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Secondary k-Kernel Symmetric Interval Valued Intuitionistic Neutrosophic Fuzzy Matrices

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Abstract

We propose the idea of secondary symmetric k-kernels with interval-valued intuitionistic neutrosophic fuzzy matrices (IVINFM) like an EP matrix within the complex field. The notion of secondary kernel IVINFM and k-kernel symmetric (KS) IVINFM is presented by providing examples. We also illustrate the graphical representation of KS adjacency IVINFM and incidence IVINFM. We found every isomorphic IVINFM and non-isomorphic IVINFM graph to be k-KS IVINFM. However, the reverse shall not be the case. The definition of k-symmetric IVINFM as k- KS IVINFM, but the reverse is not the case. The characteristics of secondary kernels IVINFM, which are symmetric IVINFM, have been explored in this research using examples. The relationship between s-k KS IVINFM, symmetric s- KS IVINFM, and KS IVINFM, and the KS IVINFMs are examined. We identify the required and sufficient criteria for the KS IVINFM and s-k KS IVINFM. The comparable requirements for the various g-inverses that make up an s-k KS IVINFM are shown. The generalized inverses of a KS IVINFM A, which correspond to the sets A{1,2}, A{1, 2 ,3} and A{1,2, 4} are described.

Keywords: IVINM, KS- IVINFM, s-k- KS IVINFM, adjacency IVINFM, incidence IVINFM.

1. Introduction

Consider A as a neutrosophic matrix (NFM). If A is a part of (NF)n it is referred to as a k-KS NFM when $N(A) = N(KA^TK)$. Matrices play a crucial role in various research fields within engineering and science. Conventional matrix theory must tackle problems involving significant uncertainty. Zadeh [1] introduced the concept of fuzzy sets (FS) using membership numbers. Atanassov [2] intuitively designed a fuzzy set that effectively provides membership and non-membership grades for an element.

Smarandache [3] proposed concepts like neutrosophic sets, a mathematical instrument to solve problems requiring indeterminacy, ambiguity, and inconsistent data.

Fuzzy matrices (FM) can be used to address specific types of problems. Numerous researchers have completed many projects. Fuzzy matrices handle only membership values. They cannot deal with values that are non-members. Khan et.al [4] were the pioneers in introducing IFMs to the academic community. Anandhkumar et al. [20-21] investigated generalized symmetric NFMs, partial ordering concepts in NFMs. Atanassov [5,6] developed significant results related to IFS and IVIFS. Hashimoto [7] investigated the canonical type of transitive matrices. Pal and Susanta Kha [22] explored interval-valued intuitionistic properties. Vidhya and Irene Hepzibah [23] presented IVNFMs. Kim and Roush [8] explored generalized FM. Lee [9] focused on secondary skew-symmetric and secondary orthogonal matrices. Hill and Waters [10] addressed Hermitian matrices. Meenakshi [11] delved into the concept of fuzzy matrices and their properties.

Meenakshi and Jaya Shree [12] explored symmetric k-kernel matrices, while Meenakshi and Krishanmoorthy [13] discussed the properties of secondary k-Hermitian matrices. Additionally, Meenakshi and Jaya Shree [14] have been investigating k-RS matrices. Morteza Yazdani et al. [24] applied interval-valued neutrosophic concepts to decision-making in supplier selection. Jaya Shree [15] delved into secondary K-KS fuzzy concepts, and Shyamal and Pal [16] presented findings on IVFM. Meenakshi and Kalliraja [17] provided a summary of regular interval-valued matrices. Anandhkumar and colleagues [18] conducted research on pseudo similarity neutrosophic fuzzy matrices.Anandhkumar et.ai[19] have studied on various Inverse of Neutrosophic Fuzzy Matrices. Jaya Shree [25] have discussed Secondary k-range symmetric fuzzy matrices. Vidhya and R. Irene Hepzibah [26] have focused On Interval Valued Neutrosophic Fuzzy Matrices.

Anandhkumar et.al [27-33] have studied on Kernel and K-Kernel Symmetric Intuitionistic Fuzzy Matrices, IV Secondary k-RS Neutrosophic fuzzy matrices, secondary k-CS neutrosophic fuzzy matrices, Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices, Reverse Tilde (T) and Minus Partial Ordering on fuzzy matrices, Secondary K-Range Symmetric Neutrosophic Fuzzy Matrices,Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices.

We present the secondary K-KS IVINM and provide a few basic operators on IVINMs. Section 2 highlights on preliminaries. Section 3 on Graphical Representation of KS Adjacency IVINM is given. Section 4 discusses on s - k KS IVINMs and regular IVINMs. In Section 5, we have discussed on various generalized inverses of matrices in IVINM. The generalized inverses of a s−ks IVINFM equivalent to the sets A={1,2}, A={1,2,3}, A={1,2,4} are considered.

Table: 1 Review of the literatures

References	**Extension of Fuzzy Matrices.**	**Year**
[12]	On k- KSFM	2009
[14]	On k -RSFM	2009
[15]	Secondary κ- KSFM	2014
[25]	Secondary κ-RSFM	2018
[28]	IV Secondary k-RS Neutrosophic fuzzy matrices	2024

[29]	secondary k-CS neutrosophic fuzzy matrices	2024
[32]	Secondary K-Range Symmetric Neutrosophic Fuzzy Matrices	2024
Proposed	Secondary k- KS IVINFMs	2024

Based on literature review that reflects no research has been carried out on Secondary k-KS IVINFM to overcome the research gap.

1.1 Research Gap

Jayashri detailed on k-range symmetric fuzzy matrices (FMs). Meenakshi and Jayashri established the results of KS in FMs.Anandhkumar established the results of IV Secondary k-RS Neutrosophic fuzzy matrices. In this paper, we have used the idea of secondary k-KS IVINFM. We have implemented the properties in IVINFM. We first present similar characterizations of the Secondary k-KS IVINFM. We then define the case of a Secondary k-KS IVINFM. We have looked at various g-inverses that are associated with regular matrices. Then, we have the definition of a set of all inverses using secondary symmetric s-k-Kernel IVINFM.

1.2 Notations:

$[A_\mu, A_\lambda, A_\nu]_L^T$ - Transpose of the IVINM $[A_\mu, A_\lambda, A_\nu]_L$,

$[A_\mu, A_\lambda, A_\nu]_U^T$ - Transpose of the IVINM $[A_\mu, A_\lambda, A_\nu]_U$,

$[A_\mu, A_\lambda, A_\nu]_L^+$ - Moore-Penrose inverse (MPI) of IVINM $[A_\mu, A_\lambda, A_\nu]_L$,

$[A_\mu, A_\lambda, A_\nu]_U^+$ - MPI of IVINM $[A_\mu, A_\lambda, A_\nu]_U$,

$R\left([A_\mu, A_\lambda, A_\nu]_L\right)$- Row space of $[A_\mu, A_\lambda, A_\nu]_L$

$R\left([A_\mu, A_\lambda, A_\nu]_U\right)$- Row space of $[A_\mu, A_\lambda, A_\nu]_U$,

$C\left([A_\mu, A_\lambda, A_\nu]_L\right)$-Column space of $[A_\mu, A_\lambda, A_\nu]_L$,

$C\left([A_\mu, A_\lambda, A_\nu]_U\right)$-Column space of $[A_\mu, A_\lambda, A_\nu]_U$

2. Preliminaries

Consider V a permutation matrix with secondary diagonal units. The function $\kappa(x)=(z_{k[1]}, z_{k[2]}, z_{k[3]},\ldots, z_{k[n]})\in F_{n\times 1}$ for $z = z_1, z_2,..,z_n \in F_{[1\times n]}$, K is involuntary, The subsequent properties holds good by using the Definition of V [15].

