Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 1s (2025)

The Development of IV-VIKOR Method Under Hesitant Fuzzy Sets in MCDM

Rafi Raza[1], Ahmad Termimi Ab Ghani[2*], Lazim Abdullah[3]
[1,2,3]Faculty of Computer Science and Mathematics, University Malaysia Terengganu,
210300 Kuala Nerus, Kuala Terengganu, Malaysia.
[*]Corresponding author: termimi@umt.edu.my

Article History:
Received: *13-08-2024*
Revised: *27-09-2024*
Accepted: *14-10-2024*

Abstract:
Due to its solution correctness and ease of evaluation, the VIKOR approach has already established itself as a popular multi-criterion decision-making (MCDM) method. This strategy focuses on picking and ranking from a list of viable options in order to assist the decision-maker in determining a final course of action. Additionally, it looks for a middle ground to resolve a problem with competing demands. It produces a ranking list of compromises based on a precise measurement of distance from the ideal response. The hesitant fuzzy set (HFS) has furthermore come under increased study because of its potency and ability in capturing ambiguity and uncertainty. This paper introduces the inversion of VIKOR method (IV-VIKOR) to take uncertain fuzzy conditions into account. These novel concepts serve as the foundation for an IV-VIKOR method that we suggest, and a real-world application is given to demonstrate how well our method works when dealing with MCDM problem with hesitant fuzzy preference data.

Keywords: MCDM, F-VIKOR method, Decision Making, Hesitant fuzzy sets.

1 Introduction

Finding the best option from a list of possibilities while taking into account the decision maker’s expectations and various considerations is the act of decision making. Every choice is made inside an environment, which is the set of facts, options, standards, and preferences that are at hand at the time, the choice needs to be made. The diversity of criteria established for evaluating the options is the decision-making process’s most challenging component. Therefore, MCDM is the process of choosing amongst several, frequently at odds criteria. In daily life, MCDM issues occur frequently Taherdoost and Madanchian [1]. As more decision-makers from various groups participate in the process, the problem of decision-making becomes increasingly complicated. Despite the constant prevalence of MCDM issues, the field of MCDM as a whole is rather young, having only been for around 30 years. The rapid development of computer technology, which permits the methodical analysis of difficult MCDM problems, is closely related to the growth of the MCDM discipline [2]. But MCDM is becoming more and more significant and helpful in assisting business decision-making as more and more individuals use information technology and produce massive volumes of data. The advantage of MCDM is that it strives to remove emotion from the decision-making process while providing a balanced assessment of how appropriate each alternative. It also prevents one component from dominating the others. The majority of MCDM issues fall into one of two categories; that is a finite an infinite number of solutions. The number of potential solutions is typically constrained in tasks involving selection and evaluation. The typical way to represent an MCDM problem is a decision matrix as can be written as given below:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ a_{31} & a_{32} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

where each entry a_{ij} denotes how effectively the i^{th} alternative meets the j^{th} criterion. Therefore, a family of tools known as MCDM techniques has developed in response to the requirement for a formalised strategy to aid decision making in scenarios containing many variables [3]. This facilitates in determining the optimal course of action. How attribute data should be evaluated in order to make the right choice is determined by an MCDM technique. To far, a number of MCDM approaches have been put out and used with success to address complex decision-making issues that arise in various management and engineering fields. The VIKOR method has become a very well-liked among those involved in decision-making because of its straightforward and understandable computational steps [4]. First establishing the “positive ideal solutions (PIS)” and “negative ideal solutions (NIS)” forms the foundation of VIKOR technique. When there are conflicting and incomparable criteria at play, it concentrates on assessing and choosing among a small number of plausible options. Depending on how closely the response adheres to the ideal response, a multi-criteria ranking index is evaluated. The ranking of the compromise options can be determined by comparing the relative closeness measure to the ideal alternative after each compromise option has been assessed in light of each criterion. The compromise option thus represents a viable alternative that is the most remote from the unfavorable ideal solution but the least nearer the favorable ideal solution [5, 6].

