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Minimal and Maximal $g\eta$-Continuous Functions in Topological Spaces

D. Subbulakshmi

Associate Professor, Department of Mathematics, RVS College of Arts and Science, Tamil Nadu, India, Email: subbulakshmi169@gmail.com.

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Abstract:
The concept of maximal and minimal gη-continuous functions and some new results are given.

Keywords: minimal gη-continuous, maximal gη-continuous functions.

1. INTRODUCTION

Levine [2] proposed some properties in 1963, s-open sets were introduced into topological spaces. In 1984, Andrijevic [1] described some of the topological properties of alpha sets. The concept of generalized closed sets in topological spaces was presented by Norman Levine [3]. [7, 8, 9] introduced the concept of $g\eta$-closed sets and $g\eta$-continuous functions and their various characterizations.

Nakaoka and Oda developed two subclasses of open sets: maximal and minimal open sets [4,5,6]. Later, numerous authors concentrated on this subject, developing the concept of minimal and maximal open sets. Following these improvements, we investigate minimal and maximal gη-continuous functions.

2. MINIMAL $g\eta$-CONTINUOUS FUNCTIONS

This section introduces and establishes various properties of minimal $g\eta$-continuous topological spaces.

Definition 2.1[10]: A minimal gη-open is a proper, nonempty gη-open subset U' of]= (X',τ') if any of its $g\eta$-opens are U' is φ' or U'.

Definition 2.2: If $\dot{a}^{-1}(M')$ is a $g\eta$-open in (X',τ') for any minimal open M' in (Y',σ') then a function $\dot{a}:(X',\tau') \to (Y',\sigma')$ is minimal $g\eta$-continuous.

Example 2.3: Let$X' = Y' = \{e', f', g'\}$, $\tau' = \{X', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{f'\}, \{e', f'\}\}$. Define $\dot{a}:(X',\tau') \to (Y',\sigma')$ by $\dot{a}(e') = f', \dot{a}(f') = g', \dot{a}(g') = e'$. Here $\{e'\}, \{f'\}$ are minimal open in(Y',σ'). Therefore $\dot{a}$ is minimal $g\eta$-continuous.

Theorem 2.4: All $g\eta$-continuonus is minimal $g\eta$-continuous.

Proof: Let M' be a minimal open in (Y',σ') and $\dot{a}:(X',\tau') \to (Y',\sigma')$ be a $g\eta$-contenuous. M' Is an open in (Y',σ') as all minimal opens are open. Consequently, $\dot{a}$ is $g\eta$-continuous, $\dot{a}$ is minimal $g\eta$-continuous as a result.

However, the converse of this theorem is not necessarily true, as shown by the following example.

Example 2.5: Let $X' = Y' = \{e', f', g'\}$, $\tau' = \{X', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{f'\}, \{e', f'\}\}$. Define $\dot{a}: (X', \tau') \to (Y', \sigma')$ as $\dot{a}(e') = f', \dot{a}(f') = g'$, $\dot{a}(g') = e'$. Then $\dot{a}^{-1}(e') = g', \dot{a}^{-1}(f') = e'$. Therefore $\dot{a}$ is minimal $g\eta$-continuous. Hence $\dot{a}$ is not $g\eta$-continuous.

Theorem 2.6: If any maximal closed in's inverse image $Y', \sigma')$ is $g\eta$-closed in (X', τ') then let $\dot{a}: (X', \tau') \to (Y', \sigma')$ be minimal $g\eta$-continuous

Proof: $\dot{a}$ be a minimal $g\eta$-continuous and let N' be a maximal closed in (Y', σ'). $(Y', \sigma') - N'$ is a minimal open in (Y', σ'). When $\dot{a}$ is minimal $g\eta$-continuous, $\dot{a}^{-1}((Y', \sigma') - N')$ is $g\eta$-open in (X', τ'). So $\dot{a}^{-1}((Y', \sigma') - N') = (X', \tau') - \dot{a}^{-1}(N')$ is $g\eta$-open in (X', τ'). $\dot{a}^{-1}(N')$ is $g\eta$-closed in (X', τ').

