Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 3s (2025)

A Novel case on Fuzzy Resolving Sets on Interval-valued Fuzzy Graph and its Application on Signal Processing Unit

R. Shanmugapriya[1] & Vasuki M[1]

[1,1]Department of Mathematics, Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology, Chennai.

Email id: spriyasathish11@gmail.com[1] & vasukimani1997@gmail.com[1]

Article History:

Received: *23-09-2024*

Revised: *09-11-2024*

Accepted: *27-11-2024*

Abstract:

A fuzzy graph generalisation known as an interval-valued fuzzy graph (IVFG) reflects uncertainty more flexibly by representing the membership values of vertices and edges as intervals rather than fixed numbers. We have defined Fuzzy Resolving Set to interval-valued fuzzy graphs such as interval-valued fuzzy resolving set and interval-valued fuzzy super resolving set. We also derived the properties of isomorphism in this topic and explained an application based on them.

Keywords: Fuzzy Graph, Fuzzy resolving set, Interval-valued fuzzy graph, interval valued fuzzy resolving set

1. Introduction

The degree of certainty or uncertainty can be expressed using degrees of membership in fuzzy sets. Instead of dealing with precise distances, we deal with fuzzy distances or fuzzy sets of distances in an FRS. In summary, an FRS provides a more flexible framework for resolving network vertices when precise information is not easily accessible. It is an extension of traditional resolving sets that accounts for measurement uncertainty in the vertex-to-vertex distance. FRS can be used to model the transmission of illnesses when precise distance measurements between sites or between individuals are difficult to obtain. An Interval-Valued Fuzzy Graph (IVFG) is a sophisticated mathematical framework that models uncertainty in real-world systems by fusing fuzzy logic and graph theory. IVFGs represent uncertainty in the degree of membership of vertices and edges using intervals, as opposed to standard graphs or fuzzy graphs, where memberships are discrete or single values.

While Euler announced graph theory, Zadeh[13] evaluated in 1965. Kauffman developed the fuzzy set, which served as the foundation for the fuzzy graph, in 1973. Moderson and associates [6] created a fuzzy graph that has numerous uses. Resolvability and an upper dimension of graphs were first suggested by Chartrand[2] in 2000. Shanmugapriya and Mary Jiny [9] subsequently extended this concept to resolvability in fuzzy graphs. Additionally, they both introduced modified FRN and FRS characteristics. Furthermore, Atanassov[1] extended the FG to create an intuitive fuzzy graph (IFG). An interval-valued fuzzy graph has been found by Muhammad akram and A. Dudek in 2011[7]. It has been further elaborated by Pramanik and others in 2020[10]. Also, the properties of interval-valued fuzzy graph has been developed by Xiaoli Qiang and others in the year 2022[12]. They also collaborated with numerous others to create a number of research works on fuzzy graphs, including studies on the size, order, and strong and weak domination of fuzzy graphs. The parameter of clustering the sets is used in the development of numerous applications. In 2023, fuzzy resolving domination set has been introduced by Shanmugapriya and Co [8][11].

Let G be a connected simple finite FG with the number of vertices greater than 3. Standard fuzzy graph and interval-valued fuzzy graph definitions were covered in section 2, whereas IVFRS, IVFSRS and its theorems were presented in section 3 and 4. We have created an application based on the idea in section 5. We are keen to explore into further fuzzy resolving set-related subjects in the future studies.

Names	**Notations**
Fuzzy Resolving Set	FRS
Interval-valued Fuzzy Graph	IVFG
Interval-valued Fuzzy Resolving Set	IVFRS
Interval-valued Fuzzy Super Resolving Set	IVFSRS
Interval-valued Fuzzy Resolving Matrix	IVFRM
Interval-valued Fuzzy Super Resolving Matrix	IVFSRM

Table 1

2. Preliminaries

In this section, we have defined a basic definition to deal with the IVFRS.

Definition 2.1.

A FG $G(V, \sigma, \mu)$ where μ is a symmetric fuzzy relation on $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(x, z) \leq \sigma(x) \wedge \sigma(z)$, $\forall\, x, z \in V$.

Definition 2.2.

Consider an ordered fuzzy subset $H = \{(u_1, \sigma(u_1)), (u_2, \sigma(u_2)), \ldots (u_k, \sigma(u_k))\}$, $|H| \geq 2$, the representation of $(z, \sigma(z)) \epsilon \sigma - H = \{(u_{k+1}, \sigma(u_{k+1})), (u_{k+2}, \sigma(u_{k+2})), \ldots (u_n, \sigma(u_n))\}$ and $\{w(z, u_1), w(z, u_2), \ldots w(z, u_k)\}$, where the significance of the link between z and y is denoted by $w(z, y)$. The set is called a FRS if every pair of items in the fuzzy subset H has a different representation with respect to H of G. The FRN is represented as Fr(G), the minimal cardinality of FRS.