Establishing the VIKOR technique in the presence of HFSs and an inversion of the VIKOR method is the primary goal of this article. An MCDM method for complex structures called VIKOR was first introduced by Opricovic [7] with the intention of dealing with crisp information. Subsequently, it was extended to rationalize a wider range of informational settings. Finding a compromise solution to the MCDM problem that fulfils the criteria of “maximum group utility” and “minimal individual regret” among opponents is the primary goal of this method. Opricovic used the following two defining essential elements of a feasible compromise solution. L_p measure and an aggregation function. L_p metric

$$L_i^p = \left\{ \sum_{j=1}^{n} \left[\frac{F_{ij} - F_j^*}{F_j^* - F_j^-} \right]^p \right\}^{\frac{1}{p}}$$

Where $F_j^* = \max_i(F_{ij})$ and $F_j^- = \min_i(F_{ij})$.

But in this chapter, the IV-VIKOR method based on L_p metric can be written as L_p

$$L_i^p = \left\{ \sum_{j=1}^{n} \left[(F_{ij} - F_j^*) \times (F_j^* - F_j^-) \right]^p \right\}^{\frac{1}{p}}$$

Where $F_j^* = \max_i(F_{ij})$ and $F_j^- = \min_i(F_{ij})$.

Despite the false premise that a decision-maker shows exact instances of the determining preferences circumstances, traditional MCDM methods use crisp information to evaluate the alternatives and qualities in typical unexpected and uncertain situations. Because priorities are a very vague and ambiguous aspect of human character [8], as a result, a decision-maker should not use precise information to assess the tendency of his issue. Bellmen and Zadeh [9] were the first to use fuzzy numbers (FNs) for the MCDM approaches. Fuzzy membership values are chosen from the range $[0, 1]$ while making decisions in the FS domain. One of the best tools for making decisions is the fuzzy set theory, which Zadeh [10] first developed. Many writers have since used the FS theory to illustrate real-world situations. The fuzzy VIKOR was introduced by Wang and Chang [11] to capture MCDM. Chang [12] classified Taiwanese hospitals based on their amenities and services using the F-VIKOR method. Other F-VIKOR method applications to address MAGDM issues with supplier selection were put forth [13-15]. Opricovic [16] modified the conventional F-VIKOR method of managing the Mlava River's water resources. An expansion of the VIKOR method for trapezoidal FNs was introduced by Ju and Wang [17]. Wang, Liang and Ho [18] created an F-VIKOR method using triangular FNs as linguistic variables to determine the best software company. To evaluate the Saudi Arabian power networks for investment, Taylan, Alamoudi [19] used various decision-making methods, including F-VIKOR, fuzzy AHP, as well as fuzzy TOPSIS. All of these VIKOR method variations could only handle the data in a way that benefited the object in question in accordance with FS's capabilities. As a result, these techniques are ineffective when there is a high level of discontent.

Zadeh [20] introduced the fuzzy sets theory. It can deal with ambiguous and imprecise information. Whenever working with insufficient and fuzzy data when several or so more sourcesof uncertainty are present simultaneously, the classic fuzzy set has a number of drawbacks Rodriguez, Martinez and Herrera [21]. Torra and Narukawa were firstly develop the HFS Torra and Narukawa [22], Torra [23] providing up new directions for future studies on decisionmaking under unclear circumstances. This generalised type of FS founded on aforementioned extensional forms of FSs. The HFS illustrates fuzziness by presenting all possible values when calculating an element's membership degree in a certain set, despite the fact that it does not offer an accurate membership function.

The article's remaining sections are organized as follows. The FS, HFSs, distance between hesitant fuzzy element inverse of any element in Section 2 along with a quick overview of some fundamental ideas that are used throughout the article. We present the VIKOR approach of HFSs for MCDM issues in Section 3. In Section 4, we go through how the IV-VIKOR method was developed. An illustration of the suggested technique is provided in Section 5 for clarity. The main findings of the paper are then briefly explored.