(P.2.1) $KK^T = K^T K = I_n,\ K = K^T, K^2 = I$

(P.2.2) $V = V^T, VV^T = V^TV = I_n$ and $V^2 = I$

(P.2.3) $N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L\right)V, N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L\right)K$

$N\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U\right)V, N\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U\right)K$

(P.2.4) $N\left([A_\mu, A_\lambda, A_\nu]_L\ \mathrm{V}\right)^T = N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_L{}^T\right)$, $N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_L\right)^T$ $=N\left([A_\mu, A_\lambda, A_\nu]_L{}^T V\right)$

$N\left([A_\mu, A_\lambda, A_\nu]_U\ \mathrm{V}\right)^T = N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_U{}^T\right)$, $N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_U\right)^T$ $=\left([A_\mu, A_\lambda, A_\nu]_U{}^T V\right)$

Definition:2.1[26] An IVNFM A = $[x_{ij}, < a_{ij\mu}, a_{ij\lambda}\ a_{ij\nu} >]_{m\times n}$ wherever $a_{ij\mu}$, $a_{ij\lambda}$ and $a_{ij\nu}$ are the subsets of [0,1] which are represented by $a_{ij\mu} = [a_{ij\mu L}, a_{ij\mu U}]$, $a_{ij\lambda} = [a_{ij\lambda L}, a_{ij\lambda U}]$ and $a_{ij\nu} = [a_{ij\nu L}, a_{ij\nu U}]$ with the conditions $0 \le a_{ij\mu U} + a_{ij\lambda U} + a_{ij\nu U} \le 3$, $0 \le a_{ij\mu L} + a_{ij\lambda L} + a_{ij\nu L} \le 3$, $0 \le a_{\mu L} \le a_{\mu U} \le 1$, $0 \le a_{\lambda L} \le a_{\lambda U} \le 1$, $0 \le a_{\nu L} \le a_{\nu U} \le 1$ for i = 1,2,⋯,m and j = 1,2,⋯,n.

Example2.1 Consider an IVNFM

$$A = \begin{bmatrix} <[1,1],[1,1],[0,0]> & <[0.2,0.5],[0.3,0.6],[0.4,0.4]> \\ <[0.2,0.5],[0.3,0.6],[0.4,0.4]> & <[1,1],[1,1],[0,0]> \end{bmatrix}$$

Lower Limit IVNFM, $[A_\mu, A_\lambda, A_\nu]_L = \begin{bmatrix} <1,1,0> & <0.2,0.3,0,4> \\ <0.2,0.3,0.4> & <1,1,0> \end{bmatrix}$,

Upper Limit IVNFM, $[A_\mu, A_\lambda, A_\nu]_U = \begin{bmatrix} <1,1,0> & <0.5,0.6,0.4> \\ <0.5,0.6,0.4> & <1,1,0> \end{bmatrix}$

and $B = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.3,0.5],[0.1,0.5]> \\ <[0.2,0.4],[0.3,0.5],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

Then, $A + B = \begin{bmatrix} <[1,1],[0,0],[1,1]> & <[0.2,0.5],[0.3,0.5],[0.1,0.5]> \\ <[0.2,0.5],[0.3,0.5],[0.1,0.5]> & <[1,1],[0,0],[1,1]> \end{bmatrix}$

Definition 2.2 [20] (Null IVNFM) IVNFM is said to be Null if the entries of true and indeterminacy are zero and the entries of false is one i.e., ([0,0],[0,0],[1,1]).

Example: 2.2 Consider IVNFM $A = \begin{bmatrix} ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \\ ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) \end{bmatrix}$,

Note:1 [11] For IVNFM A∈ F_n with det A > ([0,0], [0,0],[1,1]) where no rows or columns are zero, we have N(A) = ([0,0],[0,0],[1,1]) = N(A^T). Additionally, for a symmetric matrix A = A^T, it follows that N(A)= N(A^T).

Note:2 [12] Let A is k-Symmetric IVNFM implies it implies that it is also a k-KS IVNFM, such that A = K(A^T)K which leads to N(A) = N(KA^T K) .Example 2.3. Example 2.3 demonstrates that the converse is not necessarily true.

Example: 2.3 Let us Consider IVNFM

$$A = \begin{bmatrix} <[0,1],[0,1],[0.5,0.5]> & <[0,0.5],[0,0.6],[0.4,0.4]> & <[0.3,0.5],[0.4,0.6],[0.5,0.5]> \\ <[0.5,0.5],[0.4,0.6],[0.5,0.5]> & <[0.1,0.3],[0.4,0.6],[0.6,0.6]> & <[0,0.5],[0,0.6],[0.4,0.5]> \\ <[0.4,0.5],[0.5,0.6],[0.3,0.5]> & <[0.3,0.5],[0.4,0.4],[0.5,0.5]> & <[0,0.5],[0,0.6],[0.3,0.5]> \end{bmatrix}$$

$$K=\begin{bmatrix}(0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (1,1,0) & (0,0,1)\\ (1,1,0) & (0,0,1) & (0,0,1)\end{bmatrix},\ A_L=\begin{bmatrix}(0,0,0.5) & (0,0,0.4) & (0.3,0.4,0.5)\\ (0.5,0.4,0.5) & (0.1,0.4,0.6) & (0,0,0.4)\\ (0.4,0.5,0.3) & (0.3,0.4,0.5) & (0,0,0.3)\end{bmatrix}$$

Therefore, $A_L \neq KA_L^T K$, But, $N(A_L) = N(KA_L^T K) = (0,0,1)$

Definition 2.3. [20] For IVNFM P is KS fuzzy matrix iff $N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L^{\,T}\right)$ and $N\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U^{\,T}\right)$.

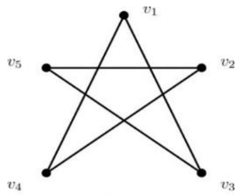
Definition 2.4. [27] An IVINFM $A = [x_{ij}, < a_{ij\mu}, a_{ij\lambda}\ a_{ij\nu} >]_{m\times n}$ where $a_{ij\mu}$, $a_{ij\lambda}$ and $a_{ij\nu}$ are the subsets of [0,1] which are represented by $a_{ij\mu} = [a_{ij\mu L}, a_{ij\mu U}]$, $a_{ij\lambda} = [a_{ij\lambda L}, a_{ij\lambda U}]$ and $a_{ij\nu} = [a_{ij\nu L}, a_{ij\nu U}]$ with the conditions $0 \le a_{ij\mu U} + a_{ij\lambda U} + a_{ij\nu U} \le 2$, $0 \le a_{ij\mu U} \le a_{ij\nu U} \le 1$, $0 \le a_{ij\lambda U} \le 1$ for i = 1,2,⋯,m and j = 1,2,⋯,n.