Definition 2.3.

A fuzzy graph G's FRS is considered a fuzzy super resolving set (FSRS) if any two of its elements have unique representations with respect to H. We also write the FSRN, denoted by Sr(G), and the super resolving matrix, denoted by $S_{n \times k}$, which is the lowest cardinality of a FG G.

Definition 2.4.

A set C of vertices V is a dominating set if every vertex of $V \backslash C$ is adjacent to any vertex of C. The smallest cardinality of this set is called the fuzzy domination number(FDN) and it is denoted by $F_\gamma(G)$.

Definition 2.5

An interval-valued fuzzy relation Q is a mapping, $Q: XxY \rightarrow D\,[0,1]$ such that $Q\,(x, y) = [Q^{-}\,(x, y), Q^{+}\,(x, y)] \in D\,[0,1]$ for all pairs $(x, y) \epsilon XxY$.

Definition 2.6

We define $G = [E, F]$, where $E = [\mu_E^{-}, \mu_E^{+}]$ and $F = [\mu_F^{-}, \mu_F^{+}]$ is called an interval-valued fuzzy graph (IVFG), if E is an interval-valued fuzzy set on V and F is an interval-valued fuzzy relation such that

$\mu_F^{-}\,(ef) \leq \min\,(\mu_E^{-}\,(e), \mu_E^{-}\,(f)), \mu_F^{+}\,(ef) \leq \min\,(\mu_E^{+}\,(e), \mu_E^{+}\,(f)),\ \forall\, ef \in E.$

Definition 2.7

The order and size of IVFG is defined by

$$O(G) = \sum_{r \in V} \frac{1 + \mu_E^{+}\,(r) - \mu_E^{-}\,(r)}{2}$$

$$\mathcal{S}(G) = \sum_{r \in V} \frac{1 + \mu_F^{+}\,(rq) - \mu_F^{-}\,(rq)}{2}$$

Definition 2.8

An IVFG is called complete if $\mu_F^{-}(ef) = \min\,(\mu_E^{-}\,(e), \mu_E^{-}\,(f))$, $\mu_F^{+}\,(ef) = \max\,(\mu_E^{+}\,(e), \mu_E^{+}\,(f))$, $\forall\, ef \in E$.

Definition 2.9

An isomorphism $\varphi: G_1 \rightarrow G_2$ is a bijective mapping of two IVFGs, $\varphi: X_1 \rightarrow X_2$, $\ni\ \mu_E^{-}\,(r) = \mu_E^{-'}(\varphi\,(r)), \mu_E^{+}\,(r) = \mu_E^{+'}\,(\varphi(r)), \mu_F^{-}\,(r, u) = \mu_F^{-'}\,(\varphi(r), \varphi(u)),\ \mu_F^{+}\,(r, u) = \mu_F^{+'}\,(\varphi(r), \varphi(u))\ \forall r, u \in E$.

Definition 2.10

A homomorphism $\varphi: G_1 \rightarrow G_2$ is a mapping $\varphi: X_1 \rightarrow X_2$ of two IVFGs such that $\mu_E^{-}\,(r) \leq \mu_E^{-'}(\varphi\,(r)),\ \mu_E^{+}\,(r) \leq \mu_E^{+'}\,(\varphi(r)), \mu_F^{-}\,(r, u) \leq \mu_F^{-'}\,(\varphi(r), \varphi(u)),\ \mu_F^{+}\,(r, u) \leq \mu_F^{+'}\,(\varphi(r), \varphi(u))\quad \forall r, u \in E$.

Definition 2.11

A co-weak isomorphism $\varphi: G_1 \rightarrow G_2$ is a bijective mapping, $\varphi: X_1 \rightarrow X_{2,}$ such that

(i) φ is homomorphism

(ii) $\mu_F^{-}\,(r, u) = \mu_F^{-'}\,(\varphi(r), \varphi(u)), \mu_F^{+}\,(r, u) = \mu_F^{+'}\,(\varphi(r), \varphi(u))\ \forall r, u \in E$.

3. Fuzzy Resolving Sets of Interval-valued Fuzzy Graphs

Consider a fuzzy subset $\mathscr{H}$, then the representation of $\mathscr{H}$ and $(u, \sigma^{-}(u), \sigma^{+}(u)) \in \sigma - \mathscr{H}$ are all distinct, where $\mathscr{H} = \{(x_1, \sigma^{-}(x_1),\ \sigma^{+}(x_1)), (x_2, \sigma^{-}(x_2),\ \sigma^{+}(x_2)), \ldots (x_k, \sigma^{-}(x_2), \sigma^{+}(x_2))\}$ and $\sigma - \mathscr{H} = \{(x_{k+1}, \sigma^{-}(x_{k+1}),\ \sigma^{+}(x_{k+1})), (x_{k+2}, \sigma^{-}(x_{k+2}),\ \sigma^{+}(x_{k+2})), \ldots (x_n, \sigma^{-}(x_n), \sigma^{+}(x_n))\}$, then $\mathscr{H}$ is known as an interval-valued fuzzy resolving set (IVFRS). The minimum cardinality of this concerned set is called as Interval-valued Resolving Number (IVRN), denoted by $\mathscr{J}r(G)$.