2 Preliminaries

Definition 2.1 [24] Fuzzy set

Suppose X is the universal set. A fuzzy set X is defined by $\mu_B(\tilde{x}): X \to [0,1]$ is the membership function of B, and it can be written as given below.

$$B = \{(\tilde{x}, \mu_B(\tilde{x})), \tilde{x} \epsilon X\}$$

Where $\mu_B(\tilde{x})$ is the degree of membership of $\tilde{x}$. In B each pair $(\tilde{x}, \mu_B(\tilde{x}))$ is a singleton set.

Definition 2.2 [22] Hesitant Fuzzy set

Suppose X is the universal set; then an HFS as B on X is defined by a function $\rho_B(\tilde{x})$ that X returns to a subset of [0,1]. Xia and Xu [25] indicated the HFS mathematically as given below.

$$B = \{\langle \tilde{x}, \rho_B(\tilde{x}) \rangle | \tilde{x} \epsilon X\}$$

Where $\rho_B(\tilde{x})$ is defined as a set of membership degrees for an element under a subset of[0,1], indicating the membership degree of an element $\tilde{x} \epsilon X$. Xia and Xu [25] called $\rho = \rho_B(\tilde{x})$ hesitant fuzzy element (HFE). A hesitant fuzzy element (HFE) is a finite and non-empty subset of [0, 1].

Example 2.1 Let $X = \{\tilde{a}, \tilde{b}, \tilde{c}\}$ be a universal set, $\rho_B(\tilde{a}) = \{0.1,0.9,0.6\}, \rho_B(\tilde{b}) = \{0.6,0.7\}, \rho_B(\tilde{c}) = \{0.7,0.3\}$, and then HFSs can be written as.

$$B = \{< \tilde{a}, \{0.1,0.9,0.6\} >, < \tilde{b}, \{0.6,0.7\} >, < \tilde{c}, \{0.7,0.3\} >\}$$

Definition 2.3 [26] Hesitant Normalized Hamming Distance

Suppose h_1 and h_2 be two HFSs on $X = \{x_1, x_2, x_3, \dots, x_n\}$ then the hesitant normalized Hamming distance measure between h_1 and h_2 is defined as.

$$d(h_1, h_2) = \frac{1}{l} \sum_{j=1}^{l} \left| h_{1\sigma(j)} - h_{2\sigma(j)} \right|$$

Where l is the number of the elements in the h, in most cases, $l_1 \neq l_2$ and for convenience, let $l = \max(l_1, l_2)$.

3 Extended VIKOR Method Under HFSs

Opricovic [16] adopted the F-VIKOR method to solve problems in an uncertain way where the criteria and weight characterize fuzzy sets [27].

Step 1. Determine PIS and NIS as below

$$A^* = \{h_1^*, h_2^*, h_3^*, \dots, h_n^*\},$$

$$\text{Where } h_i^* = \cup_i^m h_{ij}$$

$$A^- = \{h_1^-, h_2^-, h_3^-, \dots, h_n^-\},$$

$$\text{Where } h_i^- = \cap_i^m h_{ij}, \quad j = 1,2,3, \dots, m.$$

Step 2. Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} w_j \frac{||h_j^* - h_{ij}||}{||h_j^* - h_j^-||}$$

Calculate regret measure

$$R_i = \max_j(S_i) = w_j \frac{||h_j^* - h_{ij}||}{||h_j^* - h_j^-||}, \text{where } i = 1,2,3, \dots, m.$$

Step 3. Evaluate Q_i as given below

$$Q_i = v \frac{S_i - S^-}{S^* - S^-} + (1 - v) \frac{R_i - R^-}{R^* - R^-}$$

And v is introduced weight, where $S^* = \min_i(S_i)$, $S^- = \max_i(S_i)$, $R^* = \min_i(R_i)$ and $R^- = \max_i(R_i)$ $i = 1,2,3, \dots, m$.

Step 4. Classify the alternatives, categorizing through $S, R,$ and Q values from largest to smallest. The outcome is in three grades.

Step 5. Provide the alternative (A_1), which is graded top by the smallest value Q, as a compromise solution if the two factors persist, as given below.

C1. Acceptable advantage.