On the contrary, $\dot{a}^{-1}(N')$ is $g\eta$-closed in (X', τ') for all maximal closed N' in (Y', σ'). Let M' be a minimal open in (Y', σ'). So $\dot{a}^{-1}((Y', \sigma') - M') = (X', \tau') - \dot{a}^{-1}(M')$ is $g\eta$-closed in (X', τ'). Therefore $\dot{a}$ is minimal $g\eta$-continuous.

Theorem 2.7: Let $\dot{a}: (X', \tau') \to (Y', \sigma')$ is minimal $g\eta$-continuous iff, $q' \in (X', \tau')$ and minimal open M' in (Y', σ') holding $\dot{a}(q')$, there is $g\eta$-open N' in (X', τ') like that $q' \in N'$, $\dot{a}(N') \subset M'$.

Proof: M' be minimal open in (Y', σ') holding $\dot{a}(q')$, $q' \in N'$ where N' is $g\eta$-open in (X', τ'). Since $\dot{a}$ is minimal $g\eta$-continuous, $\dot{a}^{-1}(N')$ is $g\eta$-open in(X', τ'). And $N' = \dot{a}^{-1}(M')$. Therefore $\dot{a}(N') \subset M'$.

On the contrary, M' be minimal open in (Y', σ'). Then there is $g\eta$-open N' in (X', τ'), such that $q' \in N'$, $\dot{a}(q') \in \dot{a}(N') \subset M'$, $q \in \dot{a}^{-1}(\dot{a}(N')) \subset \dot{a}^{-1}(M')$. Therefore $\dot{a}^{-1}(M')$ is $g\eta$-open in (X', τ'). Hence $\dot{a}$ is minimal $g\eta$-continuous.

Theorem 2.8: Let B' be a subset of (X', τ'). If $\dot{a}: (X', \tau') \to (Y', \sigma')$ is minimal $g\eta$-continuous then the restriction function $\dot{a} \mid B': B' \to (Y', \sigma')$ is minimal $g\eta$-continuous. where B' has the relative topology.

Proof: Assume A' is subset of (X', τ') and M' be minimal open in (Y', σ'). When $\dot{a}$ is minimal $g\eta$-continuous, $\dot{a}^{-1}(M')$ is $g\eta$-open in (X', τ'). So $(\dot{a}|B')^{-1}(M') = B' \cap \dot{a}^{-1}(M')$. Hence $B' \cap \dot{a}^{-1}(M')$ is $g\eta$-open in B'. Thus $\dot{a}|B'$ is minimal $g\eta$-continuous.

Remark 2.9: Minimal $g\eta$-continuous do not always have to be minimal $g\eta$-continuous in composition.

Theorem 2.10: If $\dot{a}: (X', \tau') \to (Y', \sigma')$ and $\dot{b}: (Y', \sigma') \to (Z', \mu')$ be $g\eta$-continuous and minimal $g\eta$-continuous so $\dot{b} o \dot{a}: (X', \tau') \to (Z', \mu')$ is minimal $g\eta$-continuous.

Proof: A' be minimal open in(Z', μ'), when $\dot{b}$ is minimal $g\eta$-continuous, $\dot{b}^{-1}(A)$ is $g\eta$-open in (Y', σ'). So $\dot{a}$ is $g\eta$-continuous, $\dot{a}^{-1}(\dot{b}^{-1}(A)) = (\dot{b} o \dot{a})^{-1}(A)$ is $g\eta$-open in (X', τ'). Hence $\dot{b} o \dot{a}$ be $g\eta$-continuous.

3. MAXIMAL $g\eta$-CONTINUOUS FUNCTIONS

Introducing maximal $g\eta$-continuous is the goal of this section. Examples are used to obtain some attributes of such functions.

