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Analysis and Implementation of Quantum Gates (X, Z, Y, Hadamard CNOT and CCNOT) by using matrices for Quantum Computing

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Abstract:

The constraints of present supercomputers in addressing exponentially intricate problems are apparent, as these jobs sometimes necessitate hundreds or even thousands of years due to the exponential increase in computational requirements. Quantum computing, which utilizes concepts from quantum physics and operates with qubits, offers a promising solution by significantly decreasing problem-solving durations to only seconds or minutes. Using IBM's quantum computing platform, this work investigates the construction and analysis of fundamental quantum gates, including X, Z, Y, Hadamard, CNOT, and CCNOT. This work presents a comprehensive analysis of these gates using matrix representations, demonstrating their operational mechanics and algebraic functions, and fundamental role of matrix theory in quantum gate operations and its contribution to the future development of quantum computing.

Keywords— Quantum Computing, matrices, Hadamard gate, qubits, CNOT, CCNOT

1. Introduction

The modern subject of quantum computing presents entirely new avenues for information processing. Physicist Richard Feynman made one of the first suggestions for quantum computing in the 1980s, claiming that quantum computers might be able to imitate physical systems more effectively than classical computers. Researchers Peter Shor and Grover created algorithms in the early 1990s that showed how quantum computers may outperform classical computers at specific tasks, such factoring big numbers and searching databases, by an exponential margin. Researchers Shor & Grover created algorithms in the early 1990s that demonstrated how quantum computers may outperform classical computers at specific tasks, such factoring big numbers and searching databases, by an exponential margin [2–5].

Unlike classical computers, which utilize bits to encode information as either 0 or 1, quantum computers use quantum bits, or qubits, which can represent and handle a significantly larger amount of information due to their unique quantum properties. Qubits, also known as quantum bits, are the fundamental building blocks of information in quantum computing, much like classical bits in traditional computing [6–9]. However, qubits have unique capabilities that enable them to conduct complicated computations significantly more quickly and efficiently than conventional computers since they are founded on the concepts of quantum mechanics. Quantum computing leverages quantum phenomena such as superposition, interference, and entanglement to achieve computational feats beyond the scope of traditional computers. The ultimate goal is to build a quantum computer that can, under certain situations, perform tasks significantly faster than a classical computer [10–13].

Superposition is a fundamental concept in quantum mechanics and a fundamental idea in quantum computing. One of the main factors contributing to quantum computers' higher computing power than classical computers is their capacity for simultaneous existence in many states, or qubits. Because qubits can exist in a superposition of states, a quantum computer has the capacity to handle a vast number of possibilities simultaneously. For example, if a quantum computer contains n qubits, it may concurrently represent and control 2n different states. Because of their exponential parallelism, quantum computers can solve some tasks significantly faster than classical computers. Superposition is an essential component of many quantum algorithms, to assess every potential entry at once, providing a quadratic speedup over traditional search algorithms. Quantum interference, in which the probability amplitudes of various quantum states interfere with one another, is also made possible by superposition. Quantum algorithms take advantage of this interference to increase the probability of right responses and decrease the probability of wrong replies. possibility of choosing the wrong ones [14–17].

Entanglement, which is the state in which two or more quantum particles get so entangled that their states are instantly influenced by one another regardless of their distance from one another, is another fundamental idea in quantum physics. A key component of quantum computing is entanglement, which enables quantum machines to do complex calculations and solve problems more quickly than conventional computers. Entanglement is necessary for several quantum algorithms that perform better than classical algorithms [18–19]. For example, entanglement is used in quantum teleportation to transfer a qubit's state without actually transferring the qubit. In quantum cryptography techniques such as Quantum Key Distribution (QKD), entanglement is an essential resource. QKD offers an unparalleled degree of security by using entangled qubits to guarantee that any eavesdropping on the communication channel would be detected. Entanglement is used in quantum error correction to shield quantum data from errors brought on by decoherence and other quantum noise. In error correction codes, entangled qubits are used to identify and fix mistakes without actually measuring the quantum information, which would cause the quantum state to collapse [20-21].