$Q(A_2) - Q(A_1) \geq DQ$ Where (A_2) is the alternative with 2^{nd} position in the grading list $Q, DQ = \frac{1}{m-1}$ where m denotes the possible alternatives.

C2."Acceptable stability in decision making".

An alternative (A_1) should also possess the highest position S or R. In a decision-making procedure, such as voting by majority rule $v > 0.5$ or by consensus $v \approx 0.5$ or with veto $v < 0.5$, this compromise solution is stable.

As suggested, the following compromise solution is if one of the requirements is not fulfilled.

- Alternative (A_1) and (A_2) if only C2 is not satisfied, or
- Alternatives (A_i) where $i = 1,2,3, \dots, m$. if C1 is not satisfied; (A_m) It is determined by the relation $Q(A_m) - Q(A_1) \leq DQ$ by maximum m.

4 An IV-VIKOR method from the extended VIKOR method under HFSs

IV-VIKOR method means an inversion of the VIKOR method in the HFS information. The values of the difference of PIS and NIS should more appropriately be considered as hesitant fuzzy elements in this method because, in the hesitant fuzzy VIKOR method, the difference of the values of PIS and NIS is in division form, but in this chapter, as in multiplication. Because of this, in the current research, we extend the IV-VIKOR under the HFSs approach to solve the MCDM problem. The following structure is used for hesitant fuzzy elements.

Step 1. Determine PIS and NIS.

$$A^* = \{h_1^*, h_2^*, h_3^*, \dots, h_n^*\},$$

$$\text{Where } h_i^* = \cup_i^m h_{ij}$$

$$A^- = \{h_1^-, h_2^-, h_3^-, \dots, h_n^-\},$$

$$\text{Where } h_i^- = \cap_i^m h_{ij}, \quad j = 1,2,3, \dots, m.$$

Step 2. Compute S_i and R_i as below

$$S_i = \sum_{j=1}^{n} w_j \left\| \left(h_j^* - h_{ij}\right). \left(h_j^* - h_j^-\right) \right\|$$

Calculate regret measure

$$R_i = \max_j (S_i) = w_j \left\| \left(h_j^* - h_{ij}\right). \left(h_j^* - h_j^-\right) \right\| \quad \text{where } i = 1,2,3, \dots, m.$$

Step 3. Evaluate Q_i as given below

$$Q_i = v(S_i - S^-)(S^* - S^-) + (1 - v)(R_i - R^-).(R^* - R^-)$$

And v is introduced weight, where $S^* = \min_i(S_i)$, $S^- = \max_i(S_i)$, $R^* = \min_i(R_i)$ and $R^- = \max_i(R_i)$ $i = 1,2,3, \dots, m$.

Step 4. Classify the alternatives, categorizing through $S, R,$ and Q values from largest to smallest. The outcome is in three grades.

Step 5. Provide the alternative (A_1), which is graded top by the smallest value Q, as a compromise solution if the two factors persist, as given below.

C1. Acceptable advantage.

$Q(A_2) - Q(A_1) \le DQ$

Where (A_2) is the alternative with 2^{nd} position in the grading list by $Q, DQ = \frac{1}{m-1}$ where m denotes the possible alternatives.

C2."Acceptable stability in decision making".

An alternative should also possess the highest position from S or$R$. In a decision-making procedure, such as voting by majority rule $v > 0.5$ or by consensus $v \approx 0.5$ or with veto $v < 0.5$, this compromise solution is stable.

As suggested, the following compromise solution is if one of the requirements is not fulfilled.

- Alternative (A_1) and (A_2) if only C2 is not satisfied, or
  Alternatives A_i where $i = 1,2,3, \dots, m$. if C1 is not satisfied; (A_m) it is determined by the relation $Q(A_m) - Q(A_1) \ge DQ$ by maximum m.

5 Numerical Example

Suppose we have to find out the best airlines as $A, B, C,$ and D with criteria weight $w = (0.21,0.32,0.41,0.06)$ on the basis of multiple criteria as given, κ_1: Customer service, κ_2: Comfort and Space, κ_3: Great price, great deals, κ_4: Inflight meals.

