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Open Maps via δ-open Sets in Pythagorean Fuzzy Topological Spaces and its Applications

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Abstract:

In this paper, we introduce the concept of Pythagorean fuzzy δ (resp. $\delta\alpha$, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & $\delta\beta$ or e^{*})-open mappings are introduced and their properties are discussed. In current scenario people with symptom of Covid-19 like fever, cough, sneezing, sore throat, loss of taste and smell etc., were panic about the disease, and the diagnosis of Covid-19 takes many hours and people cannot go for the test frequently. Some other diseases like flu, pneumonia, cold etc., also has the same symptoms. Each patients has unique experience of that particular symptom and some time they may not experience that symptom even though they were affected by the Covid-19. Also, in this paper we tried to diagnosis Covid-19 with the help of picture fuzzy sets which helps to record all symptoms in prÃ©cised manner.

Introduction: This paper introduces the concept of Pythagorean fuzzy open mappings and their relevance to COVID-19 symptom diagnosis. Due to the overlapping symptoms of COVID-19 with other illnesses, a Pythagorean fuzzy set approach is utilized to more accurately capture and analyze symptom patterns.

Objectives: The objective is to develop a precise mapping model that utilizes Pythagorean fuzzy sets to differentiate between COVID-19 symptoms and other similar conditions, aiming to improve diagnostic speed and accuracy.

Methods: The method involves formulating fuzzy sets based on COVID-19 symptoms, computing distances (Hamming, Euclidean) between patients' symptoms and ideal symptom patterns, and identifying cases with higher COVID-19 risk.

Results: The approach successfully identifies patients with symptoms closest to COVID-19 indicators, offering a more systematic and timely diagnosis. Results suggest that this fuzzy set model can enhance early detection and intervention for affected individuals

Conclusions: In this paper, Some new notions of strongly Pythagorean fuzzy open (closed) maps called Pythagorean fuzzy δ-open and Pythagorean fuzzy δ-closed maps are introduced and discussed their relationship between their near mappings with examples. Also, we have tried to diagnosis Covid-19 with the help of Pythagorean fuzzy sets which helps to record all symptoms in prÃ©cised manner. In future, researchers can extend this model to other extensions of fuzzy sets such as rough sets and utilize the interdependency among the various evaluation criteria for better judgement.

Keywords: Pythagorean fuzzy δ-open mappings, distance between fuzzy sets, Covid-19 patient- record all symptoms in precise manner.

1. Introduction

Traditional logic, which is interpreted as either true or false, found to be difficult to solve uncertain real-life problems. As a counter measure, Zadeh (1965) [35] invented fuzzy set theory, where the involvement of elements in a set is characterized by membership grade, which belongs to [0,1]. To handle much uncertainty, fuzzy sets were extended by the different researchers in different ways such as vague set (Gau and Buehrer 1993) [15], intuitionistic fuzzy set (IFS) (Atanassov 1986a, 1986b) [1, 2], fuzzy soft set (Das et al. 2018) [12], rough set (Pawlak 1982) [25], fuzzy interval theory (Gorzalczany 1987) [18], intuitionistic multi fuzzy set (Das et al. 2013) [11], interval-valued intuitionistic fuzzy set (Park et al. 2008) [26] , intuitionistic fuzzy soft set (Deng 1982) [14] and neutrosophic soft set (Das et al. 2019) [13] . Consequently, the application of fuzzy set theory and its extensions increased rapidly in the decision-making methods in various domains like medical diagnosis (Das et al. 2013) [11], pattern recognition (Wei and Lan 2008) [30], data analysis (Zou and Xiao 2008) [36], forecasting (Xiao et al. 2011) [31], optimization (Kov-kov et al. 2007) [20], simulation (Kalayathankal and Singh 2010) [19] and texture classification (Mushrif et al. 2006) [23]. Recently in 2014, Cuong (2014)[10] developed the picture fuzzy set (PFS) as the generalized form of fuzzy set and IFS. The PFS approaches are found to be more appropriate in those cases when the views of someone contain more option types like yes, abstain, no and refusal. The general election of a country is noted as a good example to describe PFS, where a voter can cast his vote in favour of the candidate (yes), against the candidate (no), may not cast his vote (abstain) or may refuse to cast his vote in favour of the given candidates and prefer for nota (refusal) (Cong and Son 2015) [9].

