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New algebraic structures such as q-rung interval-valued intuitionistic fuzzy set applied to logarithm operator and its extension

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Abstract

We introduce the interval-valued intuitionistic fuzzy set used with the q-rung logarithimic operator (q-LIVIFS). A q-rung interval-valued intuitionistic fuzzy set may be developed by expanding the Pythagorean interval-valued fuzzy set (PIVFS). We discuss the q-logarithimic operator applied to interval-valued intuitionistic fuzzy weighted averaging (q-LIVIFWA), interval-valued intuitionistic fuzzy weighted geometric (q-LIVIFWG), extended q-logarithimic operator applied to interval-valued intuitionistic fuzzy weighted averaging (q-ELIVIFWA), and extended q-logarithimic operator applied to interval-valued intuitionistic fuzzy weighted geometric (q-ELIVIFWG). Several algebraic properties of q-LIVIFSs, such as distributivity, idempotency, and associativity, have been identified.

Keywords: q-LIVIFWA; q-LIVIFWG; q-ELIVIFWA; q-ELIVIFWG.

1 Introduction

Several authors have contributed to this field of research using a variety of methods. Fuzzy set (FS),[1] intuitionistic FS (IFS),[2] interval valued FS (IVFS),[3] vague set (VS),[4] Pythagorean FS (PFS),[5] IVPFS,[6] spherical FS (SFS),[7] neutrosophic set (NS)[8] can all arise from uncertainties. There are several theories about uncertainty that have been proposed. For example, fuzzy set (FS)[1] has membership grade (MG) ranging from 0 to 1. Atanassov[2] developed an intuitionistic fuzzy set (IFS) where the two MGs of each component, positive β and negative α, satisfy $0 \leq \beta + \alpha \leq 1$, for $\beta, \alpha \in [0, 1]$. Yager[5] invented Pythagorean fuzzy sets (PFS). With the assumption that $\beta + \alpha \geq 1$ to $\beta^2 + \alpha^2 \leq 1$, they are distinguished by their MG and non-membership grade (NMG). Numerous studies have been conducted on the use of PFSs and IFSs in different fields of research. They are still not adept at explaining concepts. Because of this, the experts were still having problems interpreting the data in these sets and the supporting data. Cuong et al.[9] developed the concept of a picture fuzzy set as an alternative for this information.

As a result, it has been noted that the picture fuzzy set is an expanded version of IFS with more ambiguity handling capabilities. As $0 \leq \beta + \alpha + \zeta \leq 1$, it was noted that in the picture fuzzy set, MG β, NMG ζ, and neutral grade α; for $\beta, \alpha, \zeta \in [0, 1]$. It will be feasible to ensure that expert assessments like ”yes,” ”abstain,” ”no,” and ”refusal” are expressed by following to the PFS definition. Furthermore, there will be consistency between the outcome data and the actual decision-making environment, and no evaluative detail will be missed. Although picture fuzzy sets have been used and studied extensively, less emphasis has been dedicated to their concept. In their discussion of the new aggregating operator, Palanikumar et al.[10]-.[15] Aggregation operators (AO) were introduced in addition to extended PFS to handle issues with multiple truth membership values (TMD), false membership values (FMD), and indeterminacy membership values (IMD) that are handled by the DM method, as explained by Liu et al.[16] The novel aggregating operators have been studied by Palanikumar et al.[17]-.[18] The concept of neutrosophic sets was proposed by Smarandache et al.[19] The capacity for impartial thought is one of the primary differences between FS and IFS. Neutroposophy is the study of neutral cognition. TMD, IMD, and FD are used in this reasoning to calculate a value for every proposition. A strategy for MCDM under

interval NS based on AOs was given by.[20] This universe has all elements with values between 0 and 1, totaling 1. In several applications, AOs and their algebraic structures are explained by Palanikumar et al.[21]
Xu et al.[22] proposed using various IFS averaging operators to manage IFS data. In addition, weighted, ordered weighted, and hybrid geometric operators based on IFSs were developed by Xu et al.[23] In order to construct the IF ordered weighted distance (IFOWD) operators that are covered in[25] by Zeng et al., Li et al.[24] introduced the generalized ordered weighted averaging (GOWA) operators. Peng et al.[26] used AOs to study the basic characteristics of PFS. Spherical fuzzy Dombi AOs have recently been suggested by Ashraf et al.[27] This new power-moirhead mean was defined in the SFS in 2022 by Temel et al.,[28] and it also applies to MADM. New aggregating operators have been developed recently by.[29–31] Section 1 has an introduction. Section 2 for information on FS and PNS. Section 3 contains a definition and an explanation of some of the uses for q-LIVIFNs. The article draws two main conclusions: 1) It has been shown that the q-rungs of LIVIFS exhibit the following algebraic properties: distributive, idempotent and associative. (2) We examine the q-LIVIFWA, q-LIVIFWG, q-ELIVIFWA and q-ELIVIFWG.

2 Preliminaries

In this section, we will go over PFS and PIVFS concepts.

Definition 2.1. [5] Let $\mathscr{X}$ be the universal set. The PFS $M = \Big\{u, \langle \mu_M^{\mathscr{T}}(u), \mu_M^{\mathscr{F}}(u)\rangle \big| u \in \mathscr{X}\Big\}$, $\mu_M^{\mathscr{T}} : \mathscr{X} \to (0,1)$ and $\mu_M^{\mathscr{F}} : \mathscr{X} \to (0,1)$ denotes MD and NMD of $u \in \mathscr{X}$ to M, respectively and $0 \leq (\mu_M^{\mathscr{T}}(u))^2 + (\mu_M^{\mathscr{F}}(u))^2 \leq 1$. For convenience, $M = \langle \mu_M^{\mathscr{T}}, \mu_M^{\mathscr{F}}\rangle$ is called the Pythagorean fuzzy number(PFN).

Definition 2.2. [6] The Pythagorean IVFS (PIVFS) $M = \Big\{u, \Big\langle \widetilde{\mu_M^{\mathscr{T}}(u), \mu_M^{\mathscr{F}}(u)} \Big\rangle \Big| u \in \mathscr{X}\Big\}$, where $\widetilde{\mu_M^{\mathscr{T}}} : \mathscr{X} \to Int((0,1))$ and $\widetilde{\mu_M^{\mathscr{F}}} : \mathscr{X} \to Int((0,1))$ denotes MD and NMD of $u \in \mathscr{X}$ to M, respectively, and $0 \leq (\mu_M^{\mathscr{T}+}(u))^2 + (\mu_M^{\mathscr{F}+}(u))^2 \leq 1$. For convenience, $M = \Big\langle \Big(\mu_M^{\mathscr{T}-}, \mu_M^{\mathscr{T}+}\Big), \Big(\mu_M^{\mathscr{F}-}, \mu_M^{\mathscr{F}+}\Big)\Big\rangle$ is called the PIVFN.