Table 1. Hesitant fuzzy decision matrix (vi)

Alternatives	C_1	C_2	C_3	C_4
A	$(0.3, 0.4)$	$(0.1, 0.2, 0.4)$	$(0.1, 0.2, 0.3,0.5)$	$(0.4, 0.5, 0.6)$
B	$(0.1, 0.2, 0.3)$	$(0.1, 0.2, 0.3)$	$(0.2, 0.3)$	$(0.2, 0.3)$
C	$(0.1, 0.2, 0.3)$	$(0.2, 0.4, 0.7)$	$(0.3, 0.4, 0.5)$	$(0.1, 0.8)$
D	$(0.3, 0.4)$	$(0.4, 0.5, 0.6, 0.7)$	$(0.4, 0.5, 0.6)$	$(0.5, 0.6, 0.7, 0.8)$

IV-VIKOR method from extended VIKOR method under HFS.

Step 1. Determined PIS and NIS as below.

$$A^* = [0.4, 0.7, 0.6, 0.8]$$

$$A^- = [0.1, 0.1, 0.1, 0.1]$$

Step 2. Computed S_i and R_i as below

$$S_i = \sum_{j=1}^{n} w_j \left\| \left(h_j^* - h_{ij}\right). \left(h_j^* - h_j^-\right) \right\|$$

$$S_1 = 0.17035, S_2 = 0.20345, S_3 = 0.1195, S_4 = 0.08605.$$

Calculated regret measure

$$R_i = \max_j(S_i) = w_j \left\| \left(h_j^* - h_{ij}\right). \left(h_j^* - h_j^-\right) \right\|$$

$R_1 = 0.0896, R_2 = 0.096, R_3 = 0.0512, R_4 = 0.0478.$

Step 3. Evaluated Q_i as given below

$$Q_i = v(S_i - S^-)(S^* - S^-) + (1 - v)(R_i - R^-). (R^* - R^-)$$

$Q_1 = 0.0059, Q_2 = 0.0080, Q_3 = 0.00204, Q_4 = 0.000.$

Step 4. Classified the alternatives, categorizing through $S, R,$ and Q values, from largest to smallest. The outcome is in three grades, as listed in Table 2.

Step 5. Provide the alternative (A), which is graded top by smallest values by Q, as a compromise solution if the factor persists, as given below.

C1. Acceptable advantage.

C1. $Q(B) - Q(A) \leq \frac{1}{3}$ satisfied. It means that the alternative A is a compromise solution.

Table 2. Ranking the alternatives through the IV-VIKOR method under hesitant fuzzy sets

Alternatives	S	R	Q	Ranking
A	0.17035	0.0896	0.00594	1
B	0.20345	0.096	0.0080	2
C	0.1195	0.0512	0.00204	4
D	0.08605	0.0478	0.000	3

Conclusion.

In this study, a novel concept for IV-VIKOR technique under hesitant fuzzy information was devised, and multi-criteria issues with various and incommensurable criteria were resolved, notably accounting for the complicated subjective nature of the decision maker. The difference between the PIS and NIS in this IV-VIKOR method under HFSs is in multiplication form, however in the VIKOR method under HFSs, is it in division form. This approach demonstrates the VIKOR method's inversion because we multiply their difference in this way rather than dividing it. Complex MCDM problems can be solved with this strategy extremely effectively.

References.

[1] Taherdoost, H. and M. Madanchian, *Multi-criteria decision making (MCDM) methods and concepts.* Encyclopedia, 2023. **3**(1): p. 77-87.

[2] Yin, S., et al., Practical teaching method innovation decisions related to labor and reading at agricultural colleges based on entropy-fuzzy AHP combination weights. AIMS Mathematics, 2023. **8**(4): p. 7641-7661.

[3] Velasquez, M. and P. Hester, An Analysis of Multi-Criteria Decision Making Methods International Journal of Operations Research Vol. 10. 2013.

[4] Opricovic, S. and G.-H. Tzeng, *Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS.* European journal of operational research, 2004. **156**(2): p. 445-455.