3.1 Definition

The representation of $\sigma - \mathscr{H}$ with respect to $\mathscr{H}$ is written as $[A_{ij}^{-}, A_{ij}^{+}]$, where

$$A_{ij}^{-} = [w^{-}(u_j, x_1), w^{-}(u_j, x_2), \dots w^{-}(u_j, x_k)],$$

$A_{ij}^{+} = [w^{+}(u_j, x_1), w^{+}(u_j, x_2), \dots w^{+}(u_j, x_k)]$ for $j = k+1, k+2, \dots n$ are written in a form $A_{n-k \times k}$. This matrix is called Interval-valued Fuzzy Resolving Matrix(IVFRM).

3.2 Illustration

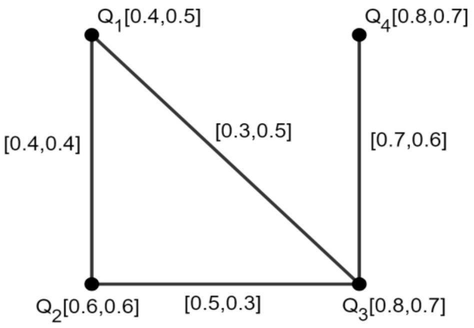

Fig 1 : IVFG

μ_E	Q_1	Q_2	Q_3	Q_4
μ_E^{-}	0.4	0.6	0.8	0.8
μ_E^{+}	0.5	0.6	0.7	0.7

μ_F	Q_1Q_2	Q_2Q_3	Q_1Q_3	Q_3Q_4
μ_F^{-}	0.4	0.5	0.3	0.7
μ_F^{+}	0.4	0.3	0.5	0.6

Let $\mathscr{H} = \{Q_1, Q_3\}$, then $\sigma - \mathscr{H} = \{Q_2, Q_4\}$

where $Q_1 = \{\sigma_1^{-}, \sigma_1^{+}\}, Q_2 = \{\sigma_2^{-}, \sigma_2^{+}\},\ Q_3 = \{\sigma_3^{-}, \sigma_3^{+}\}\ and\ Q_4 = \{\sigma_4^{-}, \sigma_4^{+}\}$

then, $\sigma_2^{-}/\mathscr{H} = \{\mu^{\infty}(\sigma_2^{-}, \sigma_1^{-}), \mu^{\infty}(\sigma_2^{-}, \sigma_3^{-})\} = (0.4, 0.5)$

$$\sigma_2^{+}/\mathscr{H} = \{\mu^{\infty}(\sigma_2^{+}, \sigma_1^{+}), \mu^{\infty}(\sigma_2^{+}, \sigma_3^{+})\} = (0.4, 0.4)$$

$$\sigma_4^{-}/\mathscr{H} = \{\mu^{\infty}(\sigma_4^{-}, \sigma_1^{-}), \mu^{\infty}(\sigma_4^{-}, \sigma_3^{-})\} = (0.4, 0.7)$$

$$\sigma_4^{+}/\mathscr{H} = \{\mu^{\infty}(\sigma_4^{+}, \sigma_1^{+}), \mu^{\infty}(\sigma_4^{+}, \sigma_3^{+})\} = (0.5, 0.6)$$

Therefore, $A_{ij}^{-} = \begin{bmatrix} 0.4 & 0.5 \\ 0.4 & 0.7 \end{bmatrix}, A_{ij}^{+} = \begin{bmatrix} 0.4 & 0.4 \\ 0.5 & 0.6 \end{bmatrix}$

Hence, the values are all distinct

Therefore, $\mathscr{J}r(G) = 2$

3.3 Theorem

If G and G′ are isomorphic, then $\mathscr{J}r(G) = \mathscr{J}r(G')$

Proof:

Let us assume that $V = \{r_1, r_2, \ldots, r_n\}$, $\mathscr{J}r(G) = k$, and let $\mathscr{H} = \{(\sigma_1^-, \sigma_1^+), (\sigma_2^-, \sigma_2^+), \ldots, (\sigma_k^-, \sigma_k^+)$ is the corresponding FRS. We denote $(r_1, \sigma^-(r_1)) = \sigma_1^-$, $(r_1, \sigma^+(r_1)) = \sigma_1^+$, $(\varphi(r_1), \sigma^-(\varphi(r_1)) = \sigma_1^{-'}$, $(\varphi(r_1), \sigma^+(\varphi(r_1)) = \sigma_1^{+'}$. The representation, $\sigma_{k+i}^-/\mathscr{H} = (w^-(r_{k+i}, r_1), w^-(r_{k+i}, r_2), \ldots, w^-(r_{k+i}, r_k)$,