Definition 2.3. The Pythagorean NS $M = \Big\{u, \langle \mu_M^{\mathscr{T}}(u), \mu_M^{\mathcal{I}}(u), \mu_M^{\mathscr{F}}(u)\rangle \big| u \in \mathscr{X}\Big\}$, where $\mu_M^{\mathscr{T}} : \mathscr{X} \to (0,1)$, $\mu_M^{\mathcal{I}} : \mathscr{X} \to (0,1)$ and $\mu_M^{\mathscr{F}} : \mathscr{X} \to (0,1)$ denotes TMD, IMD and FMD of $u \in \mathscr{X}$ to M, respectively and $0 \leq (\mu_M^{\mathscr{T}}(u))^2 + (\mu_M^{\mathcal{I}}(u))^2 + (\mu_M^{\mathscr{F}}(u))^2 \leq 2$. For convenience, $M = \langle \mu_M^{\mathscr{T}}, \mu_M^{\mathcal{I}}, \mu_M^{\mathscr{F}}\rangle$ is called the Pythagorean neutrosophic number.

3 q-LIVIFN and its fundamental operations

The q-LIVIFN has several intriguing basic operations connected to it.

Definition 3.1. The q-LIVIFS $M = \Big\{u, \Big\langle \Big(log\mathscr{T}_M^l(u), log(\mathscr{T}_M^u(u))\Big)$
$\Big(log\mathscr{F}_M^l(u), log(\mathscr{F}_M^u(u))\Big)\Big\rangle \Big| u \in \mathscr{X}\Big\}$, $\widetilde{\mu_M^{\mathscr{T}}} : \mathscr{X} \to Int((0,1))$ and $\widetilde{\mu_M^{\mathscr{F}}} : \mathscr{X} \to Int((0,1))$ denotes TMD and FMD of $u \in \mathscr{X}$ to M, respectively and $0 \leq (log_{\prod_i} \mathscr{T}_M^u(u))^q + (log_{\prod_i} \mathscr{F}_M^u(u))^q \leq 1$, where q are positive integers and $\prod = \boxdot(\mathscr{T}_M^l, \mathscr{T}_M^u), (\mathscr{F}_M^l, \mathscr{F}_M^u)$.
For convenience, $M = \Big\langle \Big(log\mathscr{T}_M^l, log(\mathscr{T}_M^u)\Big), \Big(log\mathscr{F}_M^l, log(\mathscr{F}_M^u)\Big)\Big\rangle$ is called the q-LIVIFN, where $q \geq 1$.

Definition 3.2. Let $M = \Big\langle \Big(log\mathscr{T}_M^l, log(\mathscr{T}_M^u)\Big), \Big(log\mathscr{F}_M^l, log(\mathscr{F}_M^u)\Big)\Big\rangle$ be the q-LIVIFN, the score function of M is defined as $\mathbb{S}(M) = \frac{\mathbb{S}_1(M)+\mathbb{S}_2(M)}{2}$, $-1 \leq \mathbb{S}(M) \leq 1$. where

$$\mathbb{S}_1(M) = \left(\frac{L_1}{2} + 1 - \frac{L_2}{2}\right), \mathbb{S}_2(M) = \left(\frac{L_1}{2} + 1 - \frac{L_2}{2}\right),$$

The accuracy function of M is $\mathbb{A}(M) = \frac{\mathbb{A}_1(M)+\mathbb{A}_2(M)}{2}$, where $0 \leq \mathbb{A}(M) \leq 1$.

$$\mathbb{A}_1(M) = \left(\frac{L_1}{2} + 1 + \frac{L_2}{2}\right), \mathbb{A}_2(M) = \left(\frac{L_1}{2} + 1 + \frac{L_2}{2}\right),$$

where $L_1 = (log_{\prod_i} \mathscr{T}_M^l)^2 + (log_{\prod_i}(\mathscr{T}_M^u))^2, L_2 = (log_{\prod_i} \mathscr{F}_M^l)^2 + (log_{\prod_i}(\mathscr{F}_M^u))^2$

Definition 3.3. Let $M=\left\langle(log\mathscr{T}^l_M,log(\mathscr{T}^u_M)),(log\mathscr{F}^l_M,log(\mathscr{F}^u_M))\right\rangle$,
$B=\left\langle(log\mathscr{T}^l_B,log(\mathscr{T}^u_B)),(log\mathscr{F}^l_B,log(\mathscr{F}^u_B))\right\rangle$
and $C=\left\langle(log\mathscr{T}^l_C,log(\mathscr{T}^u_C)),(log\mathscr{F}^l_C,log(\mathscr{F}^u_C))\right\rangle$ be any three q-LIVIFNs and
$\prod=\boxdot(T_{M_i},\mathscr{T}^u_{M_i}),(F_{M_i},\mathscr{F}^u_{M_i})$. Their following operations are defined as follows:

1. $B\bigvee C=$
$$\left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{(log_{\prod_i}\mathscr{T}^l_B)^q+(log_{\prod_i}\mathscr{T}^l_C)^q-(log_{\prod_i}\mathscr{T}^l_B)^q\cdot(log_{\prod_i}\mathscr{T}^l_C)^q,}\\ \sqrt[q]{(log_{\prod_i}(\mathscr{T}^u_B))^q+(log_{\prod_i}(\mathscr{T}^u_C))^q-(log_{\prod_i}(\mathscr{T}^u_B))^q\cdot(log_{\prod_i}(\mathscr{T}^u_C))^q}\end{array}\right),\\ \left(log_{\prod_i}(\mathscr{F}^l_B)^q\cdot log_{\prod_i}(\mathscr{F}^l_C)^q,log_{\prod_i}(\mathscr{F}^u_B)^q\cdot log_{\prod_i}(\mathscr{F}^u_C)^q\right)\end{array}\right),$$

2. $B\bigwedge C=$
$$\left(\begin{array}{c}\left(log_{\prod_i}(\mathscr{T}^l_B)^q\cdot log_{\prod_i}(\mathscr{T}^l_C)^q,log_{\prod_i}(\mathscr{T}^u_B)^q\cdot log_{\prod_i}(\mathscr{T}^u_C)^q\right),\\ \left(\begin{array}{c}\sqrt[q]{(log_{\prod_i}\mathscr{F}^l_B)^q+(log_{\prod_i}\mathscr{F}^l_C)^q-(log_{\prod_i}\mathscr{F}^l_B)^q\cdot(log_{\prod_i}\mathscr{F}^l_C)^q,}\\ \sqrt[q]{(log_{\prod_i}(\mathscr{F}^u_B))^q+(log_{\prod_i}(\mathscr{F}^u_C))^q-(log_{\prod_i}(\mathscr{F}^u_B))^q\cdot(log_{\prod_i}(\mathscr{F}^u_C))^q}\end{array}\right)\end{array}\right),$$

3. $$\aleph\cdot M=\left(\begin{array}{c}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{T}^l_M)^q\right)^{\aleph}},\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{T}^u_M))^q\right)^{\aleph}}\ \right),\\ \left((log_{\prod_i}\mathscr{F}^l_M)^{q\aleph},(log_{\prod_i}(\mathscr{F}^u_M))^{q\aleph}\right)\end{array}\right),$$

4. $$M^{\aleph}=\left(\begin{array}{c}\left((log_{\prod_i}\mathscr{T}^l_M)^{q\aleph},(log_{\prod_i}(\mathscr{T}^u_M))^{q\aleph}\right),\\ \left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^l_M)^q\right)^{\aleph}},\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^u_M))^q\right)^{\aleph}}\ \right)\end{array}\right).$$

4 q-LIVIFS concept

The weighed averaging operators for q-LIVIFN are given, based on the operational rules of q-LIVIFNs.