3. Graphical Representation of KS Adjacency IVINFM.

Definition 3.1. Adjacency IVINFM: An IVINM is a square matrix representing a finite graph. The elements of this matrix indicate whether pairs of vertices in the graph are connected. In the case of a finite simple graph, the IVINFM can be represented as a binary matrix, typically denoted as ([1,1],[1,1],[0,0]) and ([0,0],[0,0],[1,1]), where the diagonal elements are consistently set to ([0,0],[0,0],[1,1]) .Let G(V, E) denote a graph with n vertices. The adjacency matrix A = $[a_{ij}]$ is a symmetric matrix defined by $A=[a_{ij}]=\begin{cases}([1,1],[1,1],[0,0]) & \text{when } v_i \text{ is adjacent to } v_j\\ ([0,0],[0,0],[1,1]) & \text{otherwise}\end{cases}$, denoted by A(G) or A_G

Example: 3.1 Consider an IVINM and a equivalent graph

$$\begin{matrix} & v_1 & v_3 & v_4 & v_2 & v_5\\ v_1 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1])\\ v_3 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0])\\ v_4 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1])\\ v_2 & ([0,0],[0,0],[1,1]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0])\\ v_5 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1])\end{matrix}$$



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Definition 3.2. Incidence IVINFM

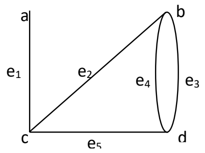
Let G(V, E) represent a simple graph with n vertices. Let V = {V_1, V_2, …, V_n} and E = {e_1, e_2, ..., e_m}. Then, the incidence IVINFM I = $[m_{ij}]$ is a $n\times m$ matrix defined by

$$I=[m_{ij}]=\begin{cases}([1,1],[1,1],[0,0]) & \text{when } v_i \text{ is incident to } e_j\\ ([0,0],[0,0],[1,1]) & \text{otherwise}\end{cases}$$ **,** denoted by A(G) or A_G.

Example:3.2 Consider an incidence IVINFM and a equivalent graph. The incidence IVINFM is

$A =$

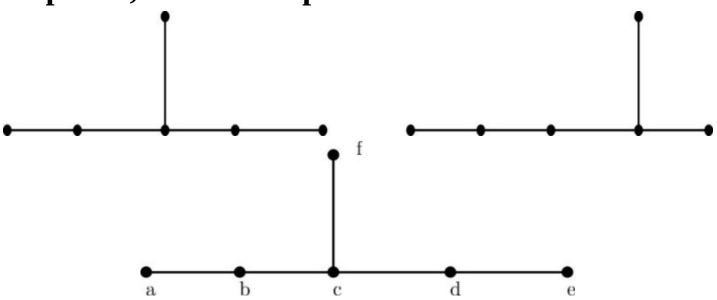
	e_1	e_2	e_3	e_4	e_5
a	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
b	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])
c	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])
d	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])



Definition 3.3. **Isomorphic of Graph**

Two graphs are said to be isomorphic if number of vertices, edges, degree sequence and adjacency IVINFM are equal.

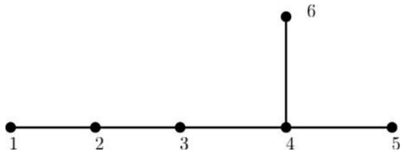
Relation between isomorphism, non-isomorphism and KS



The adjacency IVINFM of the graph is given by

	a	b	c	d	e	f
a	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
b	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
c	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])
d	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])
e	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
f	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])

Consider the graph H and name as follows

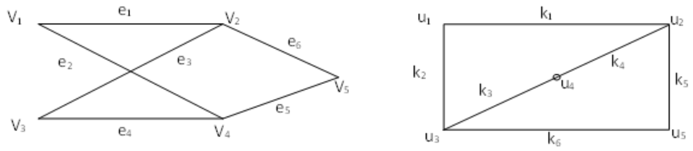


The adjacency IVINM of the graph is given by

	1	2	3	4	5	6
1	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
2	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
3	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
4	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([1,1],[1,1],[0,0])
5	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])
6	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])	([1,1],[1,1],[0,0])	([0,0],[0,0],[1,1])	([0,0],[0,0],[1,1])

The two graphs presented have identical numbers of vertices, edges, and degree sequences, yet their adjacency IVINFMs differ.

Therefore the given Graph is not isomorphic but KS.



Let us form the adjacency IVINFM AG and AH

$$A_G=\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 \\ v_1 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ v_2 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \\ v_3 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ v_4 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \\ v_5 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \end{bmatrix}$$

$$A_H=\begin{bmatrix} & u_1 & u_2 & u_3 & u_4 & u_5 \\ u_1 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ u_2 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \\ u_3 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \\ u_4 & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) \\ u_5 & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) & ([1,1],[1,1],[0,0]) & ([0,0],[0,0],[1,1]) \end{bmatrix}$$

The two graphs provided have the same number of vertices, edges, and degree sequences, and their adjacency IVINFMs are also identical. Therefore, the graphs are isomorphic and also KS IVINFM.

Every isomorphic and non-isomorphic graph is KS adjacency IVINM but converse need not be true.

4. Secondary k-KS IVINFM

Definition 4.1 For an IVINM $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U> IVINM\in_{nn}$ is a s - symmetric IVINFM iff $[A_\mu,A_\lambda,A_\nu]_L=V\left([A_\mu,A_\lambda,A_\nu]_L{}^T\right)V$ and $[A_\mu,A_\lambda,A_\nu]_U=V\left([A_\mu,A_\lambda,A_\nu]_U{}^T\right)V$.

Definition 4.2 For an IVINM $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \text{IVINM}_{nn}$ is a s- KS IVINFM iff $N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(V[A_\mu,A_\lambda,A_\nu]_L{}^TV\right)$, $N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left(V[A_\mu,A_\lambda,A_\nu]_U{}^TV\right)$.

Definition 4.3. For an IVINFM $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>$ is a s-k- KS IVINFM iff $N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(KV[A_\mu,A_\lambda,A_\nu]_L{}^TVK\right)$, $N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left(KV[A_\mu,A_\lambda,A_\nu]_U{}^TVK\right)$.

Lemma 4.1. For an IVINM $A=<[A_\mu,A_\lambda,A_\nu]_L,\ [A_\mu,A_\lambda,A_\nu]_U>\in \text{IVNFM}_{nn}$ is a s- KS IVINM $\Leftrightarrow$ $\text{VA}=<V[A_\mu,A_\lambda,A_\nu]_L,\ \text{V}[A_\mu,A_\lambda,A_\nu]_U>$ KS -IVINFM $\Leftrightarrow AV=<[A_\mu,A_\lambda,A_\nu]_L\text{V},[A_\mu,A_\lambda,A_\nu]_U\text{V}>$ is a KS-IVINFM.

Proof. Let $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \text{IVNFM}_{nn}$ be a s-KS IVINFM.

$\Leftrightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(V[A_\mu,A_\lambda,A_\nu]_L{}^TV\right)$ [Definition 3.2]

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_L \mathrm{V}\right) = N\left([A_\mu, A_\lambda, A_\nu]_L V\right)^T$

$\Leftrightarrow [A_\mu, A_\lambda, A_\nu]_L \mathrm{V}$ is KS. [By P.2.2]

$\Leftrightarrow N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_L \mathrm{VV}^T\right) = N\left(VV[A_\mu, A_\lambda, A_\nu]_L{}^T V\right)$

$\Leftrightarrow N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_L\right) = N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_L\right)^T$

$\Leftrightarrow \mathrm{V}[A_\mu, A_\lambda, A_\nu]_L$ is KS.