$\sigma_{k+i}^+/\mathscr{H} = (w^+(r_{k+i}, r_1), w^+(r_{k+i}, r_2), \ldots, w^+(r_{k+i}, r_k)$ all are distinct, $\forall\, i = 1,2, \ldots n - k$. If G and G′ are isomorphic, then ∃ a bijection $\varphi: V \to V'$ which satisfies, $\mu_E^-(r) = \mu_E^{-'}(\varphi(r))$, $\mu_E^+(r) = \mu_E^{+'}(\varphi(r))$, $\mu_F^-(r,u) = \mu_F^{-'}(\varphi(r), \varphi(u))$ and $\mu_F^+(r,u) = \mu_F^{+'}(\varphi(r), \varphi(u))$ $\forall r, u \in E$. Now, we define $\mathscr{H}_1 = \{(\sigma_1^{-'}, \sigma_1^{+'}), (\sigma_2^{-'}, \sigma_2^{+'}), \ldots, (\sigma_k^{-'}, \sigma_k^{+'})\}$ for $i = 1,2, \ldots n - k$,

$$\begin{aligned}
\sigma_{k+i}^{-'}/\mathscr{H} &= \{w^-(\varphi(r_{k+i}), \varphi(r_1)), w^-(\varphi(r_{k+i}), \varphi(r_2)), \ldots, w^-(\varphi(r_{k+i}), \varphi(r_k)\} \\
&= \{((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_1)), ((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_2)), \ldots \\
&\qquad ((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_k)) \\
&= \mu^-(r_{k+i}, r_1), \mu^-(r_{k+i}, r_2), \ldots, \mu^-(r_{k+i}, r_k) \\
&= w^-(r_{k+i}, r_1), w^-(r_{k+i}, r_2), \ldots, w^-(r_{k+i}, r_k)
\end{aligned}$$

$$\begin{aligned}
\sigma_{k+i}^{+'}/\mathscr{H} &= \{w^+(\varphi(r_{k+i}), \varphi(r_1)), w^+(\varphi(r_{k+i}), \varphi(r_2)), \ldots, w^{-+}(\varphi(r_{k+i}), \varphi(r_k)\} \\
&= \{((\mu^{+'})^\infty(\varphi(r_{k+i}), (\mu^{+'})^\infty(\varphi(r_1)), ((\mu^{+'})^\infty(\varphi(r_{k+i}), (\mu^{+'})^\infty(\varphi(r_2)), \ldots \\
&\qquad ((\mu^{+'})^\infty(\varphi(r_{k+i}), (\mu^{+'})^\infty(\varphi(r_k)) \\
&= \mu^+(r_{k+i}, r_1), \mu^+(r_{k+i}, r_2), \ldots, \mu^+(r_{k+i}, r_k) \\
&= w^+(r_{k+i}, r_1), w^+(r_{k+i}, r_2), \ldots, w^+(r_{k+i}, r_k)
\end{aligned}$$

which are all distinct for $i = 1$ to $n = k$. Therefore, $\mathscr{H}_1$ is the IVFRS of G′ and $|\mathscr{H}_1| = k$. Now to prove that $\mathscr{H}_1$ is the minimum IVFRS OF G’. Assume that there exists a FRS R of G’ such that $R = \{(\sigma_1^-, \sigma_1^+), (\sigma_2^-, \sigma_2^+), \ldots, (\sigma_{k_1}^-, \sigma_{k_1}^+)\}$ and $|\mathscr{H}_1| = k > |R| = k_1$, then σ_{k+i}^-/R and σ_{k+i}^+/R are all distinct for $n = 1, 2, \ldots, n - k_1$. Let $R_1 = \{(\sigma_1^-, \sigma_1^+), (\sigma_2^-, \sigma_2^+), \ldots, (\sigma_{k_1}^-, \sigma_{k_1}^+)\}$,

$$\begin{aligned}
\sigma_{k+i}^-/R_1 &= w^-(r_{k+i}, r_1), w^-(r_{k+i}, r_2), \ldots, w^-(r_{k+i}, r_{k_1}) \\
&= \{(\mu^-)^\infty(r_{k+i}, r_1), (\mu^-)^\infty(r_{k+i}, r_2), \ldots (\mu^-)^\infty(r_{k+i}, r_{k_1})\} \\
&= \{((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_1)), ((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_2)), \ldots \\
&\qquad ((\mu^{-'})^\infty(\varphi(r_{k+i}), (\mu^{-'})^\infty(\varphi(r_{k_1}))
\end{aligned}$$