4.1 q-LIVIF weighted averaging(q-LIVIFWA) operator

Definition 4.1. Let $M_i=\left\langle(log\mathscr{T}^l_{Mi},log\mathscr{T}^u_{Mi}),(log\mathscr{F}^l_{Mi},log\mathscr{F}^u_{Mi})\right\rangle$ be the collection of q-LIVIFNs, $\omega=(\omega_1,\omega_2,...,\omega_n)$ be the weight of M_i, $\omega_i\geq 0$ and $\boxplus^n_{i=1}\omega_i=1$ and
$\prod=\boxdot(\mathscr{T}^l_{M_i},\mathscr{T}^u_{M_i}),(F_{M_i},\mathscr{F}^u_{M_i})$. Then q-LIVIFWA operator is q-LIVIFWA $(M_1,M_2,...,M_n)=\boxplus^n_{i=1}\omega_iM_i$ for $i=1,2,...,n$.

Theorem 4.2. *Let* $M_i=\left\langle(log\mathscr{T}^l_{Mi},log\mathscr{T}^u_{Mi}),(log\mathscr{F}^l_{Mi},log\mathscr{F}^u_{Mi})\right\rangle$ *be the collection of q-LIVIFNs. Then q-LIVIFWA* $(M_1,M_2,...,M_n)=$ *(associativity property).*

$$\left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot^n_{i=1}\left(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\right)^{\omega_i}},\sqrt[q]{1-\boxdot^n_{i=1}\left(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\right)^{\omega_i}}\right),\\ \left(\boxdot^n_{i=1}\,(log_{\prod_i}\mathscr{F}^l_{Mi})^{q\omega_i},\boxdot^n_{i=1}(log_{\prod_i}(\mathscr{F}^u_{Mi}))^{q\omega_i}\right)\end{array}\right).$$

Proof. If $n=2$, then q-LIVIFWA $(M_1=B,M_2=C)=\omega_1B\bigvee\omega_2C$, where

$$\omega_1B=\left(\begin{array}{c}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{T}^l_B)^q\right)^{\omega_1}},\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{T}^u_B))^q\right)^{\omega_1}}\right),\\ \left((log_{\prod_i}\mathscr{F}^l_B)^{q\omega_1},(log_{\prod_i}(\mathscr{F}^u_B))^{q\omega_1}\right)\end{array}\right)$$

and

$$\omega_2 C = \left(\begin{array}{c}\left(\sqrt[q]{1-\Big(1-(log_{\prod_i}\mathscr{T}^l_C)^q\Big)^{\omega_2}}, \sqrt[q]{1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_C))^q\Big)^{\omega_2}}\right),\\ \Big((log_{\prod_i}\mathscr{F}^l_C)^{q\omega_2},(log_{\prod_i}(\mathscr{F}^u_C))^{q\omega_2}\Big)\end{array}\right).$$

Hence,

$$\omega_1 B \bigvee \omega_2 C = \left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{\begin{array}{c}\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_B)^q\Big)^{\omega_1}\Big)+\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_C)^q\Big)^{\omega_2}\Big)\\ -\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_B)^q\Big)^{\omega_1}\Big)\cdot\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_C)^q\Big)^{\omega_2}\Big)\end{array}},\\ \sqrt[q]{\begin{array}{c}\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_B))^q\Big)^{\omega_1}\Big)+\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_C))^q\Big)^{\omega_2}\Big)\\ -\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_B))^q\Big)^{\omega_1}\Big)\cdot\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_C))^q\Big)^{\omega_2}\Big)\end{array}}\end{array}\right),\\ \Big((log_{\prod_i}\mathscr{F}^l_B)^{q\omega_1}\cdot(log_{\prod_i}\mathscr{F}^l_C)^{q\omega_2},(log_{\prod_i}(\mathscr{F}^u_B))^{q\omega_1}\cdot(log_{\prod_i}(\mathscr{F}^u_C))^{q\omega_2}\Big)\end{array}\right)$$

$$= \left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{1-\Big(1-(log_{\prod_i}\mathscr{T}^l_B)^q\Big)^{\omega_1}\cdot\Big(1-(log_{\prod_i}\mathscr{T}^l_C)^q\Big)^{\omega_2}},\\ \sqrt[q]{1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_B))^q\Big)^{\omega_1}\cdot\Big(1-(log_{\prod_i}(\mathscr{T}^u_C))^q\Big)^{\omega_2}}\end{array}\right),\\ \Big((log_{\prod_i}\mathscr{F}^l_B)^{q\omega_1}\cdot(log_{\prod_i}\mathscr{F}^l_C)^{q\omega_2},(log_{\prod_i}(\mathscr{F}^u_B))^{q\omega_1}\cdot(log_{\prod_i}(\mathscr{F}^u_C))^{q\omega_2}\Big)\end{array}\right).$$

Thus, q-LIVIFWA $(M_1 = B, M_2 = C)$

$$= \left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^n\Big(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\Big)^{\omega_i}}, \sqrt[q]{1-\boxdot_{i=1}^n\Big(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\Big)^{\omega_i}}\right),\\ \Big(\boxdot_{i=1}^n\,(log_{\prod_i}\mathscr{F}^l_{Mi})^{q\omega_i},\boxdot_{i=1}^n(log_{\prod_i}(\mathscr{F}^u_{Mi}))^{q\omega_i}\Big)\end{array}\right).$$

It is valid for $n = m$ and $m \geq 3$.
Hence, q-LIVIFWA $(M_1, M_2, ..., M_m)$

$$= \left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^m\Big(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\Big)^{\omega_i}}, \sqrt[q]{1-\boxdot_{i=1}^m\Big(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\Big)^{\omega_i}}\right),\\ \Big(\boxdot_{i=1}^m\,(log_{\prod_i}\mathscr{F}^l_{Mi})^{q\omega_i},\boxdot_{i=1}^m(log_{\prod_i}(\mathscr{F}^u_{Mi}))^{q\omega_i}\Big)\end{array}\right).$$