Similarly

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left(V[A_\mu, A_\lambda, A_\nu]_U{}^T V\right)$

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_U \mathrm{V}\right) = N\left([A_\mu, A_\lambda, A_\nu]_U V\right)^T$

$\Leftrightarrow [A_\mu, A_\lambda, A_\nu]_U \mathrm{V}$ is KS.

$\Leftrightarrow N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_U \mathrm{VV}^T\right) = N\left(VV[A_\mu, A_\lambda, A_\nu]_U{}^T V\right)$

$\Leftrightarrow N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_U\right) = N\left(\mathrm{V}[A_\mu, A_\lambda, A_\nu]_U\right)^T$

$\Leftrightarrow \mathrm{V}[A_\mu, A_\lambda, A_\nu]_U$ is KS.

Hence the theorem.

Example 4.1 Let us consider IVINFM

$$A = \begin{bmatrix} <[1,1],[0,0],[0,0]> & <[0.2,0.5],[0.3,0.6],[0.4,0.4]> \\ <[0.2,0.5],[0.3,0.6],[0.4,0.4]> & <[1,1],[0,0],[0,0]> \end{bmatrix}$$

Lower Limit IVINM, $[A_\mu, A_\lambda, A_\nu]_L = \begin{bmatrix} <1,0,0> & <0.2,0.3,0,4> \\ <0.2,0.3,0.4> & <1,0,0> \end{bmatrix}$,

Upper Limit IVINM, $[A_\mu, A_\lambda, A_\nu]_U = \begin{bmatrix} <1,0,0> & <0.5,0.6,0.4> \\ <0.5,0.6,0.4> & <1,0,0> \end{bmatrix}$

$V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$, $\mathrm{K} = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$,

$$KVA_L{}^T VK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <1,0,0> & <0.2,0.3,0,4> \\ <0.2,0.3,0.4> & <1,0,0> \end{bmatrix}$$
$$\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$KVA_L{}^T VK \neq A_L$

Similarly, $KVA_U{}^T VK \neq A_U$

$$KA_L K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <1,0,0> & <0.2,0.3,0,4> \\ <0.2,0.3,0.4> & <1,0,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$KA_L K \neq A_L$

Similarly, $A_U \neq KA_U K$

N (A$_L$) = $N(KVA_L{}^T VK) = <0,0,0>$

The main point is that while A is symmetric under both IVINFM and KS IVINFM, it does not hold the more specific symmetries of κ-symmetric and s-κ-symmetric within the IVINFM framework.

Example 4.2. Let us consider IVINFM,

$$A = \begin{bmatrix} <[0.2,0.6],[0.2,0.4],[0.3,0.6]> & <[0.4,0.5],[0.3,0.3],[0.2,0.4]> \\ <[0.4,0.5],[0.3,0.3],[0.2,0.4]> & <[0.2,0.6],[0.2,0.4],[0.3,0.6]> \end{bmatrix}$$

$$V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}, \ K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},$$

LLIVINM, $A_L = \begin{bmatrix} <0.2,0.2,0.3> & <0.4,0.3,0.2> \\ <0.4,0.3,0.2> & <0.2,0.2,0.3> \end{bmatrix}$,

ULIVINM, $A_U = \begin{bmatrix} <0.6,0.4,0.6> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.6,0.4,0.6> \end{bmatrix}$

$$KVA_L{}^T VK = \begin{bmatrix} <0.2,0.2,0.3> & <0.4,0.3,0.2> \\ <0.4,0.3,0.2> & <0.2,0.2,0.3> \end{bmatrix} = A_L$$

A is symmetric, KS, s-κ-symmetric IVINFM and hence s- k- KS IVINFM.

Theorem 4.1. Below statements are equal for $A \in \mathrm{IVINM}_n$

(i) $A = <[A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U> \in \mathrm{IVINM}_{nn}$ is s – κ KS IVINFM.

(ii) $\mathrm{KV}\,A = <KV[A_\mu, A_\lambda, A_\nu]_L, \mathrm{KV}[A_\mu, A_\lambda, A_\nu]_U>$ is KS IVINFM.

(iii) $A\mathrm{KV} = <[A_\mu, A_\lambda, A_\nu]_L KV, [A_\mu, A_\lambda, A_\nu]_U KV>$ is KS IVINFM.

(iv) $\mathrm{V}\,A = <V[A_\mu, A_\lambda, A_\nu]_L, \mathrm{V}[A_\mu, A_\lambda, A_\nu]_U>$ is k- KS IVINFM.

(v) $A\mathrm{K} = <[A_\mu, A_\lambda, A_\nu]_L \mathrm{K}, [A_\mu, A_\lambda, A_\nu]_U \mathrm{K}>$ is s- KS IVINFM.

(vi) A^T is a s-k KS IVINFM.

(vii) $\mathrm{N}([A_\mu, A_\lambda, A_\nu]_L) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T \mathrm{VK}\right), \mathrm{N}([A_\mu, A_\lambda, A_\nu]_U) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T \mathrm{VK}\right)$

(viii) $\mathrm{N}([A_\mu, A_\lambda, A_\nu]_L{}^T) = N\left([A_\mu, A_\lambda, A_\nu]_L \mathrm{VK}\right), \mathrm{N}([A_\mu, A_\lambda, A_\nu]_U{}^T) = \left(\mathrm{R}[A_\mu, A_\lambda, A_\nu]_U \mathrm{VK}\right)$

(ix) $\mathrm{N}(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_L) = N\left(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_L{}^T\right)^T, \mathrm{N}(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_U) = N\left(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_U{}^T\right)^T$

(x) $A\mathrm{VK} = N\left(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_L{}^T\right)^T \mathrm{VK}, N\left(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_U{}^T\right)^T \mathrm{VK}$ is KS IVINFM.

(xi) $A\mathrm{V} =< [A_\mu, A_\lambda, A_\nu]_L \mathrm{V}, [A_\mu, A_\lambda, A_\nu]_U \mathrm{V} >$ is KS IVINFM.

(xii) $\mathrm{VKP} =< \mathrm{VK}[A_\mu, A_\lambda, A_\nu]_L, \mathrm{VK}[A_\mu, A_\lambda, A_\nu]_U >$ is KS IVINFM.

(xiii) $\mathrm{KA} =< \mathrm{K}[A_\mu, A_\lambda, A_\nu]_L, \mathrm{K}[A_\mu, A_\lambda, A_\nu]_U >$ is IV KS IVINFM.

Proof: (i) iff (ii) iff (iv)

Let $A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \mathrm{IVNFM}_{nn}$ is s – κ KS IVINFM

Let $[A_\mu, A_\lambda, A_\nu]_L$ is a s – κ KS IVINFM.

$\Leftrightarrow N([A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L{}^T VK), \mathrm{N}([A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U{}^T VK),$

(By Definition 3.3)

$\Leftrightarrow N(KV[A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L)^T, \mathrm{N}([A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U)^T$

By (P.2.3)

$\Leftrightarrow \mathrm{KV}\, A =< KV[A_\mu, A_\lambda, A_\nu]_L, \mathrm{KV}[A_\mu, A_\lambda, A_\nu]_U >$ is KS IVINFM

$\Leftrightarrow \mathrm{VP} =< V[A_\mu, A_\lambda, A_\nu]_L, \mathrm{V}[A_\mu, A_\lambda, A_\nu]_U >$ is κ- KS IVINFM

Therefore, (i), (ii), and (iv) are all equivalent.