Similarly,

$$\begin{aligned}\sigma^{+}_{k+i}/R_1 &= w^{+}(r_{k+i}, r_1), w^{+}(r_{k+i}, r_2), \ldots, w^{+}(r_{k+i}, r_{k_1})\\ &= \{(\mu^{+})^{\infty}(r_{k+i}, r_1), (\mu^{+})^{\infty}(r_{k+i}, r_2), \ldots (\mu^{+})^{\infty}(r_{k+i}, r_{k_1})\}\\ &= \{((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_1)), ((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_2)), \ldots\\ &\qquad ((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_{k_1}))\end{aligned}$$

which are all distinct for $i = 1,2,\ldots n - k$. Therefore, R_1 is the IVFRS of G and $\mathscr{J}r(G)=k_1$ $[k_1 < k]$, which is a contradiction to our assumption that $\mathscr{J}r(G) = k$. Hence, $\mathscr{H}_1$ is the minimum IVRS of G′. Therefore,$\mathscr{J}r(G) \cong \mathscr{J}r(G')$.

3.4 Theorem

If G and G′ are homomorphic functions, then $\mathscr{J}r(G)$ and $\mathscr{J}r(G')$ are homomorphic.

Proof:

Let us assume that $V = \{r_1, r_2, \ldots, r_n\}$, $\mathscr{J}r(G)= k$, and let $\mathscr{H} = \{(\sigma_1^{-}, \sigma_1^{+}), (\sigma_2^{-}, \sigma_2^{+}), \ldots, (\sigma_k^{-}, \sigma_k^{+})$ is the corresponding FRS. We denote $(r_1, \sigma^{-}(r_1)) = \sigma_1^{-}$, $(r_1, \sigma^{+}(r_1)) = \sigma_1^{+}$, $(\varphi(r_1), \sigma^{-}(\varphi(r_1)) = \sigma_1^{-'}$, $(\varphi(r_1), \sigma^{+}(\varphi(r_1)) = \sigma_1^{+'}$. The representation, $\sigma^{-}_{k+i}/\mathscr{H}$=$(w^{-}(r_{k+i}, r_1), w^{-}(r_{k+i}, r_2), \ldots, w^{-}(r_{k+i}, r_k)$,

$\sigma^{+}_{k+i}/\mathscr{H}$=$(w^{+}(r_{k+i}, r_1), w^{+}(r_{k+i}, r_2), \ldots, w^{+}(r_{k+i}, r_k)$ all are distinct, $\forall\, i = 1,2,\ldots n - k$. If G and G′ are homomorphic, then ∃ a bijection $\varphi: V \to V'$ which satisfies, $\mu_E^{-}(r) \leq \mu_E^{-'}(\varphi(r))$, $\mu_E^{+}(r) \leq \mu_E^{+'}(\varphi(r))$, $\mu_F^{-}(r,u) \leq \mu_F^{-'}(\varphi(r), \varphi(u))$ and $\mu_F^{+}(r,u) \leq \mu_F^{+'}(\varphi(r), \varphi(u))$ $\forall r, u \in E$. Now, we define $\mathscr{H}_1 = \{(\sigma_1^{-'}, \sigma_1^{+'}), (\sigma_2^{-'}, \sigma_2^{+'}), \ldots, (\sigma_k^{-'}, \sigma_k^{+'})\}$ for $i = 1,2,\ldots n - k$,

$$\begin{aligned}\sigma^{-'}_{k+i}/\mathscr{H} &= \{w^{-}(\varphi(r_{k+i}), \varphi(r_1)), w^{-}(\varphi(r_{k+i}), \varphi(r_2)), \ldots, w^{-}(\varphi(r_{k+i}), \varphi(r_k)\}\\ &= \{((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_1)), ((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_2)), \ldots\\ &\qquad ((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_k))\\ &\geq \mu^{-}(r_{k+i}, r_1), \mu^{-}(r_{k+i}, r_2), \ldots, \mu^{-}(r_{k+i}, r_k)\\ &= w^{-}(r_{k+i}, r_1), w^{-}(r_{k+i}, r_2), \ldots, w^{-}(r_{k+i}, r_k)\end{aligned}$$

$$\begin{aligned}\sigma^{+'}_{k+i}/\mathscr{H} &= \{w^{+}(\varphi(r_{k+i}), \varphi(r_1)), w^{+}(\varphi(r_{k+i}), \varphi(r_2)), \ldots, w^{-+}(\varphi(r_{k+i}), \varphi(r_k)\}\\ &= \{((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_1)), ((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_2)), \ldots\\ &\qquad ((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_k))\\ &\geq \mu^{+}(r_{k+i}, r_1), \mu^{+}(r_{k+i}, r_2), \ldots, \mu^{+}(r_{k+i}, r_k)\\ &= w^{+}(r_{k+i}, r_1), w^{+}(r_{k+i}, r_2), \ldots, w^{+}(r_{k+i}, r_k)\end{aligned}$$