Nowadays, the whole world has become fully unbalanced and passing through an uncontrolled situation due to the dangerous and novel virus Covid-19. Most countries are totally stagnant and the people are quarantined to make themselves safe from Covid-19 (Ren et al. 2020) [27]. Many researchers are continuously contributing to developing various type of mathematical and hybrid models to predict the future trends, strength and transmission capability of Covid-19 virus, and have drawn some useful conclusions which assist the health department to take the necessary precaution to track and handle the Covid-19 situations. The authors in Melin et al. (2020) [22] introduced a novel hybrid prediction model that can mergethe ensemble architectures of fuzzy logic-based neural networks for response integration. The fundamental concept of the proposed model is to merge several fuzzy-based neural network predictors, control the uncertainty of the individual networks and try to reduce the uncertainty of the total predictions. This model was able to predict the future trends of Covid-19 up to some extent and help the authorities make the necessary decision to handle the health care system in a better manner. The authors in Sun and Wang (2020) [28] collected the Covid-19 data from a decided location within a specific time interval and trained through the ordinary differential equation model for fitting.Then, they modified the simulation by the trained model to realize the effect of the Covid-19 affected visitors. They found that the affected visitors have a great role in the newly introduced case of Covid-19. Stochastic simulations proved that the physical connections could be rapidly increased due to the affected visitors which are considered sufficient for the local outbreak of Covid-19. The confirmed case of asymptomatic patients was significantly less than the model predictions quantity. This indicated that a major portion of asymptomatic patients are not identified/found. Fuzzy-based hybrid approaches for forecasting the confirmed cases and deaths of the countries according to their time series are given in Castillo and Melin (2020) [6]. The fundamental concept of this proposed hybrid method (Castillo and Melin 2020) [6] is to combine the fractal dimension and fuzzy logic for enabling efficient and accurate forecasting of Covid-19 time series. The fractal dimension is provided to differentiate and categorize the object. They introduced a fuzzy rule-based system to represent the knowledge about the forecasting time series of the countries. The authors in Castillo and Melin (2021) [7] introduced the hybrid procedure for composing the fuzzy logic and fractal dimension which measured the uncommon activities of times series to classify countries according to their Covid-19 time series data. The proposed method generates an accurate classification of countries based on the complexity of the Covid-19 time series data. Editors (Boccaletti et al. 2020) [5] of the journal â€˜â€˜Chaos, Solitons and Fractalsâ€™â€™ analysed the impact of Covid-19 pandemic throughout the world and felt the necessity to create a unique platform for the researchers to help the society to avoid the worst effects of future pandemics. Recently, Mishra et al. (2021) [27] proposed an extended fuzzy decision-making framework using hesitant fuzzy sets for the drug selection to treat the mild symptoms of Covid-19. Although the researchers are working hard, they are still struggling to recover from this unwanted situation. The scientists from different domains are consistently trying to apply their knowledge in different perspectives such as dominating the virus, identifying the virus, isolating from the virus, protecting from the virus, and finding the treatment of the virus affected patients, to manage the superfluous situation (Kumar et al., 2020 [21], Ghosh et al. , 2020) [17], which are considered to be the long term project. As an intermediate solution, the most important aspect is to provide suitable medical service to the affected patients and recover those who are critically ill due to perilous virus Covid-19. The health department of India has classified the Covid-19 affected patients into some categories according to the patients physical condition. The extreme condition is called severe cases, and this type of patient requires quality treatment (Clinical Management Protocol 2020) [8]. In current scenario people with symptom of Covid-19 like fever, cough, sneezing, sore throat, loss of taste and smell etc., were panic about the disease, and the diagnosis of Covid-19 takes many hours and people cannot go for the test frequently. Some other diseases like flu, pneumonia, cold etc., also has the same symptoms. Each patients has unique experience of that particular symptom and some time they may not experience that symptom even though they were affected by the Covid-19.

To fill up this research gap, this paper proposes Pythagorean fuzzy δ (resp. $\delta\alpha$, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-open, closed mappings and an alternative Pythagorean fuzzy set based approach, here we tried to diagnosis Covid-19 with the help of Pythagorean fuzzy sets which helps to record all symptoms in preccised manner.

2 Preliminaries

We recall some basic notions of fuzzy sets, IFS's and pfs's .

Definition 2.1 *[35] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A\colon X \to [0,1]$. That is:*

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ ispartlyin } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X\right\}$, where the function $\mu_A(x): X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval [0,1].

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2 [1, 2, 3, 4] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $\mathrm{A} = \{< \mathrm{x}, \mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x}) > | \mathrm{x} \in \mathrm{X}\}$ or $\mathrm{A} = \left\{\left\langle \frac{\mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x})}{\mathrm{x}} \right\rangle | \mathrm{x} \in \mathrm{X}\right\}$, where the functions $\mu_\mathrm{A}(\mathrm{x}): \mathrm{X} \to [0,1]$ and $\nu_\mathrm{A}(\mathrm{x}): \mathrm{X} \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $\mathrm{x} \in \mathrm{X}$ to A, which is a subset of X, and for every $\mathrm{x} \in \mathrm{X}$: $0 \le \mu_\mathrm{A}(\mathrm{x}) + \nu_\mathrm{A}(\mathrm{x}) \le 1$. For each A in X: $\pi_\mathrm{A}(\mathrm{x}) = 1 - \mu_\mathrm{A}(\mathrm{x}) - \nu_\mathrm{A}(\mathrm{x})$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_\mathrm{A}(\mathrm{x})$ is the degree of nondeterminacy of $\mathrm{x} \in \mathrm{X}$ to the set A and $\pi_\mathrm{A}(\mathrm{x}) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $\mathrm{x} \in \mathrm{X}$ or $\mathrm{x} \notin \mathrm{X}$. Thus: $\mu_\mathrm{A}(\mathrm{x}) + \nu_\mathrm{A}(\mathrm{x}) + \pi_\mathrm{A}(\mathrm{x}) = 1$.