Quantum computing offers more efficient problem-solving than classical methods, with promising experimental outcomes suggesting commercial availability in the coming years. Quantum computers, like Shor's prime factorization algorithm, can theoretically solve decryption speeds in hours. They have physical structures resembling classical systems but with quantum registers and entangled states, requiring unique circuit design technologies. Quantum computers use reversible quantum gates for unitary operations. Although operational today, the number of available quantum computing systems remains limited. Despite limited laboratory availability, emerging fields are developing large-scale quantum computers in distributed frameworks. Technological advancements may allow them to be miniaturized into compact devices, similar to classical computers' development stages [22].

As previously said, quantum computers employ quantum effects like superposition, entanglement, and interference to encode information in quantum states using quantum information principles. These systems can be realized in various forms, including atomic energy levels, spin, and polarization. Quantum circuit computations model states as interdependent entangled systems, allowing for concurrent representation of classical values. Quantum computations rely on reversible computation, which allows for the retrieval of the initial state from the output state. This principle is applicable to classical systems, where input quantum states undergo reversible evolution through unitary operations. In quantum computation, temporary quantum systems, called ancilla states, are disregarded once output is produced. A measurement is performed on the quantum register for subsequent calculations. The DiVincenzo criteria outline the physical implementation of quantum computers, requiring scalable quantum registers, universal gates, coherence time, fidelity, and retrievable results through measurements, serving as the cornerstone for practical quantum computation [23-24].

As a unitary operator, the X gate maintains the norm of the quantum state it operates on. Sustaining the probabilistic interpretation of quantum mechanics depends on this characteristic.

A key component of quantum error correcting methods is the X gate. It rectifies faults in quantum bits by flipping them back to their intended state via protocols such as the bit-flip code. The X gate can be physically implemented using a range of quantum computing methods, including photonic qubits, trapped ions, and superconducting qubits. Research priorities include fidelity and efficiency of these implementations [28]. Error mitigation approaches have been the subject of recent research aimed at increasing the accuracy of quantum gates, such as the X gate. This entails creating strategies to lessen the effects of mistakes in quantum processes [29].

Another crucial element of the collection of Pauli matrices used in quantum computing is the Y quantum gate. Similar to the X and Z gates, this gate is essential to quantum algorithms and operations. A survey of the literature on the Y quantum gate will address its technological developments, applications, and theoretical features. Another crucial element of the collection of Pauli matrices used in quantum computing is the Y quantum gate. Similar to the X and Z gates, this gate is essential to quantum algorithms and operations. A survey of the literature on the Y quantum gate will address its technological developments, applications, and theoretical features. The goal of future research is to integrate Y-gate-using quantum systems into bigger quantum computers by scaling them up. This entails incorporating mistake correcting techniques and maximizing gate performance [30].

Prior to measurement, qubit states are altered using the Hadamard gate, enabling interference effects that are essential for quantum algorithms. The H gate guarantees that quantum algorithms can explore various computational routes by generating superposition [31].

In order to comprehend and improve the performance of CNOT gates, sophisticated theoretical models have been put forth. The objective of these models is to improve operational efficiency and gate fidelity [32]. Utilizing CNOT gates to optimize quantum circuits and enhance algorithm performance is a recent development in quantum algorithms [33].

Often referred to as the Controlled-Controlled-NOT (CCNOT) gate, the Toffoli gate is a fundamental three-qubit quantum gate used for a variety of quantum algorithms and error correction methods. This study of the literature examines important breakthroughs and research on the CCNOT gate from 2019 to 2024, with an emphasis on new technologies, applications, and theoretical advancements. Improvements to the CCNOT gate's theoretical architecture are intended to maximize its effectiveness and functionality in quantum circuits [34]. It has been investigated how to employ the CCNOT gate in quantum algorithms, and new methods and uses have been discovered [35].