[5] Mardani, A., et al., VIKOR technique: A systematic review of the state of the art literature on methodologies and applications. Sustainability, 2016. **8**(1): p. 37.

[6] Yazdani, M. and F.R. Graeml, *VIKOR and its applications: A state-of-the-art survey.* International Journal of Strategic Decision Sciences (IJSDS), 2014. **5**(2): p. 56-83.

[7] Opricovic, S., Visekriterijumska optimizacija u građevinarstvu-Multi-criteria optimization of civil engineering systems. Faculty of Civil Engineering, Belgrade, 1998.

[8] Akram, M., G. Muhiuddin, and G. Santos-García, *An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets.* Mathematical Biosciences and Engineering, 2022. **19**(7): p. 7201-7231.

[9] Bellman, R.E. and L.A. Zadeh, *Decision-making in a fuzzy environment.* Management science, 1970. **17**(4): p. B-141-B-164.

[10] Zadeh, L.A., *Information and control.* Fuzzy sets, 1965. **8**(3): p. 338-353.

[11] Wang, T. and T. Chang. Fuzzy VIKOR as a resolution for multicriteria group decision-making. in The 11th International Conference on Industrial Engineering and Engineering Management. 2005. Atlantis Press Paris, France.

[12] Chang, T.-H., Fuzzy VIKOR method: A case study of the hospital service evaluation in Taiwan. Information Sciences, 2014. **271**: p. 196-212.

[13] Mishra, S., et al., Multi-attribute group decision-making (MAGDM) for supplier selection using fuzzy linguistic modelling integrated with VIKOR method. International Journal of Services and Operations Management, 2012. **12**(1): p. 67-89.

[14] Sanayei, A., S.F. Mousavi, and A. Yazdankhah, *Group decision making process for supplier selection with VIKOR under fuzzy environment.* Expert systems with applications, 2010. **37**(1): p. 24-30.

[15] Shemshadi, A., et al., A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting. Expert systems with applications, 2011. **38**(10): p. 12160-12167.

[16] Opricovic, S., *Fuzzy VIKOR with an application to water resources planning.* Expert Systems with Applications, 2011. **38**(10): p. 12983-12990.

[17] Ju, Y. and A. Wang, Extension of VIKOR method for multi-criteria group decision making problem with linguistic information. Applied Mathematical Modelling, 2013. **37**(5): p. 3112-3125.

[18] Wang, T.-C., J.-L. Liang, and C.-Y. Ho. Multi-criteria decision analysis by using fuzzy VIKOR. in 2006 International conference on service systems and service management. 2006. IEEE.

[19] Taylan, O., et al., Assessment of energy systems using extended fuzzy AHP, fuzzy VIKOR, and TOPSIS approaches to manage non-cooperative opinions. Sustainability, 2020. **12**(7): p. 2745.

[20] Zadeh, L., *Fuzzy set theory.* Information and control, 1965. **8**(3): p. 338-353.

[21] Rodriguez, R.M., L. Martinez, and F. Herrera, *Hesitant fuzzy linguistic term sets for decision making.* IEEE Transactions on fuzzy systems, 2011. **20**(1): p. 109-119.

[22] Torra, V. and Y. Narukawa. On hesitant fuzzy sets and decision. in 2009 IEEE international conference on fuzzy systems. 2009. IEEE.

[23] Torra, V., *Hesitant fuzzy sets.* International journal of intelligent systems, 2010. **25**(6): p. 529-539.

[24] Zadeh, L.A., *Fuzzy sets.* Information and control, 1965. **8**(3): p. 338-353.

[25] Xia, M. and Z. Xu, *Hesitant fuzzy information aggregation in decision making.* International journal of approximate reasoning, 2011. **52**(3): p. 395-407.

[26] Zhang, N. and G. Wei, *Extension of VIKOR method for decision making problem based on hesitant fuzzy set.* Applied Mathematical Modelling, 2013. **37**(7): p. 4938-4947.

[27] Zeleny, M. and J. Cochrane, Multiple criteria decision making McGraw-Hill New York, 34. 1982.