Definition 3.1: A Maximal $g\eta$-open is a proper nonempty $g\eta$-open subset U' of (X', τ') if any of its $g\eta$-opens are U' is either (X', τ') or U'.

Definition 3.2: If ȧ$^{-1}(M')$ is a $g\eta$-open in (X', τ') for any maximal open M' in (Y', σ') then a function ȧ$: (X', \tau') \to (Y', \sigma')$ is maximal $g\eta$-continuous.

Example 3.3: Let $X' = Y' = \{e', f', g'\}$, $\tau' = \{X', \varphi', \{e'\}, \{f', g'\}\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$. Define ȧ$: (X', \tau') \to (Y', \sigma')$ by ȧ$(e') = e'$, ȧ$(f') = g'$, ȧ$(g') = f'$. Here $\{e', g'\}$ is maximal open in (Y', σ'). Therefore ȧ is maximal $g\eta$-continuous.

Theorem 3.4: All $g\eta$-continuous is maximal $g\eta$-continuous.

Proof: Let M' be a maximal open in (Y', σ') and ȧ$: (X', \tau') \to (Y', \sigma')$ be a $g\eta$-continuous, M' be a maximal open in (Y', σ'). All maximal open is an open when M' is an open in (Y', σ'). So ȧ is $g\eta$-continuous. Therefore ȧ is maximal $g\eta$-continuous.

The converse of the preceding theorem does not necessarily have to be true, as demonstrated by the example that follows.

Example 3.5: Take $X' = Y' = \{e', f', g'\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$, $\tau' = \{X', \varphi', \{e'\}, \{f'\}, \{e', f'\}\}$. Assign ȧ$: (X', \tau') \to (Y', \sigma')$ by ȧ$(e') = f'$, ȧ$(f') = e'$, ȧ$(g') = g'$. Then ȧ$^{-1}(\{e', g'\}) = \{f', g'\}$ is $g\eta$-open in (X', τ'). Consequently ȧ is maximal $g\eta$-continuous. So ȧ is not $g\eta$-continuous.

Remark 3.6: There is independence between maximal-continuous and minimal-continuous.

Example 3.7: Take $X' = Y' = \{e', f', g'\}$, $\tau' = \{X', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{f', g'\}\}$. Assign ȧ$: (X', \tau') \to (Y', \sigma')$ by ȧ$(e) = f$, ȧ$(f) = e$, ȧ$(g) = g$. Now $\{f', g'\}$ is maximal open in $(Y', \sigma'))$. For that reason ȧ is maximal $g\eta$-continuous. Consequently, ȧ is not minimal $g\eta$-continuous.

Example 3.8: Take $X' = Y' = \{e', f', g'\}$, $\tau' = \{X', \varphi', \{e'\}\}$, $\sigma' = \{Y', \varphi', \{e'\}, \{g'\}, \{e', g'\}\}$. Assign ȧ$: (X', \tau') \to (Y', \sigma')$ by ȧ$(e) = f'$, ȧ$(f') = e'$, ȧ$(g') = g'$. Now $\{e'\}, \{g'\}$ is minimal open in (Y', σ'). Consequently, ȧ is minimal $g\eta$-continuous and ȧ is not maximal $g\eta$-continuous.

Theorem 3.9: Assign ȧ$: (X', \tau') \to (Y', \sigma')$ is maximal $g\eta$-continuous iff the inverse image of each minimal closed in $(Y', \sigma')$is a $g\eta$-closed in (X', τ').

Proof: Take ȧ be a maximal $g\eta$-continuous, N' be a minimal closed in (Y', σ'). So $(Y', \sigma') - N'$ is a maximal open in (Y', σ'). As ȧ is maximal $g\eta$-continuous, ȧ$^{-1}((Y', \sigma') - N')$ is an $g\eta$-open in (X', τ'). But ȧ$^{-1}((Y', \sigma') - N') = (X', \tau') -$ ȧ$^{-1}(N')$ is $g\eta$-open in (X', τ'). Consequently, ȧ$^{-1}(N')$ is $g\eta$-closed in (X', τ').