If $n = l + 1$ and we apply, q-LIVIFWA $(M_1, M_2, ..., M_m, M_{m+1})$

$$= \left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{\begin{array}{c}\boxplus_{i=1}^m\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\Big)^{\omega_i}\Big)+\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{M\,m+1})^q\Big)^{\omega_{m+1}}\Big)\\ -\boxdot_{i=1}^m\,\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\Big)^{\omega_i}\Big)\cdot\Big(1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{M\,m+1})^q\Big)^{\omega_{m+1}}\Big)\end{array}},\\ \sqrt[q]{\begin{array}{c}\boxplus_{i=1}^m\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\Big)^{\omega_i}\Big)+\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M\,m+1}))^q\Big)^{\omega_{m+1}}\Big)\\ -\boxdot_{i=1}^m\,\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\Big)^{\omega_i}\Big)\cdot\Big(1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M\,m+1}))^q\Big)^{\omega_{m+1}}\Big)\end{array}}\end{array}\right),\\ \Big(\boxdot_{i=1}^m\,(log_{\prod_i}\mathscr{F}^l_{Mi})^{q\omega_i}\cdot(log_{\prod_i}\mathscr{F}^l_{M\,m+1})^{q\omega_{m+1}},\boxdot_{i=1}^m(log_{\prod_i}(\mathscr{F}^u_{Mi}))^{q\omega_i}\cdot(log_{\prod_i}(\mathscr{F}^u_{M\,m+1}))^{q\omega_{m+1}}\Big)\end{array}\right)$$

$$= \left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{m+1}\Big(1-(log_{\prod_i}\mathscr{T}^l_{Mi})^q\Big)^{\omega_i}}, \sqrt[q]{1-\boxdot_{i=1}^{m+1}\Big(1-(log_{\prod_i}(\mathscr{T}^u_{Mi}))^q\Big)^{\omega_i}}\right),\\ \Big(\boxdot_{i=1}^{m+1}\,(log_{\prod_i}\mathscr{F}^l_{Mi})^{q\omega_i},\boxdot_{i=1}^{m+1}(log_{\prod_i}(\mathscr{F}^u_{Mi}))^{q\omega_i}\Big)\end{array}\right).$$

Theorem 4.3. *(idempotency property) If all* $M_i = \Big\langle(log\mathscr{T}^l_{Mi}, log\mathscr{T}^u_{Mi}),(log\mathscr{F}^l_{Mi}, log\mathscr{F}^u_{Mi})\Big\rangle (i = 1, 2, ..., n)$ *are equal and* $M_i = M$. *Then q-LIVIFWA* $(M_1, M_2, ..., M_n) = M$.

Proof. Given that $(log\mathscr{T}^l_{Mi}, log\mathscr{T}^u_{Mi}) = (log\mathscr{T}^l_M, log(\mathscr{T}^u_M))$ and $(log\mathscr{F}^l_{Mi}, log\mathscr{F}^u_{Mi}) = (log\mathscr{F}^l_M, log(\mathscr{F}^u_M))$, for $i = 1, 2, ..., n$ and $\boxplus_{i=1}^{n}\omega_i = 1$. Now, q-LIVIFWA $(M_1, M_2, ..., M_n)$

$$
\begin{aligned}
&= \left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(log_{\prod_i}\mathscr{T}^l_{M\,i})^q\Big)^{\omega_i}},\ \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M\,i}))^q\Big)^{\omega_i}}\ \right),\\ \Big(\boxdot_{i=1}^{n}\,(log_{\prod_i}\mathscr{F}^l_{M\,i})^{q\omega_i},\boxdot_{i=1}^{n}(log_{\prod_i}(\mathscr{F}^u_{M\,i}))^{q\omega_i}\Big)\end{array}\right)\\
&= \left(\begin{array}{c}\left(\sqrt[q]{1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{M})^q\Big)^{\boxplus_{i=1}^{n}\omega_i}},\ \sqrt[q]{1\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M}))^q\Big)^{\boxplus_{i=1}^{n}\omega_i}}\right),\\ ((log_{\prod_i}\mathscr{F}^l_{M})^{\boxplus_{i=1}^{n}q\omega_i},(log_{\prod_i}(\mathscr{F}^u_{M}))^{\boxplus_{i=1}^{n}q\omega_i})\end{array}\right)\\
&= \left(\begin{array}{c}\left(\sqrt[q]{1-\Big(1-(log_{\prod_i}\mathscr{T}^l_{M})^q\Big)},\ \sqrt[q]{1-\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M}))^q\Big)}\right),\\ \Big((log_{\prod_i}\mathscr{F}^l_{M})^{q},(log_{\prod_i}(\mathscr{F}^u_{M}))^{q}\Big)\end{array}\right)\\
&= M.
\end{aligned}
$$

Theorem 4.4. *Let* $M_i = \Big\langle(log\mathscr{T}^l_{M\,ij}, log(\mathscr{T}^u_{M\,ij})), (log\mathscr{F}^l_{M\,ij}, log(\mathscr{F}^u_{M\,ij}))\Big\rangle (i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *be the collection of q-LIVIFWA, where* $\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{} = \min log_{\prod_i}\mathscr{T}^l_{M\,ij}$, $\overbrace{log_{\prod_i}\mathscr{T}^l_M}^{} = \max log_{\prod_i}\mathscr{T}^l_{M\,ij}$, $\underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{} = \min log_{\prod_i}(\mathscr{T}^u_{M\,ij})$, $\overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{} = \max log_{\prod_i}(\mathscr{T}^u_{M\,ij})$, $\underbrace{log_{\prod_i}\mathscr{F}^l_M}_{} = \min log_{\prod_i}\mathscr{F}^l_{M\,ij}$, $\overbrace{log_{\prod_i}\mathscr{F}^l_M}^{} = \max log_{\prod_i}\mathscr{F}^l_{M\,ij}$, $\underbrace{log_{\prod_i}(\mathscr{F}^u_M)}_{} = \min log_{\prod_i}(\mathscr{F}^u_{M\,ij})$, $\overbrace{log_{\prod_i}(\mathscr{F}^u_M)}^{} = \max log_{\prod_i}(\mathscr{F}^u_{M\,ij})$.