which are all distinct for $i = 1$ to $n = k$. Therefore, $\mathscr{H}_1$ is the IVFRS of G′ and $|\mathscr{H}_1| = k$. Now to prove that $\mathscr{H}_1$ is the minimum IVFRS OF G'. Assume that there exists a FRS R of G' such that $R = \{(\sigma_1^{-}, \sigma_1^{+}), (\sigma_2^{-}, \sigma_2^{+}), \ldots, (\sigma_{k_1}^{-}, \sigma_{k_1}^{+})\}$ and $|\mathscr{H}_1| = k > |R| = k_1$, then σ^{-}_{k+i}/R and σ^{+}_{k+i}/R are all distinct for $n = 1, 2, \ldots, n - k_1$. Let $R_1 = \{(\sigma_1^{-}, \sigma_1^{+}), (\sigma_2^{-}, \sigma_2^{+}), \ldots, (\sigma_{k_1}^{-}, \sigma_{k_1}^{+})\}$,

$$\sigma^{-}_{k+i}/R_1 = w^{-}(r_{k+i}, r_1), w^{-}(r_{k+i}, r_2), \ldots, w^{-}(r_{k+i}, r_{k_1})$$

$$= \{(\mu^{-})^{\infty}(r_{k+i}, r_1), (\mu^{-})^{\infty}(r_{k+i}, r_2), \ldots (\mu^{-})^{\infty}(r_{k+i}, r_{k_1})\}$$

$$\leq \{((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_1)), ((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_2)), \ldots$$
$$((\mu^{-'})^{\infty}(\varphi(r_{k+i}), (\mu^{-'})^{\infty}(\varphi(r_{k_1}))$$

Similarly,

$$\sigma_{k+i}^{+}/R_1 = w^{+}(r_{k+i}, r_1), w^{+}(r_{k+i}, r_2), \ldots, w^{+}(r_{k+i}, r_{k_1})$$

$$= \{(\mu^{+})^{\infty}(r_{k+i}, r_1), (\mu^{+})^{\infty}(r_{k+i}, r_2), \ldots (\mu^{+})^{\infty}(r_{k+i}, r_{k_1})\}$$

$$\leq \{((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_1)), ((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_2)), \ldots$$
$$((\mu^{+'})^{\infty}(\varphi(r_{k+i}), (\mu^{+'})^{\infty}(\varphi(r_{k_1}))$$

which are all distinct for $i = 1,2,\ldots n-k$. Therefore, R_1 is the IVFRS of G and $\mathscr{J}r(G)=k_1$ $[k_1 < k]$, which is a contradiction to our assumption that $\mathscr{J}r(G) = k$. Hence, $\mathscr{H}_1$ is the minimum IVRS of G′. Therefore, $\mathscr{J}r(G)$ and $\mathscr{J}r(G')$ are homomorphic functions.

3.5 Corollary

If G and G′ are co-weak isomorphic, then $\mathscr{J}r(G)$ and $\mathscr{J}r(G')$ also have a co-weak isomorphism.

3.6 Theorem:

For a connected IVFG with atleast three different membership value with both σ^{-} and σ^{+}, there exists an IVFRSs, such that $2 \leq \mathscr{J}r(G) \leq n-1$. $[n \geq 3]$

Proof:

Let us take $n = 3$, then the defined three vertices are a, b, c. This implies that, $\sigma^{-}(a) = \alpha_1, \sigma^{+}(a) = \beta_1, \sigma^{-}(b) = \alpha_2, \sigma^{+}(b) = \beta_2, \sigma^{-}(c) = \alpha_3, \sigma^{+}(c) = \beta_3$, as they exist three different membership values, then the representation of any two vertices in H will be different. Hence $\mathscr{J}r(G) = 2$, for $n = 3$. Similarly, if $n = k$, then ∃ atleast three different membership values of both σ^{-} and σ^{+}. Hence the representation will be unique with atleast $(n-1)$ vertices. That is, $\mathscr{J}r(G) \leq n-1$. Hence $2 \leq \mathscr{J}r(G) \leq n-1$.

4. Fuzzy Super Resolving Sets of Interval-valued Fuzzy Graphs

An IVFRS is called Interval-valued Fuzzy Super Resolving Set (IVFSRS) if any two elements of $[\sigma^{-}, \sigma^{+}]$ have different representation with regards to $\mathscr{H}$. Here we take $w^{-}(r,r) = \sigma^{-}(r)$ and $w^{+}(r,r) = \sigma^{+}(r)$, and the least cardinality of this set is known as Interval-valued Super Resolving Number (IVSRN) which is denoted by $\mathscr{J}sr(G)$, If we arrange the obtained set in a row form, then it is named as Interval-valued Fuzzy Super Resolving Matrix denoted as $[S_{n\times k}^{-}, S_{n\times k}^{+}]$.