In this paper, Working & implementation of various quantum gates X, Y, Z, H CNOT & CCNOT are demonstrated in IBM quantum computers, also show their output. It is shown these gates work as very crucial in design of quantum computer. In quantum computing, matrices operations play very important role.

2. Quantum Gates & Its Operation

Qubits: Similar to the bit in classical computing, the qubit (quantum bit) is the fundamental unit of quantum computing. The values of bits are either 0 or 1. Put another way, a qubit's value can range from $|0\rangle$ to $|1\rangle$, or it can be somewhere in between. [25]. Qubit can also have respected by a **Bloch** sphere value of qubit may be on sphere**.** $|0\rangle$ & $|1\rangle$ can be recuperated in figure 1:

$|0\rangle$ qubit $\quad\quad$ $|1\rangle$ qubit

Figure 1: Qubit representation on sphere

A qubit is halfway between $|0\rangle$ and $|1\rangle$ if It is located anywhere along the Bloch sphere's equator.

Qubits can be produced via photonics. An isolated, trapped light ion is called a qubit. Trapped ion qubits are used in Honeywell and 1 on 0 quantum computers, while a superconductivity qubit is a current which runs freely at very low temperatures.

Quantum bits can be used as carriers for data in quantum computing. we already know that in classical bits has only two 0 or 1, whereas in quantum computing We represent quantum bits in a two-dimensional Hilbert space using Dirac notation.

$$|0\rangle = (1\ 0\ ),\quad |1\rangle = (0\ 1\ )$$

When Using the Hadamard gate on the $|0\rangle$ qubit, in Drac notation can be represented:

$$\frac{1}{\sqrt{2}}(\,|0\rangle + |1\rangle\,)$$

It means getting probability 0 is ½ & 1 is ½.

Quantum computing gates

In classical computing, output of gate is deterministic but in quantum computing the output of gates are probabilistic.it works on the concept of linear algebra, gates are equivalent to matrix multiplication. quantum gates operation are reversible, but class computing operation of gates \re not reversible. Single qubit can be represented by a probabilistic state:

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$$|q\rangle = \alpha|0\rangle + \beta|1\rangle = (\alpha\ \beta)$$

here α and β are the likelihood amplitude means

$$|\alpha^2| + |\beta^2| = 1$$

A single qubit is linear transformation a vector or it is matrix multiplication

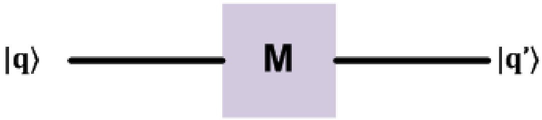


Figure 2: Quantum Gate

Where M = $[m_{00}\ m_{01}\ m_{10}\ m_{11}]$

In Dirac notation

$$|q'\rangle = M\,|q\rangle$$

Matrix is used in quantum computing must unitary

$$MM^t = I$$

$$M^t = M^{-1}$$

Means, every matrices are sued in quantum gates are unitary.

Single qubit Pauli X, Y, & Z Gates

Pauli X gates

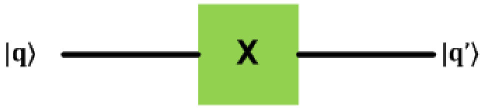


Figure 3: X Gate

$$\sigma_x = [0\ 1\ 1\ 0]$$

$$\sigma_x|0\rangle = [0\ 1\ 1\ 0][1\ 0] = [0\ 1] = |1\rangle$$

It means Pauli X gates transform qubit $|0\rangle$ to into $|1\rangle$, it rotates vectors along with X axis with π.