Then, $\Big\langle(\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{}, \underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{}), (\overbrace{log_{\prod_i}\mathscr{F}^l_M}^{}, \overbrace{log_{\prod_i}(\mathscr{F}^u_M)}^{})\Big\rangle$

$$
\begin{aligned}
&\le\ new\ type\ LIVIFWA(M_1, M_2, ..., M_n)\\
&\le\ \Big\langle(\overbrace{log_{\prod_i}\mathscr{T}^l_M}^{}, \overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{}), (\underbrace{log_{\prod_i}\mathscr{F}^l_M}_{}, \underbrace{log_{\prod_i}(\mathscr{F}^u_M)}_{})\Big\rangle.
\end{aligned}
$$

where $1 \le i \le n,\ j = 1, 2, ..., i_j$, *(boundedness property).*

Proof. Since, $\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{} = \min log_{\prod_i}\mathscr{T}^l_{M\,ij}$, $\overbrace{log_{\prod_i}\mathscr{T}^l_M}^{} = \max log_{\prod_i}\mathscr{T}^l_{M\,ij}$ $\underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{} = \min log_{\prod_i}(\mathscr{T}^u_{M\,ij})$, $\overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{} = \max log_{\prod_i}(\mathscr{T}^u_{M\,ij})$ and $\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{} \le log_{\prod_i}\mathscr{T}^l_{M\,ij} \le \overbrace{log_{\prod_i}\mathscr{T}^l_M}^{}$ and $\underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{} \le log_{\prod_i}(\mathscr{T}^u_M)_{ij} \le \overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{}$.
Now, $\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{} + \underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{}$

$$
\begin{aligned}
&= \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(\underbrace{log_{\prod_i}\mathscr{T}^l_M}_{})^q\Big)^{\omega_i}} + \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(\underbrace{log_{\prod_i}(\mathscr{T}^u_M)}_{})^q\Big)^{\omega_i}}\\
&\le \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(log_{\prod_i}\mathscr{T}^l_{M\,ij})^q\Big)^{\omega_i}} + \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(log_{\prod_i}(\mathscr{T}^u_{M\,ij}))^q\Big)^{\omega_i}}\\
&\le \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(\overbrace{log_{\prod_i}\mathscr{T}^l_M}^{})^q\Big)^{\omega_i}} + \sqrt[q]{1-\boxdot_{i=1}^{n}\Big(1-(\overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{})^q\Big)^{\omega_i}}\\
&= \overbrace{log_{\prod_i}\mathscr{T}^l_M}^{} + \overbrace{log_{\prod_i}(\mathscr{T}^u_M)}^{}.
\end{aligned}
$$

Since, $\underbrace{log_{\prod_i}\mathscr{F}^l_M}_{} = \min log_{\prod_i}\mathscr{F}^l_{M\,ij}$, $\overbrace{log_{\prod_i}\mathscr{F}^l_M}^{} = \max log_{\prod_i}\mathscr{F}^l_{M\,ij}$ $\underbrace{log_{\prod_i}(\mathscr{F}^u_M)}_{} = \min log_{\prod_i}(\mathscr{F}^u_{M\,ij})$, $\overbrace{log_{\prod_i}(\mathscr{F}^u_M)}^{} = \max log_{\prod_i}(\mathscr{F}^u_{M\,ij})$ and $\underbrace{log_{\prod_i}\mathscr{F}^l_M}_{} \le log_{\prod_i}\mathscr{F}^l_{M\,ij} \le \overbrace{log_{\prod_i}\mathscr{F}^l_M}^{}$ and $\underbrace{log_{\prod_i}(\mathscr{F}^u_M)}_{} \le log_{\prod_i}(\mathscr{F}^u_{M\,ij}) \le$

$\overbrace{log_{\prod_i}(\mathscr{F}_M^u)}$. Now,

$$\begin{aligned}
\underbrace{log_{\prod_i}\mathscr{F}_M^l}+\underbrace{log_{\prod_i}(\mathscr{F}_M^u)} &= \boxdot_{i=1}^{n}(\underbrace{log_{\prod_i}\mathscr{F}_M^l})^{q\omega_i}+\boxdot_{i=1}^{n}(\underbrace{log_{\prod_i}(\mathscr{F}_M^u)})^{q\omega_i}\\
&\leq \boxdot_{i=1}^{n}(log_{\prod_i}\mathscr{F}_{M\,ij}^l)^{q\omega_i}+\boxdot_{i=1}^{n}(log_{\prod_i}(\mathscr{F}_{M\,ij}^u))^{q\omega_i}\\
&\leq \boxdot_{i=1}^{n}(\overbrace{log_{\prod_i}\mathscr{F}_M^l})^{q\omega_i}+\boxdot_{i=1}^{n}(\overbrace{log_{\prod_i}(\mathscr{F}_M^u)})^{q\omega_i}\\
&= \overbrace{log_{\prod_i}\mathscr{F}_M^l}+\overbrace{log_{\prod_i}(\mathscr{F}_M^u)}.
\end{aligned}$$

Therefore,

$$\left(\frac{\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(\underbrace{log_{\prod_i}\mathscr{T}_M^l})^q\right)^{\omega_i}}\right)^2+\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(\underbrace{log_{\prod_i}(\mathscr{T}_M^u)})^q\right)^{\omega_i}}\right)^2}{\begin{array}{c}2_{2}\\ +1-\frac{\left(\boxdot_{i=1}^{n}(\overbrace{log_{\prod_i}\mathscr{F}_M^l})^{q\omega_i}\right)^2+\left(\boxdot_{i=1}^{n}(\overbrace{log_{\prod_i}(\mathscr{F}_M^u)})^{q\omega_i}\right)^2}{2}\end{array}}\right)$$

$$=\left(\frac{\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(log_{\prod_i}\mathscr{T}_{M\,ij}^l)^q\right)^{\omega_i}}\right)^2+\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(log_{\prod_i}(\mathscr{T}_{M\,ij}^u))^q\right)^{\omega_i}}\right)^2}{\begin{array}{c}2\\ +1-\frac{\left(\boxdot_{i=1}^{n}(log_{\prod_i}\mathscr{F}_{M\,ij}^l)^{q\omega_i}\right)^2+\left(\boxdot_{i=1}^{n}(log_{\prod_i}(\mathscr{F}_{M\,ij}^u))^{q\omega_i}\right)^2}{2}\end{array}}\right)$$

$$=\left(\frac{\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(\overbrace{log_{\prod_i}\mathscr{T}_M^l})^q\right)^{\omega_i}}\right)^2+\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-(\overbrace{log_{\prod_i}(\mathscr{T}_M^u)})^q\right)^{\omega_i}}\right)^2}{\begin{array}{c}2_{2}\\ +1-\frac{\left(\boxdot_{i=1}^{n}(\underbrace{log_{\prod_i}\mathscr{F}_M^l})^{q\omega_i}\right)^2+\left(\boxdot_{i=1}^{n}(\underbrace{log_{\prod_i}(\mathscr{F}_M^u)})^{q\omega_i}\right)^2}{2}\end{array}}\right).$$