4.1 Illustration

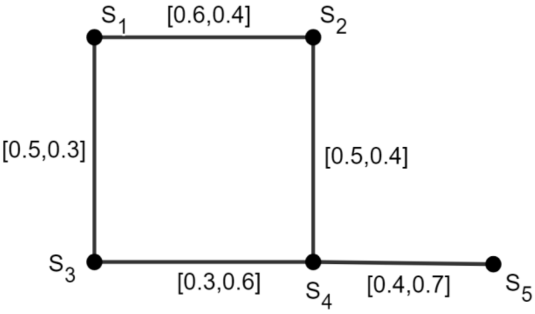

Fig 2: IVFG

M_E	S_1	S_2	S_3	S_4	S_5
M_E^-	0.7	0.8	0.6	0.5	0.4
M_E^+	0.4	0.5	0.6	0.7	0.8

M_F	S_1S_2	S_1S_3	S_4S_2	S_3S_4	S_4S_5
M_F^-	0.6	0.5	0.5	0.3	0.4
M_F^+	0.4	0.3	0.4	0.6	0.7

The strength of connectedness matrix is

$$C_{ij}^- = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{bmatrix} 0.7 & 0.6 & 0.5 & 0.5 & 0.4 \\ 0.6 & 0.8 & 0.5 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.6 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix} \end{array}$$

$$C_{ij}^- = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.5 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.6 & 0.6 & 0.6 \\ 0.4 & 0.4 & 0.6 & 0.7 & 0.7 \\ 0.4 & 0.4 & 0.6 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

The IVFSRS is $\{S_2, S_4, S_5\}$ and IVFSRM is written as, $\begin{bmatrix} 0.6 & 0.5 & 0.4 \\ 0.8 & 0.5 & 0.4 \\ 0.5 & 0.6 & 0.4 \\ 0.5 & 0.5 & 0.4 \\ 0.4 & 0.4 & 0.4 \end{bmatrix}$ and $\begin{bmatrix} 0.4 & 0.4 & 0.4 \\ 0.5 & 0.4 & 0.4 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.8 \end{bmatrix}$

Hence $\mathscr{J}\mathrm{sr}(G) = 3$.

4.2 Corollary

If G and G′ are isomorphic, then $\mathscr{J}\mathrm{sr}(G) \cong \mathscr{J}\mathrm{sr}(G')$.

4.3 Theorem

An IVERS is also an IVFRS but the converse may or may not be true.

Proof:

Let G be an IVFG with ‘n’ vertices. Let $\mathscr{H} = \{(\sigma_1^-, \sigma_1^+), (\sigma_2^-, \sigma_2^+), \dots, (\sigma_m^-, \sigma_m^+)\}$ be an IVFRS of G, then the representation of $\sigma_i^- / \mathscr{H}$ and $\sigma_i^+ / \mathscr{H}$, for $i = m+1, m+2, \dots n$ are all distinct. Also, $\sigma_i^- / \mathscr{H}$ and $\sigma_i^+ / \mathscr{H}$ need not to be distinct for $i = 1, 2, \dots n$. (i.,e) $\mathscr{H}$ does not need to be an IVFSRS.

Conversely, let $\mathscr{H} = \{(\sigma_1^-, \sigma_1^+), (\sigma_2^-, \sigma_2^+), \dots, (\sigma_m^-, \sigma_m^+)\}$ is an IVFSRS of G, then $\sigma_i^- / \mathscr{H}$ and $\sigma_i^+ / \mathscr{H}$, for $i = 1, 2, \dots m+1, \dots n$, are all distinct which means that the representation of $\sigma_i^- / \mathscr{H}$ and $\sigma_i^+ / \mathscr{H}$, for $i = m+1, m+2, \dots n$, are all distinct, Hence, $\mathscr{H}$ is also an IVFRS.

4.4 Theorem

For an interval-valued Fuzzy Star Graph (IVFSG) with distinct membership values, ∃ any two vertices ∋ r, u ∈ E, $\mathscr{J}\mathrm{sr}(G) = \mathscr{J}\mathrm{r}(G) = 2$

Proof:

By the definition of IVFSG, $\mu_E^-(r, u_i) > 0, \mu_E^+(r, u_i) > 0, \mu_E^-(u_i, u_{i+1}) = 0, \mu_E^-(u_i, u_{i+1}) = 0$ for $i = 1, 2, \dots n$. It is evident that we can find any two vertices with distinct membership values in the IVFSRM as well as IVFRM. Hence, $\mathscr{J}\mathrm{sr}(G) = \mathscr{J}\mathrm{r}(G) = 2$.

5. Application

In several domains, interval-valued fuzzy graphs (IVFGs) are employed to manage imprecision and uncertainty.Compared to crisp or normal fuzzy graphs, interval-valued fuzzy graphs (IVFGs) are better at capturing imprecision because they use intervals for edge and vertex values to depict interactions with uncertainty. They are used to efficiently handle varying or imprecise data in uncertain systems such as networks, decision-making, biology, and transportation. IVFGs use interval values to represent relationships in social networks that have unclear strengths, such as differing degrees of influence, friendship, or trust. This aids in identifying important influencers or clusters and analysing dynamic, imprecise interactions. IVFGs are used in signal processing to describe uncertain relationships between elements, such as varying noise levels or signal intensities. They help with data transmission uncertainty management, pathway optimisation, and robust system design.