Pauli Z gates

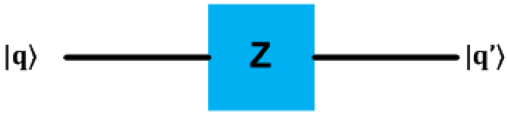


Figure 4: Z Gate

$$\sigma_z = [1\ 0\ 0\ \ -1]$$

$$\sigma_z|+\rangle = \sigma_z|-\rangle$$

Pauli Z gates rotating qubit by π along Z axis

Pauli Y Gates

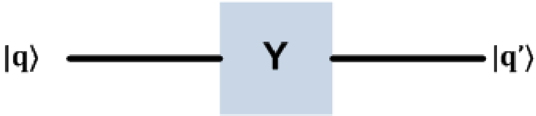


Figure 5: Y Gate

$$\sigma_y = [0 \ -i \ i \ 0 \ ]$$

Pauli Y gates rotates along with Y axis by π

Hadamard Gates H: This quantum gates converts a qubit into superposition states, it is the combination of $|0\rangle$ & $|1\rangle$.

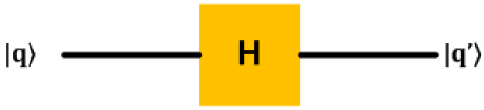
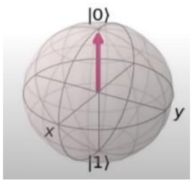


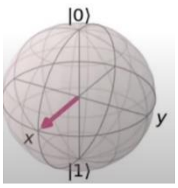
Figure 6: H (Hadamard) Gate

$$H = \frac{1}{\sqrt{2}}[1 \ 1 \ 1 \ -1 \ ]$$

When H gates are applied to $|0\rangle$:

$$\mathrm{H}\,|0\rangle = \frac{1}{\sqrt{2}}[1 \ 1 \ 1 \ -1 \ ][0 \ 1 \ ] = \frac{1}{\sqrt{2}}\ [1 \ 1 \ ]=\frac{1}{\sqrt{2}}\ (\ |0\rangle + \ |1\rangle \ )$$





Qubit in sphere $|0\rangle$ qubit in superposition

$$H\frac{1}{\sqrt{2}}\ (\ |0\rangle + \ |1\rangle \ ) \ = \ |0\rangle$$

Means Hadamard gate is use to transform Z based to X axis vice -versa qubits.

Quantum circuit makes from sequences of quantum gates. Example

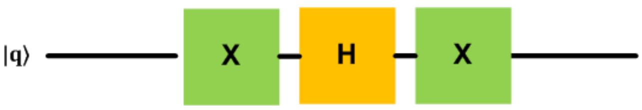


Figure 7: Quantum Circuit

Means quantum circuits matrices multiplication with

$$\sigma_{\varkappa}\mathrm{H}\sigma_{\varkappa} \ = \ [0 \ 1 \ 1 \ 0 \ ]\,\frac{1}{\sqrt{2}}[1 \ 1 \ 1 \ -1 \ ][0 \ 1 \ 1 \ 0 \ ]$$

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Control- X or CNOT gate (two qubits operation)

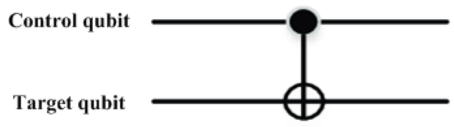


Figure 7: CNOT Gate

CNOT gate is two qubit operation, in this first qubits works as a qubit of control and 2nd Qubit functions as the intended object.

$$\text{CNOT} = [0\ 0\ 0 \quad 0\ 0\ 1\ 0 \quad 0\ 0\ 0 \quad 0 \quad 1 \quad 0\ 0\ 1 \quad 0], \quad |t\rangle = [a\ b\ c\ d\ ]$$

$$\text{CNOT}\ |t\rangle = [0\ 0\ 0 \quad 0\ 0\ 1\ 0 \quad 0\ 0\ 0 \quad 0 \quad 1 \quad 0\ 0\ 1 \quad 0] \quad [a\ b\ c\ d\ ] \quad = [a\ b\ d\ c\ ]$$

Suppose we target qubit |10⟩

$$\text{CNOT}\ |10\rangle = [0\ 0\ 0 \quad 0\ 0\ 1\ 0 \quad 0\ 0\ 0 \quad 0 \quad 1 \quad 0\ 0\ 1 \quad 0][0\ 0\ 1\ 0\ ] = [0\ 0\ 0\ 1\ ]$$

The target qubit in this qubit is |0⟩, while the control qubit is |1⟩.