Hence, $\left\langle(\underbrace{log\mathscr{T}_M^l},\underbrace{log(\mathscr{T}_M^u)}),(\overbrace{log\mathscr{F}_M^l},\overbrace{log(\mathscr{F}_M^u)})\right\rangle$

$$\begin{aligned}
&\leq q-LIVIFWA(M_1,M_2,...,M_n)\\
&\leq \left\langle(\overbrace{log\mathscr{T}_M^l},\overbrace{log(\mathscr{T}_M^u)}),(\underbrace{log\mathscr{F}_M^l},\underbrace{log(\mathscr{F}_M^u)})\right\rangle.
\end{aligned}$$

Theorem 4.5. *(monotonicity property) Let* $M_i=\left\langle(log\mathscr{T}_{M\,t_{ij}}^l,log(\mathscr{T}_{M\,t_{ij})}^u),(log\mathscr{F}_{M\,t_{ij}}^l,log(\mathscr{F}_{M\,t_{ij}}^u))\right\rangle$ *and* $\omega_i=\left\langle(log\mathscr{T}_{M\,h_{ij}}^l,log(\mathscr{T}_{M\,h_{ij}}^u)),(log\mathscr{F}_{M\,h_{ij}}^l,log(\mathscr{F}_{M\,h_{ij}}^u))\right\rangle$
$(i=1,2,...,n);(j=1,2,...,i_j)$ *be the families of q-LIVIFWAs. For any* i*, if there is* $\left(log_{\prod_i}\mathscr{T}_{M\,t_{ij}}^l\right)^2+\left(log_{\prod_i}(\mathscr{T}_{M\,t_{ij}}^u)\right)^2\leq\left(log_{\prod_i}\mathscr{T}_{M\,h_{ij}}^l\right)^2+\left(log_{\prod_i}(\mathscr{T}_{M\,h_{ij}}^u)\right)^2$ *and* $\left(log_{\prod_i}\mathscr{F}_{M\,t_{ij}}^l\right)^2+\left(log_{\prod_i}(\mathscr{F}_M^u)_{t_{ij}}\right)^2\geq\left(log_{\prod_i}\mathscr{F}_{M\,h_{ij}}^l\right)^2+\left(log_{\prod_i}(\mathscr{F}_{M\,h_{ij})}^u\right)^2$ *or* $M_i\leq\mathscr{W}_i$*. Then q-LIVIFWA* $(M_1,M_2,...,M_n)\leq$ *q-LIVIFWA* $(\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n)$*.*

Proof. For any i, $\left(log_{\prod_i}\mathscr{T}_{M\,t_{ij}}^l\right)^q+\left(log_{\prod_i}(\mathscr{T}_{M\,t_{ij}}^u)\right)^q\leq\left(log_{\prod_i}\mathscr{T}_{M\,h_{ij}}^l\right)^q+\left(log_{\prod_i}(\mathscr{T}_{M\,h_{ij}}^u)\right)^q$.
Therefore, $1-\left(log_{\prod_i}\mathscr{T}_{M\,t_i}^l\right)^q+1-\left(log_{\prod_i}(\mathscr{T}_{M\,t_i}^u)\right)^q\geq1-\left(log_{\prod_i}\mathscr{T}_{M\,h_i}^l\right)^q+1-\left(log_{\prod_i}(\mathscr{T}_{M\,h_i}^u)\right)^q$.
Hence, $\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}\mathscr{T}_{M\,t_i}^l\right)^q\right)^{\omega_i}+\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}(\mathscr{T}_{M\,t_i}^u)\right)^q\right)^{\omega_i}\geq$
$\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}\mathscr{T}_{M\,h_i}^l\right)^q\right)^{\omega_i}+\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}(\mathscr{T}_{M\,h_i}^u)\right)^q\right)^{\omega_i}$
and $\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}\mathscr{T}_{M\,t_i}^l\right)^q\right)^{\omega_i}}+\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}(\mathscr{T}_{M\,t_i)}^u\right)^q\right)^{\omega_i}}$
$\leq\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}\mathscr{T}_{M\,h_i}^l\right)^q\right)^{\omega_i}}+\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left(log_{\prod_i}(\mathscr{T}_{M\,h_i)}^u\right)^q\right)^{\omega_i}}$.

For any i, $\Big(log_{\prod_i}\mathscr{F}^l_{M\,t_{ij}}\Big)^q + \Big(log_{\prod_i}(\mathscr{F}^u_{M\,t_{ij})}\Big)^q \geq \Big(log_{\prod_i}\mathscr{F}^l_{M\,h_{ij}}\Big)^q + \Big(log_{\prod_i}(\mathscr{F}^u_{M\,h_{ij})}\Big)^q$.
Therefore, $1 - \frac{\left(\boxdot_{i=1}^n log_{\prod_i}\mathscr{F}^l_{M\,t_{ij}}\right)^q + \left(\boxdot_{i=1}^n log_{\prod_i}(\mathscr{F}^u_{M\,t_{ij})}\right)^q}{2} \leq 1 - \frac{\left(\boxdot_{i=1}^n log_{\prod_i}\mathscr{F}^l_{M\,h_{ij}}\right)^q + \left(\boxdot_{i=1}^n log_{\prod_i}(\mathscr{F}^u_{M\,h_{ij})}\right)^q}{2}$.

$$= \left(\frac{\left(\sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}\mathscr{T}^l_{M\,ti})^q\Big)^{\omega_i}}\right)^2 + \left(\sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}(\mathscr{T}^u_{M\,ti}))^q\Big)^{\omega_i}}\right)^2}{2} \atop +1 - \frac{\left(\boxdot_{i=1}^n (log_{\prod_i}\mathscr{F}^l_{M\,tij})\right)^2 + \left(\boxdot_{i=1}^n (log_{\prod_i}(\mathscr{F}^u_{M\,tij}))\right)^2}{2} \right)$$

$$= \left(\frac{\left(\sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}\mathscr{T}^l_{M\,hi})^q\Big)^{\omega_i}}\right)^2 + \left(\sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}(\mathscr{T}^u_{M\,hi}))^q\Big)^{\omega_i}}\right)^2}{2} \atop +1 - \frac{\left(\boxdot_{i=1}^n (log_{\prod_i}\mathscr{F}^l_{M\,hij})\right)^2 + \left(\boxdot_{i=1}^n (log_{\prod_i}(\mathscr{F}^u_{M\,hij}))\right)^2}{2} \right).$$

Hence, q-LIVIFWA $(M_1, M_2, ..., M_n) \leq$ q-LIVIFWA $(\mathscr{W}_1, \mathscr{W}_2, ..., W_n)$.

4.2 q-LIVIF weighted geometric(q-LIVIFWG) operator

Definition 4.6. Let $M_i = \Big\langle (log\mathscr{T}^l_{M\,i}, log\mathscr{T}^u_{M\,i}), (log\mathscr{F}^l_{M\,i}, log\mathscr{F}^u_{M\,i}) \Big\rangle$ be the collection of q-LIVIFNs. Then q-LIVIFWG operator is q-LIVIFWG $(M_1, M_2, ..., M_n) = \boxdot_{i=1}^n M_i^{\omega_i}$ $(i = 1, 2, ..., n)$.