Transportation networks, such as airline or urban transit systems, are a real-world example of an interval-valued fuzzy graph (IVFG) with a significant number of vertices. We simulate the unpredictability of passenger demand and flight connectivity between cities. Every vertex denotes a city, such as Tokyo, London, or New York. This might include hundreds or thousands of cities for a global airline network. Every edge depicts a flight path between two cities, with an interval signifying the degree of popularity or dependability of the route. The uncertainty of a city's traffic capacity or significance in the network is indicated by the vertex membership value. Uncertainty in passenger demand or flight dependability is represented by the edge membership value. The benefits includes Assisting airlines in optimising their schedules in spite of demand fluctuations, helps identify important cities or routes for investment, beneficial for managing disruptions (such as delays and bad weather). These models effectively handle big, complicated systems with a lot of uncertainty. For example, $M_E(A_1$: London) = (0.5, 0.4), $M_E(A_2$: New york) = (0.7, 0.6), $M_E(A_3$: Malaysia) = (0.8, 0.6) and so on. Also, $M_F(A_1A_2)$ = (0.4, 0.4), $M_F(A_1A_3)$ = (0.4, 0.3) and so on. Using IVFRS, we can obtain the few set of vertices such that the resulted set will have a shortest route between them considering all the traffic routes and the passenger dependancy.

5. Conclusion

One effective paradigm for simulating uncertainty in complex systems is offered by IVFGs. IVFGs are well suited for real-world applications like as social systems, biological interactions, and transportation networks because they incorporate interval-based membership values for vertices and edges, which effectively express imprecise relationships. In this article, we have defined interval-valued fuzzy resolving sets, interval-valued fuzzy super resolving sets and its properties. Also, we have defined an application based on interval-valued fuzzy resolving set. In future, we may look into more problems related to interval-valued fuzzy resolving set.

Reference

[1] Atanassov,K.T. (1986). Intuitionistic Fuzzy sets, Fuzzy sets and systems, 20(1), 87-96.

[2] Chartrand,G. Poisson,C. Zhang,P. (2000). Resolvability and the upper dimension of graphs, Comput. Mathematical Applications, 39, 19-28.

[3] Gani, A. N. and Basheer Ahamed, M., (2003), Order and size in Fuzzy graphs, Bull. Pure Appl. Sci., 22E, 145-148.

[6] Mordeson, J. N. Mathew, S., (2019), Advanced topics in Fuzzy Graph Theory, Springer science and Business media.

[7] Muhammad Akram. Wieslaw A. Dudek. (2011), Interval-valued fuzzy graphs, Computers and mathematics with Applications, 289-299.

[8] Shanmugapriya, R. and Hemalatha, P. K., (2023), A study of Independency on Fuzzy Resolving sets of Labelling graphs, Mathematics,11, 3440.

[9] Shanmugapriya,R. and Mary Jiny,D. (2021), Fuzzy super resolving number and resolving number of some special graphs, TWMS J.App. and Eng Math, 11, 459-468.

[10] Tarasankar Pramanik. Sovan Samanta. And Madhumangal Pal. (2020), Interval-valued Fuzzy Graphs, International journal of Fuzzy logic and Intelligent systems, 316-323.

[11] Vasuki, M. Shanmugapriya, R. Mahdal, M. and Robert Cep. (2023), A study on Fuzzy Resolving Domination sets and their Application in Network theory, Mathematics, 11, 317.

[12] Xiaoli Qiang. Qian-Ru Xiao. Aysha Khan, A. Talebi, A. Arun Kumar Sivaraman. Mojahedfar, M. (2022), Astudy on Interval-valued Fuzzy Graph with Application in Energy Industry Management, Discrete Dynamics in Nature and Society, 9.

[13] Zadeh,L.A. (1965), Fuzzy sets, Information and Control, 8, 338-353.

[14] Renukadevi, R. et al. “An Improved Collaborative User Product Recommendation System Using Computational Intelligence with Association Rules.” *Communications on Applied Nonlinear Analysis* (2024): n. pag. https://doi.org/10.52783/cana.v31.1243

[15] Kumar, E. Boopathi, and M. Sundaresan. "Edge detection using trapezoidal membership function based on fuzzy's mamdani inference system." *2014 International Conference on Computing for Sustainable Global Development (INDIACom)*. IEEE, 2014.

[16] Yookesh, T. L., et al. "Efficiency of iterative filtering method for solving Volterra fuzzy integral equations with a delay and material investigation." *Materials today: Proceedings* 47 (2021): 6101-6104.