(i) iff (ii) iff (v)

Let $A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \mathrm{IVNFM}_{nn}$ is s – κ KS IVINFM

$\Leftrightarrow N(KV[A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L)^T, \mathrm{N}(KV[A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U)^T,$

$\Leftrightarrow N(VK(KV[A_\mu, A_\lambda, A_\nu]_L)) = N((VK)[A_\mu, A_\lambda, A_\nu]_L{}^T VK(VK)^T),$

$\mathrm{N}(VK(KV[A_\mu, A_\lambda, A_\nu]_U)) = N((VK)[A_\mu, A_\lambda, A_\nu]_U{}^T VK(VK)^T)$

$\Leftrightarrow AKV = \left[[A_\mu, A_\lambda, A_\nu]_L KV, [A_\mu, A_\lambda, A_\nu]_U KV\right]$ is KS IVINFM

$\Leftrightarrow PK = \left[[A_\mu, A_\lambda, A_\nu]_L K, [A_\mu, A_\lambda, A_\nu]_U K\right]$ is s- KS IVINFM

Therefore, (i), (iii), and (v) are all equivalent.

(ii) $\Leftrightarrow$ (ix)

$KVA = \left[KV[A_\mu, A_\lambda, A_\nu]_L, KV[A_\mu, A_\lambda, A_\nu]_U\right]$ is KS IVINFM

$\Leftrightarrow N\left(KV[A_\mu, A_\lambda, A_\nu]_L\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_L\right)^T\right), \mathrm{N}\left(KV[A_\mu, A_\lambda, A_\nu]_U\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$

(ii) $\Leftrightarrow$ (ix) is true. (ii) $\Leftrightarrow$ (vii)

$KVA = \left[KV[A_\mu, A_\lambda, A_\nu]_L, KV[A_\mu, A_\lambda, A_\nu]_U\right]$ is KS IVINFM

$\Leftrightarrow N\left(KV[A_\mu, A_\lambda, A_\nu]_L\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_L\right)^T\right), N\left(KV[A_\mu, A_\lambda, A_\nu]_U\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T VK\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T VK\right)$

Therefore, (ii), and (vii) are equivalent.

. (iii) $\Leftrightarrow$ (viii)

$AVK = \left[[A_\mu, A_\lambda, A_\nu]_L VK, [A_\mu, A_\lambda, A_\nu]_U VK\right]$

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_L VK\right) = N\left(\left([A_\mu, A_\lambda, A_\nu]_L VK\right)^T\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U VK\right) = N\left(\left([A_\mu, A_\lambda, A_\nu]_U VK\right)^T\right)$

$\Leftrightarrow N\left([A_\mu, A_\lambda, A_\nu]_L VK\right) = N\left([A_\mu, A_\lambda, A_\nu]_L\right)^T, \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U VK\right) = N\left([A_\mu, A_\lambda, A_\nu]_U\right)^T$

Therefore, (iii), and (viii) are equivalent.

(i) $\Leftrightarrow$ (vi)

Let $P = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \mathrm{IVNFM}_{nn}$ is a s – κ KS IVINFM

$\Leftrightarrow N([A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L{}^T VK), \mathrm{N}([A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U{}^T VK),$

(By Definition 3.3)

$\Leftrightarrow (KVA)^T = (KV[A_\mu, A_\lambda, A_\nu]_L, KV[A_\mu, A_\lambda, A_\nu]_U)^T$ is KS IVINFM

$\Leftrightarrow A^T VK = ([A_\mu, A_\lambda, A_\nu]_L VK, [A_\mu, A_\lambda, A_\nu]_U VK)$ is KS IVINFM

$\Leftrightarrow A^T = \left([A_\mu, A_\lambda, A_\nu]_L{}^T, [A_\mu, A_\lambda, A_\nu]_U{}^T\right)$ is s – κ KS IVINFM

Therefore, (i), and (vi) are equivalent.

Let $A = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \mathrm{IVINM}_{nn}$ is a s – κ KS IVINFM.

Consider $[A_\mu, A_\lambda, A_\nu]_L$ is a s – κ KS IVINFM

$\Leftrightarrow N([A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L{}^T VK), N([A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U{}^T VK),$

(By Definition 3.3)

$\Leftrightarrow N([A_\mu, A_\lambda, A_\nu]_L VK) = N([A_\mu, A_\lambda, A_\nu]_L VK), N([A_\mu, A_\lambda, A_\nu]_U VK) = N([A_\mu, A_\lambda, A_\nu]_U VK)$

By (P.2.3)

$\Leftrightarrow AVK = \left[[A_\mu, A_\lambda, A_\nu]_L VK, [A_\mu, A_\lambda, A_\nu]_L VK \right]$ is KS IVINFM

$\Leftrightarrow AV = \left[[A_\mu, A_\lambda, A_\nu]_L V, [A_\mu, A_\lambda, A_\nu]_U V \right]$ is κ-KS IVINFM

Therefore, (i) ⇔ (x) ⇔ (xi) is true. (i) ⇔ (xii) ⇔ (xiii)

Let $P = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \text{IVNFM}_{nn}$ is a s – κ KS IVINFM

$\Leftrightarrow N([A_\mu, A_\lambda, A_\nu]_L) = N(KV[A_\mu, A_\lambda, A_\nu]_L{}^T VK), N([A_\mu, A_\lambda, A_\nu]_U) = N(KV[A_\mu, A_\lambda, A_\nu]_U{}^T VK),$

[By Definition 3.3]

$\Leftrightarrow N(VK[A_\mu, A_\lambda, A_\nu]_L) = N(VK[A_\mu, A_\lambda, A_\nu]_L)^T, N(VK[A_\mu, A_\lambda, A_\nu]_U) = N(VK[A_\mu, A_\lambda, A_\nu]_U)^T,$

By (P.2.3)

$\Leftrightarrow N(KV(VK[A_\mu, A_\lambda, A_\nu]_L)) = N((KV)[A_\mu, A_\lambda, A_\nu]_L{}^T KV(KV)^T),$

$N(KV(VK[A_\mu, A_\lambda, A_\nu]_U)) = N((KV)[A_\mu, A_\lambda, A_\nu]_U{}^T KV(KV)^T)$

$\Leftrightarrow N(VK[A_\mu, A_\lambda, A_\nu]_L) = N(VK[A_\mu, A_\lambda, A_\nu]_L)^T, N(VK[A_\mu, A_\lambda, A_\nu]_U) = N(VK[A_\mu, A_\lambda, A_\nu]_U)^T$

[By Lemma. 2.2]

$\Leftrightarrow VKA = \left[VK[A_\mu, A_\lambda, A_\nu], VK[A_\mu, A_\lambda, A_\nu]_U \right]$ is a KS IVINM

$\Leftrightarrow KA = \left[K[A_\mu, A_\lambda, A_\nu]_L, K[A_\mu, A_\lambda, A_\nu]_U \right]$ is a KS s- KS IVINM

Therefore, (i), (xii),(xiii) are equivalent.

Corollary:4.1 Below conditions are equal for $A \in \text{IVINM}_{nn}$

(i) $A = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \text{IVINM}_{nn}$ is s-KS IVINFM.

(ii) $VA = < V[A_\mu, A_\lambda, A_\nu]_L, \text{V}[A_\mu, A_\lambda, A_\nu]_U >$ is KS IVINFM.

(iii) $AV = < [A_\mu, A_\lambda, A_\nu]_L \text{ V}, [A_\mu, A_\lambda, A_\nu]_U \text{ V} >$ is KS IVINFM.