On the contrary, ȧ$^{-1}(N')$ is $g\eta$-closed in (X', τ') for all minimal closed N' in (Y', σ'). Let M' be maximal open in (Y', σ'). So $(Y', \sigma') - M'$ is minimal closed in (Y', σ'). But

$\dot{a}^{-1}((Y',\sigma') - M') = (X',\tau') - \dot{a}^{-1}(M')$ is $g\eta$-closed in (X',τ'). Hence $\dot{a}^{-1}(M')$ is $g\eta$-open in (X',τ'). Thus $\dot{a}$ is maximal $g\eta$-continuous.

Theorem 3.10: Assign $\dot{a}: (X',\tau') \to (Y',\sigma')$ is maximal $g\eta$-continuous iff $q' \in (X',\tau')$ and maximal open M' in (Y',σ') holding $\dot{a}(q')$, there is $g\eta$-open N' in (X',τ') in order that $q' \in N'$, $\dot{a}(N') \subset M'$.

Proof: Take M' be maximal open in (Y',σ') holding $\dot{a}(q')$, $q' \in N'$ where N' is an $g\eta$-open in (X',τ'), $\dot{a}$ is maximal $g\eta$-continuous and $\dot{a}^{-1}(N')$ is $g\eta$-open in (X',τ'). Then $N' = \dot{a}^{-1}(M')$. So $\dot{a}(N') \subset M'$.

On the contrary, M' be a maximal open in (Y',σ'). Then there is $g\eta$-open N' in (X',τ'), $q' \in N'$, $\dot{a}(q') \in \dot{a}(N') \subset M'$, $q' \in \dot{a}^{-1}(\dot{a}(N')) \subset \dot{a}^{-1}(M')$. So $\dot{a}^{-1}(M')$ is $g\eta$-open in (X',τ'). Consequently, $\dot{a}$ is maximal $g\eta$-continuous.

Theorem 3.11: Take B' be a non-empty subset of (X',τ') and $\dot{a}: (X',\tau') \to (Y',\sigma')$ is maximal $g\eta$-continuous then the restriction $\dot{a} \mid B': B' \to Y'$ is maximal $g\eta$-continuous. When B' has the relative topology.

Proof: Conclude A' is a non-empty subset of a $(X',\tau')$and M' be any maximal open in (Y',σ'). So $\dot{a}$ is maximal $g\eta$-continuous, $\dot{a}^{-1}(M')$ is $g\eta$-open in (X',τ'). Using relative topology, $(\dot{a} \mid B')^{-1}(M') = B' \cap \dot{a}^{-1}(M')$. So $B' \cap \dot{a}^{-1}(M')$ is $g\eta$-open in B'. Thus $\dot{a} \mid B'$ is maximal $g\eta$-continuous.

Remark 3.12: Maximal $g\eta$-continuous functions do not always have to be maximal $g\eta$-continuous in composition.

Theorem 3.13: The maximal gη-continuousness is $\dot{b} o \dot{a}: (X',\tau') \to (Z',\mu')$. If $\dot{a}: (X',\tau') \to (Y',\sigma')$ is gη-continuous and $\dot{b}: (Y',\sigma') \to (Z',\mu')$ is maximal gη-continuous.

Proof: Since $\dot{b}$ is a maximal $g\eta$-continuous, $\dot{b}^{-1}(A')$ is $g\eta$-open in (Y',σ') , assuming that A' be a maximal open in (Z',μ'). Every open is $g\eta$-open. But $\dot{a}$ is $g\eta$-continuous, $\dot{a}^{-1}(\dot{b}^{-1}(A')) = (\dot{b} o \dot{a})^{-1}(A')$ is $g\eta$-open in (X',τ'). Hence $\dot{b} o \dot{a}$ is $g\eta$-continuous.

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