Control (qubit)	Target(Qubit)	Output (Qubit)
0	0	0
0	1	0
1	0	1
1	1	0

Table 1: CNOT

It is very similar X-OR gate in classical computing.

CNOT gate changes the amplitude of |10⟩ & |11⟩ states, it is very similar to classical computing when we apply CNOT in Hadamard gate.

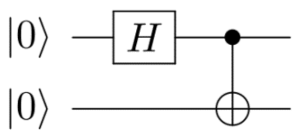


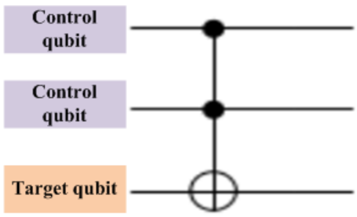
Figure 8: Hadamard with CNOT gate

$$\text{CNOT}\ |{+0}\rangle = \frac{1}{\sqrt{2}}(\ |10\rangle + |11\rangle\ )$$

This situation create entanglement.

Three Qubit Controlled (CNOT) or Toffoli Gate:. Toffoli gates can therefore be used to implement any typical classical circuit in a quantum computing environment. Two-qubit

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 1s (2025)



gates can also be used to implement the Toffoli gate.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

CCNOT matrix

INPUT			OUTPUT		
C QUBIT 1	C QUBIT 2	TARGET QUBIT	C QUBIT 1	C QUBIT 2	TARGET QUBIT
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

TABLE 2: CCNOT QUBIT

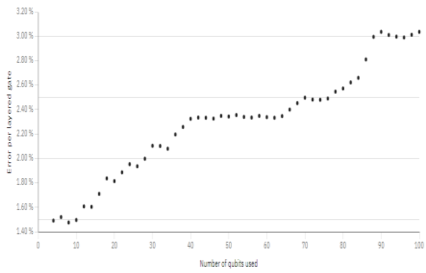
Table 2 depict that target will change when bot control qubits are $|1\rangle$, otherwise target qubit will remain same. In the table 2, C stand for control.

Gates	Qubit types	Operation	Real life applications
X	One qubit	Flips state of qubit	Qubit manipulation
Y	One Qubit	Phase flip	Qubit manipulation
Z	One Qubit	Phase flip	Qubit manipulation
H	One Qubit	Convert into superposition qubit	Key in creating superposition, used in quantum algorithms
CNOT	Two qubits	Flips target qubit for control is $\|1\rangle$	quantum entanglement, quantum error correction, and quantum teleportation.
CCNOT	Three Qubit	Flips target qubit for both control is $\|1\rangle$	reversible computing, logic gate operations,

Table 3: summary about quantum gates

3. IMPLEMENTATION OF X, Z, Y, H, CNOT & CCNOT GATES IN IBM-KYOTO QUANTUM COMPUTER

IBM -Kyoto Quantum computing details [27] on which we run quantum computing gates, these are details information about this quantum computer.