(iv) $A^T = < [A_\mu, A_\lambda, A_\nu]_L{}^T, [A_\mu, A_\lambda, A_\nu]_U{}^T >$ is s – KS IVINFM.

(v) $N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T V\right), \text{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T V\right)$

(vi) $N\left([A_\mu, A_\lambda, A_\nu]_L{}^T\right) = N\left([A_\mu, A_\lambda, A_\nu]_L V\right), \text{N}\left([A_\mu, A_\lambda, A_\nu]_U{}^T\right) = N\left([A_\mu, A_\lambda, A_\nu]_U \text{ V}\right)$

(vii) $N\left(\text{KV}[A_\mu, A_\lambda, A_\nu]_L\right) = N\left(\text{V}[A_\mu, A_\lambda, A_\nu]_L\right)^T, \text{N}\left(\text{KV}[A_\mu, A_\lambda, A_\nu]_U\right) = N\left(\text{V}[A_\mu, A_\lambda, A_\nu]_U\right)^T$

Proof: (i) and (ii) implies (iii)

Let $A = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \text{IVINM}_{nn}$ is a s – κ KS IVINFM

$\Rightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T VK\right), N\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T VK\right)$

$\Rightarrow N\left(K[A_\mu, A_\lambda, A_\nu]_L K\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_L{}^T K\right),$

[By Theorem 3.1]

$$N\left(K[A_\mu,A_\lambda,A_\nu]_U K\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_U{}^T K\right)$$

(i) & (ii) $\Rightarrow$ (iii) is correct

(i)& (iii) $\Rightarrow$ (ii)

$A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>$ is a κ-KS IVINFM

$$\Rightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_L{}^T K\right), N[A_\mu,A_\lambda,A_\nu]_U=N\left(K[A_\mu,A_\lambda,A_\nu]_U{}^T K\right)$$

$$\Rightarrow N\left(K[A_\mu,A_\lambda,A_\nu]_L K\right)=N\left(\left([A_\mu,A_\lambda,A_\nu]_L\right)^T\right), N\left(K[A_\mu,A_\lambda,A_\nu]_U K\right)=N\left(\left([A_\mu,A_\lambda,A_\nu]_U\right)^T\right)$$

Therefore, (i) & (iii)

$$\Rightarrow N\left(K[A_\mu,A_\lambda,A_\nu]_L K\right)=N\left(\left(V[A_\mu,A_\lambda,A_\nu]_L K\right)^T\right), N\left(K[A_\mu,A_\lambda,A_\nu]_U K\right)=N\left(\left(V[A_\mu,A_\lambda,A_\nu]_U K\right)^T\right)$$

$$\Rightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left([A_\mu,A_\lambda,A_\nu]_L{}^T VK\right), N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left([A_\mu,A_\lambda,A_\nu]_U{}^T VK\right)$$

$$\Rightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)^T\right), N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_U\right)^T\right)$$

$A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ is a s-k-KS IVINM

$\Rightarrow$ (ii) is true

(ii) & (iii) implies (i)

$A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ is a s-k-KS IVINM

$$\Rightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left([A_\mu,A_\lambda,A_\nu]_L{}^T VK\right), N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left([A_\mu,A_\lambda,A_\nu]_U{}^T VK\right)$$

$$\Rightarrow N\left(K[A_\mu,A_\lambda,A_\nu]_L K\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_L{}^T K\right), N\left(K[A_\mu,A_\lambda,A_\nu]_U K\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_U{}^T K\right)$$

Therefore, (ii) and (iii)

$$\Rightarrow N\left(K[A_\mu,A_\lambda,A_\nu]_L K\right)=N\left([A_\mu,A_\lambda,A_\nu]_L{}^T\right), N\left(K[A_\mu,A_\lambda,A_\nu]_U K\right)=N\left([A_\mu,A_\lambda,A_\nu]_U{}^T\right)$$

$$\Rightarrow N\left([A_\mu,A_\lambda,A_\nu]_L\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_L{}^T K\right), N\left([A_\mu,A_\lambda,A_\nu]_U\right)=N\left(K[A_\mu,A_\lambda,A_\nu]_U{}^T K\right)$$

$A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ is a κ-KS IVINFM.

Therefore, (i) is true, hence the theorem is proved.

Theorem 4.2. For $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ Then, any two of the following conditions imply the third.

(i) $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ is a κ- KS IVINFM.

(ii) $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$ is a s-κ- KS IVINFM.

(iii) $N\left([A_\mu,A_\lambda,A_\nu]_L\right)^T=N\left(\mathrm{VK}[A_\mu,A_\lambda,A_\nu]_L\right)^T, \mathrm{N}\left([A_\mu,A_\lambda,A_\nu]_U\right)^T=N\left(\mathrm{VK}[A_\mu,A_\lambda,A_\nu]_U{}^T\right)$

Proof: (i) and (ii) implies (iii)

Let $A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U >\in \text{IVINM}_{nn}$ is a s – κ KS IVINFM

$$\Rightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T VK\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T VK\right)$$

[By Theorem 3.1]

$$\Rightarrow N\left(K[A_\mu, A_\lambda, A_\nu]_L K\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_L{}^T K\right), \mathrm{N}\left(K[A_\mu, A_\lambda, A_\nu]_U K\right)$$

$$= N\left(K[A_\mu, A_\lambda, A_\nu]_U{}^T K\right)$$

$$\mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right)^T = N\left(\left(VK[A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$$

(i) & (ii) $\Rightarrow$ (iii) is correct

(i)& (iii) $\Rightarrow$ (ii)

$A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U >$ is an IV κ- KS

$$\Rightarrow N\left(K[A_\mu, A_\lambda, A_\nu]_L K\right) = N\left(\left([A_\mu, A_\lambda, A_\nu]_L\right)^T\right), \mathrm{N}\left(K[A_\mu, A_\lambda, A_\nu]_U K\right) = N\left(\left([A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$$

Therefore, (i) & (iii)

$$\Rightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left([A_\mu, A_\lambda, A_\nu]_L{}^T VK\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left([A_\mu, A_\lambda, A_\nu]_U{}^T VK\right)$$

$$\Rightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_L\right)^T\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left(\left(KV[A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$$

$A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U >\in \text{IVINM}_{nn}$ is a s-k-KS IVINM

$\Rightarrow$ (ii) is correct

(ii) & (iii) $\Rightarrow$(i)

$A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U >\in \text{IVINM}_{nn}$ is a s- κ- KS IVINM

$$\Rightarrow N\left(K[A_\mu, A_\lambda, A_\nu]_L K\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_L{}^T K\right), \mathrm{N}\left(K[A_\mu, A_\lambda, A_\nu]_U K\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_U{}^T K\right)$$

Therefore,(ii) and (iii)

$$\Rightarrow N\left([A_\mu, A_\lambda, A_\nu]_L\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_L{}^T K\right), \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U\right) = N\left(K[A_\mu, A_\lambda, A_\nu]_U{}^T K\right)$$

$A =< [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U >\in \text{IVINM}_{nn}$ is a κ- KS IVINFM.

Consequently, (i) is true,

Therefore the theorem is proved.

5. s – κ KS regular IVINFM

In this segment, we have explored different generalized inverses of matrices within the IVINFM framework. We have also defined the criteria for various g-inverses of an s-k KS IVINFM to qualify as s-k KS IVINFM. The generalized inverses of an s-κ KS IVINFM A equivalent to the sets A{1, 2}, A{1, 2, 3}, and A{1, 2, 4} have been specifically considered.

