2, with mean return 0.12.

In the sequel, we articulate additional optimizational principle and for this determination, we put in practice our identifiable quantitative parametric entropic model of order α and type β recently developed by Parkash, Singh and Sharma [13] and well established the subsequent quantitative manifestation:

$$H_{\alpha,\beta}(P)=\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[1-p_i^{(\beta-\alpha)p_i}\right];\alpha\neq\beta,\beta-\alpha>0 \tag{2.15}$$

Accordingly, in the present circumstances, the above model (2.15) takes the subsequent arrangement:

$$H_{\alpha,\beta}(P)=\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[1-\left\{\frac{p_j\rho_j}{\sum_{k=1}^{m}p_k\rho_k}\right\}^{(\beta-\alpha)\frac{p_j\rho_j}{\sum_{k=1}^{m}p_k\rho_k}}\right]=Constant-\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left\{\frac{p_j\rho_j}{\bar{\rho}}\right\}^{(\beta-\alpha)\frac{p_j\rho_j}{\bar{\rho}}}$$

Accordingly, we articulate the subsequent generalized entropy based principle as:

Optimizational Principle-II

Choose $y_1,y_2,\ldots,y_n$ so as to maximize

$$Z=-\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left\{\frac{p_j\rho_j}{\bar{\rho}}\right\}^{(\beta-\alpha)\frac{p_j\rho_j}{\bar{\rho}}} \tag{2.16}$$

subject to the subsequent constraints:

$$\sum_{j=1}^{m}p_j\left(y_1\rho_{1j}+y_2\rho_{2j}+\cdots+y_n\rho_{nj}\right)=Constant \tag{2.17}$$

and

$$y_1+y_2+\ldots+y_n=1 \tag{2.18}$$

$$y_1,y_2,\ldots,y_n\geq 0 \tag{2.19}$$

Conclusion

Our findings conceal that by means of parametric and non-parametric information theoretic entropy models, we can deliver numerous advancements towards several optimizational principles worthwhile in countless disciplines of Operations Research. Additionally, since these parametric models having their own merits induce flexibility in the system under study, entropy models can be made proficient to deliver their solicitations in portfolio analysis. The newly proposed discrete entropy models provide an alternative method to make decision-making in uncertain portfolio selection problem. Such a comprehensive study can be stretched by engaging other parametric models of information entropy for the discrete as well as continuous probability distributions.

Compliance with Ethical Standard

Disclosure of potential conflicts of interest:

The authors affirm the absence of any potential conflicts of interest concerning the research, authorship, and publication of this paper.

Moreover, No external financial support was sought or received from any funding agency, organization, or institution for the research, development, or publication of this work.

Research involving Human Participants and/or Animals:

In ensuring the safety and welfare of human participants and/or animals, this study strictly adhered to ethical guidelines and research conducted in this study did not involve the use of human participants or animals.

Informed consent:

Informed consent was diligently obtained, underscoring the commitment to prioritize the rights and well-being of all participants throughout the research process.

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