QUANTUM COMPUTER DETAILS	
ERROR PER LAYERED GATE	

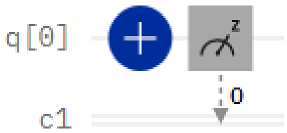
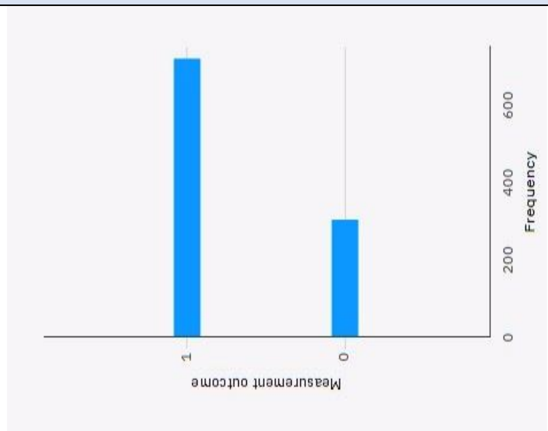
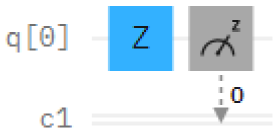
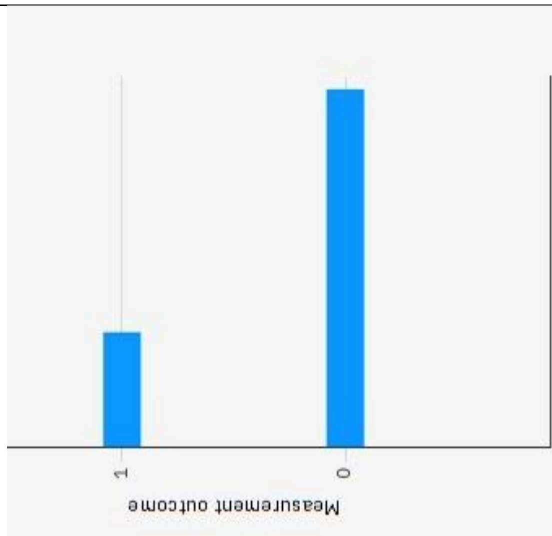
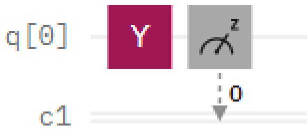
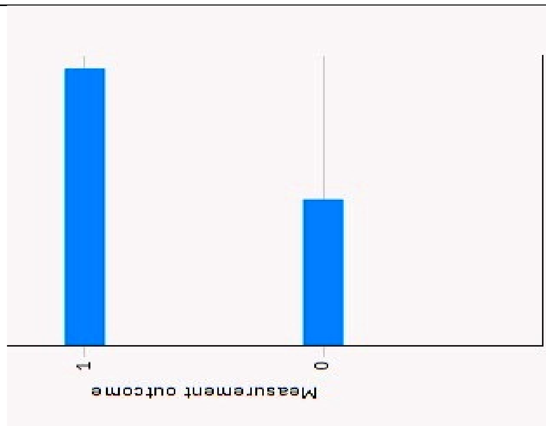
X gate is working like not gate in classical computing but the output on real quantum computer is in the form probabilistic form, we have run 1024 times output of qubit $|1\rangle = 720$ & $|0\rangle = 304$ out 1024 times. It is happened due to error noise in quantum computer. In quantum circuits and the X gate is frequently employed for a number of functions, such as error correction, quantum algorithm implementation, and qubit initialization.

Pauli Z gates, when apply any qubit it rotates qubit by π along Z axis. when it run on quantum computing it shows $|1\rangle = 774$ & $|0\rangle = 250$ out 1024 times. In quantum circuits, the Z gate is employed for different tasks, including phase shifts, the creation of increasingly intricate quantum algorithms, and error-correcting codes.

Pauli Y gates revolve by π with respect to the Y axis. when it run on quantum computing it shows $|1\rangle = 670$ & $|0\rangle = 354$ out of 1024 times. The Y gate is a flexible tool in quantum computation that may be utilized for manipulating qubit states in a way that involves both real and imaginary components. It mixes bit-flipping and phase-flipping with a complex coefficient.

This quantum gate, which is the product of $|0\rangle$ & $|1\rangle$, transforms a qubit into superposition states. when it run on quantum computing it shows $|1\rangle = 482$ & $|0\rangle = 542$ out of 1024 times. In order to explore quantum parallelism and apply different quantum algorithms, the Hadamard gate is essential for generating superposition states in quantum computing.