Theorem 5.1: Let $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$, Z belongs to A{1,2} and AZ, ZA are a s-κ- KS IVINM. Then A is a s- κ - KS IVINM iff $Z=<[Z_\mu,Z_\lambda,Z_\nu]_L,[Z_\mu,Z_\lambda,Z_\nu]_U>$ is a s- κ – KS IVINFM.

Proof: Let $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>\in \mathrm{IVINM}_{nn}$

$$N\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)=N\left(KV[A_\mu,A_\lambda,A_\nu]_L\ \mathrm{Z}[A_\mu,A_\lambda,A_\nu]\right)\subseteq N\left(\mathrm{Z}[A_\mu,A_\lambda,A_\nu]_L\right)$$

$$=N\left(ZVV[A_\mu,A_\lambda,A_\nu]_L\right)\subseteq N\left(ZVKKV[A_\mu,A_\lambda,A_\nu]_L\right)\subseteq N\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)$$

Hence, $\mathrm{N}\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)=N\left(\mathrm{Z}[A_\mu,A_\lambda,A_\nu]_L\right)$

$$=\ \mathrm{N}\left(KV\left(\mathrm{Z}[A_\mu,A_\lambda,A_\nu]_L\right)^T VK\right) \qquad \left[\text{ZA is s- κ-KS IVINM}\right]$$

$$=\ \mathrm{N}\left([A_\mu,A_\lambda,A_\nu]_L{}^T[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L{}^T VK\right)$$

$$=\ \mathrm{N}\left([\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L{}^T VK\right)=\ \mathrm{N}\left(\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L\right)^T\right)$$

$$N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)^T\right)=N\left([A_\mu,A_\lambda,A_\nu]_L{}^T VK\right)$$

$$=\ \mathrm{N}\left(KV[A_\mu,A_\lambda,A_\nu]_L[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L\right) \qquad \left[\text{ VP is s- κ} - K\text{SIVINM}\right]$$

$$=\ \mathrm{N}\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L\right)$$

Similarly,

Hence, $\mathrm{N}\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U\right)=N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_U\right)^T\right)$ (KVZ is a KS IVINM)

$$\Leftrightarrow N\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)=N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_L\right)^T\right),\ N\left(KV[A_\mu,A_\lambda,A_\nu]_U\right)=N\left(\left(KV[A_\mu,A_\lambda,A_\nu]_U\right)^T\right)$$

$$\Leftrightarrow N\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L\right)=N\left(\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L\right)^T\right),$$

$$N\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U\right)=N\left(\left(KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U\right)^T\right)$$

$\Leftrightarrow KVX=\left[KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L,KV[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U\right]$ is a KS IVINFM

$Z=<[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L,[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U>$ is a KS IVINFM.

Theorem 5.2:, Let $A=<[A_\mu,A_\lambda,A_\nu]_L,[A_\mu,A_\lambda,A_\nu]_U>$, $Z=<[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_L,[\mathrm{Z}_\mu,\mathrm{Z}_\lambda,\mathrm{Z}_\nu]_U>$ ∈A{1,2,3},

$\mathrm{N}(\mathrm{KV}[A_\mu, A_\lambda, A_\nu]_L) = N\left(\mathrm{KV}[\mathrm{X}_\mu, \mathrm{X}_\lambda, \mathrm{X}_\nu]_L\right)^T$, $\mathrm{N}(\mathrm{KV}[\mathrm{P}_\mu, \mathrm{P}_\lambda, \mathrm{P}_\nu]_U) = N\left(\mathrm{KV}[\mathrm{Z}_{\mu U}, \mathrm{Z}_{\lambda U}, \mathrm{Z}_{\nu U}]\right)^T$ .Then $A = < [A_\mu, A_\lambda, A_\nu]_L, [A_\mu, A_\lambda, A_\nu]_U > \in \mathrm{IVINM}_{nn}$ is s-κ- KS IVINFM $\Leftrightarrow Z = < [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L, [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U >$ is s- κ – KS IVINFM.

Proof: Given A{1,2,3}, Hence , $[A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L [A_\mu, A_\lambda, A_\nu]_L = [A_\mu, A_\lambda, A_\nu]_L$, $[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L [A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L = [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L$,

$\left([A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)^T = [A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L$

Consider, $\mathrm{N}\left(\left(KV[A_\mu, A_\lambda, A_\nu]_L\right)^T\right) = \mathrm{N}\left([\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L{}^T [A_\mu, A_\lambda, A_\nu]_L{}^T VK\right)$ $\left[\text{By using A = AZA}\right]$

$= \mathrm{N}\left(KV\left([A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)^T\right)$

$= \mathrm{N}\left(\left([A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)^T\right)$ $\left[By\ P._{2.3}\right]$

$= \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)$

$= \mathrm{N}\left([\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)$

$\left[\text{By using } [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L = [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L [A_\mu, A_\lambda, A_\nu]_L [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right]$

$= \mathrm{N}\left(KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)$ $\left[By\ P_{2.3}\right]$

Similarly, we can consider, $\mathrm{N}\left(\left(KV[A_\mu, A_\lambda, A_\nu]_U\right)^T\right) = \mathrm{N}\left([\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U{}^T [A_\mu, A_\lambda, A_\nu]_U{}^T VK\right)$

$= \mathrm{N}\left(KV\left([A_\mu, A_\lambda, A_\nu]_U [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U\right)^T\right) = \mathrm{N}\left(\left([A_\mu, A_\lambda, A_\nu]_U [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U\right)^T\right)$ $\left[By\ P._{2.3}\right]$

$= \mathrm{N}\left([A_\mu, A_\lambda, A_\nu]_U [\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U\right)$ $\left[(PZ)^T = \mathrm{PZ}\right]$

$= \mathrm{N}\left([\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U\right)$ $\left[\text{By using Z = ZA}Z\right]$

$= \mathrm{N}\left(KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U\right)$ $\left[\text{By } P_{2.3}\right]$

If KVA is a KS IVINM

$\Leftrightarrow \mathrm{N}\left(KV[A_\mu, A_\lambda, A_\nu]_L\right) = \mathrm{N}\left(\left(KV[A_\mu, A_\lambda, A_\nu]_L\right)^T\right)$,

$\mathrm{N}\left(KV[A_\mu, A_\lambda, A_\nu]_U\right) = \mathrm{N}\left(\left(KV[A_\mu, A_\lambda, A_\nu]_U\right)^T\right)$

$\Leftrightarrow \mathrm{N}\left(KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right) = \mathrm{N}\left(\left(KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L\right)^T\right)$, $KVX = [KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_L, KV[\mathrm{Z}_\mu, \mathrm{Z}_\lambda, \mathrm{Z}_\nu]_U]$ is a

KS IVINM.

$Z = <[Z_\mu, Z_\lambda, Z_v]_L, [Z_\mu, Z_\lambda, Z_v]_U>$ is a s-k KS IVINM.