Theorem 4.7. *Let $M_i = \Big\langle (log\mathscr{T}^l_{M\,i}, log\mathscr{T}^u_{M\,i}), (log\mathscr{F}^l_{M\,i}, log\mathscr{F}^u_{M\,i}) \Big\rangle$ be the collection of q-LIVIFNs. Then q-LIVIFWG $(M_1, M_2, ..., M_n)$*

$$= \left({ \left(\boxdot_{i=1}^n (log_{\prod_i}\mathscr{T}^l_{M\,i})^{q\omega_i}, \boxdot_{i=1}^n (log_{\prod_i}(\mathscr{T}^u_{M\,i}))^{q\omega_i}\right), \atop \left(\sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}\mathscr{F}^l_{M\,i})^q\Big)^{\omega_i}}, \sqrt[q]{1 - \boxdot_{i=1}^n \Big(1 - (log_{\prod_i}(\mathscr{F}^u_{M\,i}))^q\Big)^{\omega_i}} \right)} \right).$$

Theorem 4.8. *If all $M_i = \Big\langle (log\mathscr{T}^l_{M\,i}, log\mathscr{T}^u_{M\,i}), (log\mathscr{F}^l_{M\,i}, log\mathscr{F}^u_{M\,i}) \Big\rangle$ are equal and $M_i = M$, for $i = 1, 2, ..., n$. Then q-LIVIFWG $(M_1, M_2, ..., M_n) = M$.*

Corollary 4.9. *The q-LIVIFWG operator is used to satisfy the boundedness and monotonicity properties.*

4.3 Extended q-LIVIFWA (q-ELIVIFWA) operator

Definition 4.10. Let $M_i = \Big\langle (log\mathscr{T}^l_{M\,i}, log\mathscr{T}^u_{M\,i}), (log\mathscr{F}^l_{M\,i}, log\mathscr{F}^u_{M\,i}) \Big\rangle$ be the collection of q-LIVIFN. Then q-ELIVIFWA $(M_1, M_2, ..., M_n) = \Big(\boxplus_{i=1}^n \omega_i M_i^{\aleph} \Big)^{1/\aleph}$ is called the q-ELIVIFWA operator.

Theorem 4.11. *Let $M_i = \Big\langle (log\mathscr{T}^l_{M\,i}, log\mathscr{T}^u_{M\,i}), (log\mathscr{F}^l_{M\,i}, log\mathscr{F}^u_{M\,i}) \Big\rangle$ be the collection of q-LIVIFNs. Then q-ELIVIFWA $(M_1, M_2, ..., M_n)$*

$$= \left(\begin{array}{c} \left(\left(\sqrt[q]{1 - \boxdot_{i=1}^n \left(1 - \Big((log_{\prod_i}\mathscr{T}^l_{M\,i})^q\Big)^q\right)^{\omega_i}} \right)^{1/\aleph}, \left(\sqrt[q]{1 - \boxdot_{i=1}^n \left(1 - \Big((log_{\prod_i}(\mathscr{T}^u_{M\,i}))^q\Big)^q\right)^{\omega_i}} \right)^{1/\aleph} \right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \left(1 - \left(\boxdot_{i=1}^n \left(\sqrt[q]{1 - \Big(1 - (log_{\prod_i}\mathscr{F}^l_{M\,i})^q\Big)^q}\right)^{\omega_i}\right)^q\right)^{1/\aleph}}, \\ \sqrt[q]{1 - \left(1 - \left(\boxdot_{i=1}^n \left(\sqrt[q]{1 - \Big(1 - (log_{\prod_i}(\mathscr{F}^u_{M\,i}))^q\Big)^q}\right)^{\omega_i}\right)^q\right)^{1/\aleph}} \end{array} \right) \end{array} \right).$$

Proof. We have, $\boxplus_{i=1}^{n}\omega_i M_i^{\aleph}$

$$=\left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{M\,i})^{q}\right)^{q}\right)^{\omega_i}},\ \sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{M\,i}))^{q}\right)^{q}\right)^{\omega_i}}\right),\\ \left(\boxdot_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,i})^{q}\right)^{q}}\right)^{\omega_i},\ \boxdot_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{M\,i}))^{q}\right)^{q}}\right)^{\omega_i}\right)\end{array}\right).$$

If $n=2$, then $\omega_1 B\bigvee\omega_2 C$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{B})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{C})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q},\\ -\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{B})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{C})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\end{array}}\\ \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{B}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{C}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\\ -\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{B}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{C}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\end{array}}\end{array}\right),\\ \left(\begin{array}{c}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{B})^{q}\right)^{q}}\right)^{\omega_1}\cdot\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{C})^{q}\right)^{q}}\right)^{\omega_1},\\ \left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{B}))^{q}\right)^{q}}\right)^{\omega_1}\cdot\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{C}))^{q}\right)^{q}}\right)^{\omega_1}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{q}\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{B})^{q}\right)^{q}\right)^{\omega_i}},\ \sqrt[q]{1-\boxdot_{i=1}^{q}\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{B}))^{q}\right)^{q}\right)^{\omega_i}}\right),\\ \left(\boxdot_{i=1}^{q}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,i})^{q}\right)^{q}}\right)^{\omega_i},\boxdot_{i=1}^{q}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{M\,i}))^{q}\right)^{q}}\right)^{\omega_i}\right)\end{array}\right).$$

It is valid for $n=m$ and $m\geq 3$.
Hence, $\boxplus_{i=1}^{m}\omega_i M_i^{\aleph}=$

$$\left(\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{B})^{q}\right)^{q}\right)^{\omega_i}},\ \sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{B}))^{q}\right)^{q}\right)^{\omega_i}}\right),\\ \left(\boxdot_{i=1}^{m}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,i})^{q}\right)^{q}}\right)^{\omega_i},\boxdot_{i=1}^{m}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,i})^{q}\right)^{q}}\right)^{\omega_i}\right)\end{array}\right).$$