CNOT gate is using two qubits; in this gate first qubit works as control qubit & second qubit works target. When it is implemented on quantum computing, $|00\rangle = 718$, $|01\rangle = 39$, $|10\rangle = 75$

SNO	GATES	CIRCUIT DIAGRAM	OUTPUT ON IBM-KYOTO	RFREQUENCY OUTCOMES OUT OF 1024
1	X			|1⟩ = 720 |0⟩ = 304
2	Z			|1⟩ = 774 |0⟩ = 250
3	Y			|1⟩ = 670 |0⟩ = 354

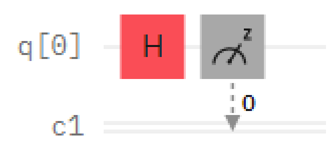
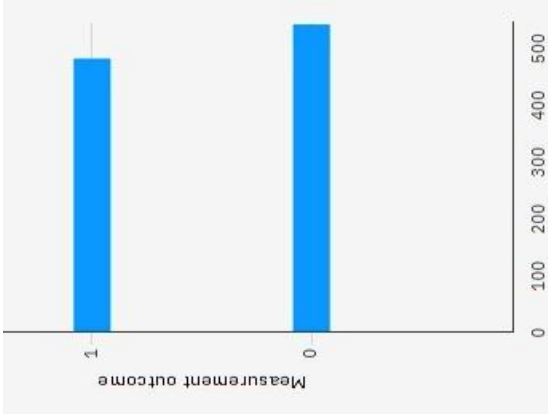
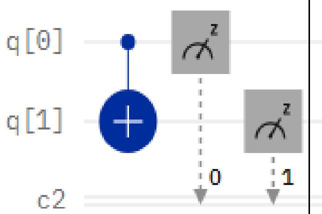
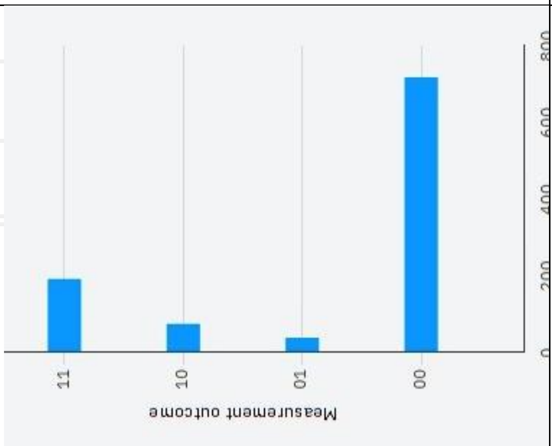
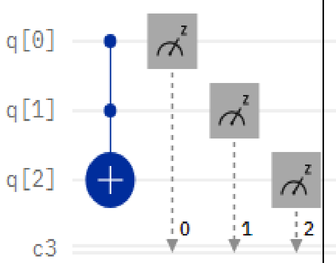
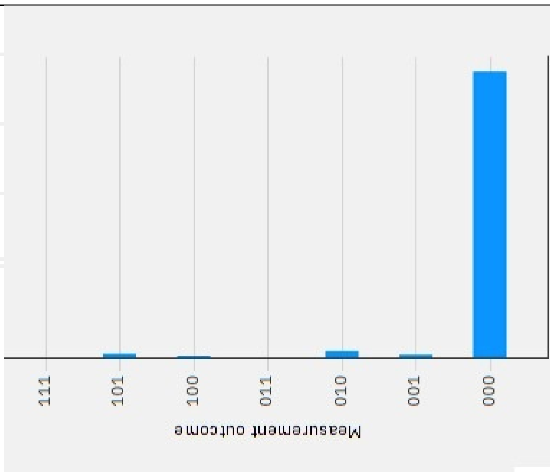
4	H			|1⟩ = 482 |0⟩ = 542
5	CNOT			|00⟩ = 718 |01⟩ = 39 |10⟩ = 75 |11⟩ = 192
6	CCNOT			|000⟩ = 953 |001⟩ = 14 |010⟩ = 26 |011⟩ = 03 |100⟩ = 10 |101⟩ = 17 |111⟩ = 01

Table 5: Gates output

|11⟩=192.For many Quantum computing jobs, especially those involving entanglement and quantum information processing, CNOT quantum gate is important two qubits. in that that permits conditional operations.