Theorem 5.3: Let $A = <[A_\mu, A_\lambda, A_v]_L, [A_\mu, A_\lambda, A_v]_U> \in \text{IVINM}_{nn}$, Z ∈ A {1, 2, 4}, $\text{N}(\text{KV}[A_\mu, A_\lambda, A_v]_L)^T = \text{N}\left(\text{KV}[Z_\mu, Z_\lambda, Z_v]_L\right), \text{N}(\text{KV}[A_\mu, A_\lambda, A_v]_U)^T = \text{N}\left(\text{KV}[Z_\mu, Z_\lambda, Z_v]_U\right)$. Then KVP is an s- κ-KS IVINM iff $Z = <[Z_\mu, Z_\lambda, Z_v]_L, [Z_\mu, Z_\lambda, Z_v]_U>$ is a s- κ- KS IVINFM.

Proof: Given, A {1, 2, 4}, Hence $[A_\mu, A_\lambda, A_v]_L [Z_\mu, Z_\lambda, Z_v]_L [A_\mu, A_\lambda, A_v]_L = [A_\mu, A_\lambda, A_v]_L$,

$[Z_\mu, Z_\lambda, Z_v]_L [A_\mu, A_\lambda, A_v]_L [Z_\mu, Z_\lambda, Z_v]_L = [Z_\mu, Z_\lambda, Z_v]_L$,

$\left([Z_\mu, Z_\lambda, Z_v]_L [A_\mu, A_\lambda, A_v]_L\right)^T = [Z_\mu, Z_\lambda, Z_v]_L [A_\mu, A_\lambda, A_v]_L$

Consider, $\text{N}\left(\left(KV[A_\mu, A_\lambda, A_v]_L\right)^T\right) = \text{N}\left([Z_\mu, Z_\lambda, Z_v]_L{}^T [A_\mu, A_\lambda, A_v]_L{}^T VK\right)$ $\left[\text{By using A = AZA}\right]$

$= \text{N}\left(KV\left([A_\mu, A_\lambda, A_v]_L [Z_\mu, Z_\lambda, Z_v]_L\right)^T\right)$

$= \text{N}\left(\left([A_\mu, A_\lambda, A_v]_L [Z_\mu, Z_\lambda, Z_v]_L\right)^T\right)$ $\left[By\ P_{2.3}\right]$

$= \text{N}\left([A_\mu, A_\lambda, A_v]_L [Z_\mu, Z_\lambda, Z_v]_L\right) = \text{N}\left([Z_\mu, Z_\lambda, Z_v]_L\right) = \text{N}\left(\text{KV}[Z_\mu, Z_\lambda, Z_v]_L\right)$ $\left[By\ P_{2.3}\right]$

$\text{N}\left(\left(KV[A_\mu, A_\lambda, A_v]_U\right)^T\right) = \text{N}\left([Z_\mu, Z_\lambda, Z_v]_U{}^T [A_\mu, A_\lambda, A_v]_U{}^T VK\right)$ $\left[\text{By using A = AZA}\right]$

$= \text{N}\left(KV\left([A_\mu, A_\lambda, A_v]_U [Z_\mu, Z_\lambda, Z_v]_U\right)^T\right)$

$= \text{N}\left(\left([A_\mu, A_\lambda, A_v]_U [Z_\mu, Z_\lambda, Z_v]_U\right)^T\right)$ $\left[By\ P_{2.3}\right]$

$= \text{N}\left([A_\mu, A_\lambda, A_v]_U [Z_\mu, Z_\lambda, Z_v]_U\right)$ $\left[\ (\text{PZ})^T = PZ\right]$

$= N\left([X_{\mu U}, X_{\lambda U}, X_{vU}]\right) = \text{N}\left(\text{KV}[Z_\mu, Z_\lambda, Z_v]_U\right)$ $\left[By\ P_{2.3}\right]$

If KVP is a KS IVINM

$\Leftrightarrow \text{N}\left(KV[A_\mu, A_\lambda, A_v]_L\right) = \text{N}\left(\left(KV[A_\mu, A_\lambda, A_v]_L\right)^T\right)$,

$\text{N}\left(KV[A_\mu, A_\lambda, A_v]_U\right) = \text{N}\left(\left(KV[A_\mu, A_\lambda, A_v]_U\right)^T\right)$

$\Leftrightarrow N\left(KV[Z_\mu, Z_\lambda, Z_v]_L\right) = \text{N}\left(\left(KV[Z_\mu, Z_\lambda, Z_v]_L\right)^T\right)$,

$KVX = [KV[Z_\mu, Z_\lambda, Z_v]_L, KV[Z_\mu, Z_\lambda, Z_v]_U]$ is a KS IVINM.

$Z=<[Z_\mu, Z_\lambda, Z_v]_L,[Z_\mu, Z_\lambda, Z_v]_U>$ is a s-k KS IVINM.

Corollary 5.1: For $A=<[A_\mu, A_\lambda, A_v]_L,[A_\mu, A_\lambda, A_v]>\in \text{IVNFM}_{nn}$, Z∈ A {1, 2} and $AZ=<[A_\mu, A_\lambda, A_v]_L[Z_{\mu L}, Z_{\lambda L}, Z_{vL}],[A_\mu, A_\lambda, A_v]_U[Z_\mu, Z_\lambda, Z_v]_U>$, $ZA=<[Z_\mu, Z_\lambda, Z_v]_L[A_\mu, A_\lambda, A_v]_L,[Z_\mu, Z_\lambda, Z_v]_U[A_\mu, A_\lambda, A_v]_U>$, is a s- KS IVINFM. Then A is a s- KS IVINM iff $Z=<[Z_\mu, Z_\lambda, Z_v]_L,[Z_\mu, Z_\lambda, Z_v]_U>$ is a s- KS IVINFM.

Corollary 5.2: For $A=<[A_\mu, A_\lambda, A_v]_L,[A_\mu, A_\lambda, A_v]_U>\in \text{IVINM}_{nn}$, Z∈ A {1, 2, 3}, $\text{N}(\text{KV}[A_\mu, A_\lambda, A_v]_L)$ $=N\left(\text{V}[Z_\mu, Z_\lambda, Z_v]_L\right)^T$, $\text{N}(\text{KV}[A_\mu, A_\lambda, A_v])=N\left(\text{V}[Z_\mu, Z_\lambda, Z_v]_U\right)^T$.

Then A is a s- RS IVINM iff $Z=<[Z_\mu, Z_\lambda, Z_v]_L,[Z_\mu, Z_\lambda, Z_v]_U>$ is a s- KS IVINFM.

Corollary 5.3: For $A=<[A_\mu, A_\lambda, A_v]_L,[A_\mu, A_\lambda, A_v]_U>\in \text{IVINM}_{nn}$, Z∈ A {1, 2, 4} , $\text{N}(\text{V}[A_\mu, A_\lambda, A_v]_L)^T$ $=N\left(\text{V}[Z_\mu, Z_\lambda, Z_v]_L\right)^T$, $\text{N}(\text{V}[A_\mu, A_\lambda, A_v]_U)^T=N\left(\text{V}[Z_\mu, Z_\lambda, Z_v]_U\right)$. Then A is a s- KS IVINM iff Z is a s-KS IVINFM.

5. Conclusion:

We present equivalent definitions of a k-KS IVINFM, KS IVINFM, and the s-KS variant of IVINFM and s-k KS IVINFM. We also give an example of s-k-symmetric IVINFM and s-k KS IVINFM, but the converse may not be valid. We review various g-inverses related to regular matrices and characterize the set of all inverted inverses. Statements for different G-inverses of a s-k-KS IVINFM and s-kernel IVINFM that is symmetric are identified. In the future, we will determine some properties related to secondary IVINFMs with k-KS.

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