If $n=m+1$ and we apply, then $\boxplus_{i=1}^{m}\omega_i M_i^{\aleph}+\omega_{m+1}M_{m+1}^{\aleph}=\boxplus_{i=1}^{m+1}\omega_i M_i^{\aleph}$.
Now,$\boxplus_{i=1}^{m}\omega_i M_i^{\aleph}+\omega_{m+1}M_{m+1}^{\aleph}=\omega_1 M_1^{\aleph}\bigvee\omega_2 M_2^{\aleph}\bigvee...\bigvee\omega_m M_m^{\aleph}\bigvee\omega_{m+1}M_{m+1}^{\aleph}$

$$=\left(\begin{array}{c}\left(\begin{array}{c}\sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{M\,i})^{q}\right)^{q}\right)^{\omega_i}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{M\,m+1})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q},\\ -\left(\sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{M\,i})^{q}\right)^{q}\right)^{\omega_i}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}\mathscr{T}^{l}_{M\,m+1})^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\end{array}}\\ \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{M\,i}))^{q}\right)^{q}\right)^{\omega_i}}\right)^{q}+\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{M\,m+1}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\\ -\left(\sqrt[q]{1-\boxdot_{i=1}^{m}\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{M\,i}))^{q}\right)^{q}\right)^{\omega_i}}\right)^{q}\cdot\left(\sqrt[q]{1-\left(1-\left((log_{\prod_i}(\mathscr{T}^{u}_{M\,m+1}))^{q}\right)^{q}\right)^{\omega_1}}\right)^{q}\end{array}}\end{array}\right),\\ \left(\begin{array}{c}\boxdot_{i=1}^{m}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,i})^{q}\right)^{q}}\right)^{\omega_i}\cdot\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}^{l}_{M\,m+1})^{q}\right)^{q}}\right)^{\omega_1},\\ \boxdot_{i=1}^{m}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{M\,i}))^{q}\right)^{q}}\right)^{\omega_i}\cdot\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}^{u}_{M\,m+1}))^{q}\right)^{q}}\right)^{\omega_1}\end{array}\right)\end{array}\right).$$

Thus,

$$\boxplus_{i=1}^{m+1}\omega_i M_i^{\aleph} = \left(\begin{array}{c} \left(\sqrt[q]{1-\boxdot_{i=1}^{m+1}\left(1-\left((log_{\prod_i}\mathscr{T}_{Mi}^{l})^q\right)^q\right)^{\omega_i}}, \sqrt[q]{1-\boxdot_{i=1}^{m+1}\left(1-\left((log_{\prod_i}(\mathscr{T}_{Mi}^{u}))^q\right)^q\right)^{\omega_i}}\right), \\ \boxdot_{i=1}^{m+1}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}_{Mi}^{l})^q\right)^q}\right)^{\omega_i}, \boxdot_{i=1}^{m+1}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}_{Mi}^{l})^q\right)^q}\right)^{\omega_i} \end{array}\right).$$

Hence,
$\left(\boxplus_{i=1}^{m+1}\omega_i M_i^{\aleph}\right)^{1/\aleph}$

$$= \left(\begin{array}{c} \left(\left(\sqrt[q]{1-\boxdot_{i=1}^{m+1}\left(1-\left((log_{\prod_i}\mathscr{T}_{Mi}^{l})^q\right)^q\right)^{\omega_i}}\right)^{1/\aleph}, \left(\sqrt[q]{1-\boxdot_{i=1}^{m+1}\left(1-\left((log_{\prod_i}(\mathscr{T}_{Mi}^{u}))^q\right)^q\right)^{\omega_i}}\right)^{1/\aleph}\right), \\ \left(\begin{array}{c} \sqrt[q]{1-\left(1-\left(\boxdot_{i=1}^{m+1}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{F}_{Mi}^{l})^q\right)^q}\right)^{\omega_i}\right)\right)^q}^{1/\aleph}, \\ \sqrt[q]{1-\left(1-\left(\boxdot_{i=1}^{m+1}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{F}_{Mi}^{u}))^q\right)^q}\right)^{\omega_i}\right)\right)^q}^{1/\aleph} \end{array}\right) \end{array}\right).$$

It is valid for $m \geq 1$.

Remark 4.12. If $\omega_i = 1$, then q-ELIVIFWA operator is modified to the q-LIVIFWA operator.

Theorem 4.13. *If all* $M_i = \left\langle(log\mathscr{T}_{Mi}^{l}, log\mathscr{T}_{Mi}^{u}), (log\mathscr{F}_{Mi}^{l}, log\mathscr{F}_{Mi}^{u})\right\rangle$ *are equal and* $M_i = M$. *Then q-ELIVIFWA* $(M_1, M_2, ..., M_n) = M$.

Remark 4.14. We use the q-ELIVIFWA operator to satisfy boundedness and monotonicity conditions.

4.4 Extended q-LIVIFWG (q-ELIVIFWG) operator

Definition 4.15. Let $M_i = \left\langle(log\mathscr{T}_{Mi}^{l}, log\mathscr{T}_{Mi}^{u}), (log\mathscr{F}_{Mi}^{l}, log\mathscr{F}_{Mi}^{u})\right\rangle$ be the collection of q-LIVIFNs. Then q-ELIVIFWG $(M_1, M_2, ..., M_n) = \frac{1}{\aleph}\Big(\boxdot_{i=1}^{n}(\aleph M_i)^{\omega_i}\Big)$ is called the q-ELIVIFWG operator.

Theorem 4.16. *Let* $M_i = \left\langle(log\mathscr{T}_{Mi}^{l}, log\mathscr{T}_{Mi}^{u}), (log\mathscr{F}_{Mi}^{l}, log\mathscr{F}_{Mi}^{u})\right\rangle$ *be the collection of q-LIVIFNs. Then q-ELIVIFWG*$(M_1, M_2, ..., M_n)$

$$= \left(\begin{array}{c} \left(\begin{array}{c} \sqrt[q]{1-\left(1-\left(\boxdot_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}\mathscr{T}_{Mi}^{l})^q\right)^q}\right)^{\omega_i}\right)\right)^q}^{1/\aleph}, \\ \sqrt[q]{1-\left(1-\left(\boxdot_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(log_{\prod_i}(\mathscr{T}_{Mi}^{u}))^q\right)^q}\right)^{\omega_i}\right)\right)^q}^{1/\aleph} \end{array}\right), \\ \left(\left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left((log_{\prod_i}\mathscr{F}_{Mi}^{l})^q\right)^q\right)^{\omega_i}}\right)^{1/\aleph}, \left(\sqrt[q]{1-\boxdot_{i=1}^{n}\left(1-\left((log_{\prod_i}(\mathscr{F}_{Mi}^{u}))^q\right)^q\right)^{\omega_i}}\right)^{1/\aleph}\right) \end{array}\right).$$

Remark 4.17. If $\omega_i = 1$, then q-ELIVIFWG operator is converted to the q-LIVIFWG operator.

Remark 4.18. q-ELIVIFWG operators satisfy boundedness and monotonicity properties.

Corollary 4.19. *If all* $M_i = \left\langle(log\mathscr{T}_{Mi}^{l}, log\mathscr{T}_{Mi}^{u}), (log\mathscr{F}_{Mi}^{l}, log\mathscr{F}_{Mi}^{u})\right\rangle$ *are equal and* $M_i = M$, *for* $i = 1, 2, ..., n$. *Then q-ELIVIFWG*$(M_1, M_2, ..., M_n) = M$.

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