One sort of controlled CNOT (CCNOT) gate is the Toffoli gate, which consists of two control qubits and one target qubit. Stated differently, if both the first and second qubits are |1⟩, then the target qubit, or third qubit, will be inverted. An n-Toffoli gate, a CNOT gate with n controlling qubits, is occasionally encountered. The Toffoli gate can replicate any classical gate. We have designed CCNOT on quantum computer & run after that output is |000⟩ = 953 |001⟩ = 14, |010⟩ = 26, |011⟩ = 03,|100⟩ = 10, |101⟩ = 17,|111⟩ = 01

4. Comparison with Research Work

Gate Analysis	Matrix Representation	Platform Used	Focus	Contribution	Comparison with Current Work	References
X, Y, Z, H, CNOT, CCNOT	Complete matrix-based analysis for each gate	IBM Kyoto, IBM 5-qubit	Comprehensive matrix-based gate analysis for universal gates	Demonstrates matrix operations for quantum gates with IBM platform integration	Integrates matrix operations, IBM Kyoto performance, multi-gate analysis, and gate interdependencies	Current work
CNOT, CCNOT	Basic matrix analysis of QFT gates	IBM 16-qubit	QFT and error rates in multi-qubit gates	Studied QFT-specific gates and errors	Focuses on specific algorithm gates, not universal gates	[36]
Hadamard gate	Extensive matrix-based analysis	Rigetti Aspen	Matrix representation in quantum state evolution	Detailed matrix-based gate evolution	Concentrates on Hadamard only; no multi-gate integration	[37]
X, Y, Z, H gates	Algebraic function analysis	IBM Q Experience	Quantum algorithms with universal gates	Algebraic function analysis of universal gates	No use of IBM Kyoto and matrix comparison across all gates	[38]
X, Y, Z gates	Limited matrix focus on fidelity	IBM 5-qubit	Gate fidelity improvement in quantum circuits	Emphasis on gate fidelity	Lacks comprehensive matrix analysis of gate mechanics	[39]

Table 6: comparison with other research work

The comparison table highlights that implementing and analysing fundamental quantum gates—X, Y, Z, Hadamard, CNOT, and CCNOT—through matrix representations is essential for optimizing quantum computing performance. Previous research has predominantly focused on individual gate performance, error rates, or fidelity improvements across various quantum hardware, such as IBM's quantum processors and Rigetti's Aspen. Our study provides a unified, comprehensive approach to gate analysis by leveraging matrix theory, demonstrating improved consistency in gate performance, particularly for multi-qubit operations. This focus on matrix-based optimization confirms its importance in advancing quantum computing capabilities and efficiency, addressing computation complexities and operational fidelity more effectively than prior implementations.

5. Conclusion

This paper analyses the foundational role of fundamental quantum gates, such as X, Y, Z, and Hadamard (H), as integral components in executing a range of quantum algorithms and operations. IBM's quantum systems, including the Kyoto quantum computer, demonstrate both the design sophistication and operational efficacy of these gates. In particular, the Y gate enables intricate quantum state manipulations essential for advanced algorithms, while the Hadamard gate initiates qubits into superposition states, a vital process for achieving quantum parallelism and interference. The two-qubit CNOT and three-qubit CCNOT gates are pivotal in establishing entanglement and facilitating complex quantum operations such as error correction, both of which are critical for the reliable functioning of quantum circuits.

Matrices play a crucial role in this context, as they provide the mathematical framework underlying each gate's functionality. Through matrix operations, quantum gates transform quantum states, making them indispensable for constructing and analyzing quantum algorithms. This study's analysis of gate matrices not only illustrates the algebraic operations performed by these gates but also highlights how matrix theory contributes to the scalability and efficiency of quantum computing. Efficient implementation of these gates, informed by matrix representations, is therefore central to the progression of quantum technology and the future of computational problem-solving in complex domains.

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