Example 2.1 Let $\mathrm{X} = \{\mathrm{x}, \mathrm{y}, \mathrm{z}\}$ be a fixed universe of discourse and $\mathrm{A} = \left\{\left\langle \frac{0.6,0.1}{\mathrm{x}} \right\rangle, \left\langle \frac{0.8,0.1}{\mathrm{y}} \right\rangle, \left\langle \frac{0.5,0.3}{\mathrm{z}} \right\rangle\right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements $\mathrm{x}, \mathrm{y}, \mathrm{z}$ to A are as follows: $\pi_\mathrm{A}(\mathrm{x}) = 0.3, \pi_\mathrm{A}(\mathrm{y}) = 0.1$ and $\pi_\mathrm{A}(\mathrm{z}) = 0.2$.

Definition 2.3 [32, 33, 34] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $\mathrm{A} = \{< \mathrm{x}, \mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x}) | \mathrm{x} \in \mathrm{X}\}$ or $\mathrm{A} = \left\{\left\langle \frac{\mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x})}{\mathrm{x}} \right\rangle | \mathrm{x} \in \mathrm{X}\right\}$, where the functions $\mu_\mathrm{A}(\mathrm{x}): \mathrm{X} \to [0,1]$ and $\nu_\mathrm{A}(\mathrm{x}): \mathrm{X} \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $\mathrm{x} \in \mathrm{X}$ to A, which is a subset of X, and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$. We denote the set of all PFS's over X by pfs(X).

Definition 2.4 [34] Let A and B be pfs's of the forms $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ and $B = \{< a, \lambda_B(a), \mu_B(a) > | a \in X\}$. Then

1. $A \subseteq B$ if and only if $\lambda_A(a) \leq \lambda_B(a)$ and $\mu_A(a) \geq \mu_B(a)$ for all $a \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $\overline{A} = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$.
4. $A \cap B = \{< a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) > | a \in X\}$.
5. $A \cup B = \{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > | a \in X\}$.
6. $\phi = \{< a, \phi, X > | a \in X\}$ and $X = \{< a, X, \phi > | a \in X\}$.
7. $\overline{X} = \phi$ and $\overline{\phi} = X$.

Definition 2.5 [24] An Pythagorean fuzzy topology by subsets of a non-empty set X is a family τ of pfs's satisfying the following axioms. [(i)]

1. $\phi, X \in \tau$.
2. $G_1 \cap G_2 \in \tau$ for every $G_1, G_2 \in \tau$ and
3. $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i | i \in j\} \subseteq \tau$. The pair (X, τ) is called an Pythagorean fuzzy topological space (pfts in short) and any pfs G in τ is called an Pythagorean fuzzy open set (pfos in short) in X. The complement $\overline{A}$ of an Pythagorean fuzzy open set A in an pfts(X, τ) is called an Pythagorean fuzzy closed set (pfcs in short).

Definition 2.6 [24] Let (X, τ) be an pfts and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the interior and the closure of A are denoted by pfint(A) and pfcl(A) and are defined as follows: $\text{pfcl}(A) = \cap \{K | K \text{ isan pfcs and } A \subseteq K\}$ and $\text{pfint}(A) = \cup \{G | G \text{ isan pfos and } G \subseteq A\}$. Also, it can be established that pfcl(A) is an pfcs and pfint(A) is an pfos, A is an pfcs if and only if $\text{pfcl}(A) = A$ and A is an pfos if and only if $\text{pfint}(A) = A$. We say that A is pf-dense if $\text{pfcl}(A) = X$.

Lemma 2.1 [29] For any Pythagorean fuzzy set A in (X, τ), we have $X - \text{pfint}(A) = \text{pfcl}(X - A)$ and $X - \text{pfcl}(A) = \text{pfint}(X - A)$.

Definition 2.7 [29] Let (X, τ) be an pfts and A be an pfs. Then A is said to be an Pythagorean fuzzy (i) regular open set (pfros in short) if $A = \text{pfint}(\text{pfcl}(A))$. (ii) regular closed set (pfrcs in short) if $A = \text{pfcl}(\text{pfint}(A))$. By Lemma 2.1, it follows that A is an pfros iff $\overline{A}$ is an pfrcs.

Definition 2.8 [16] Let (X_1, Γ_P) & (X_2, Ψ_P) be a pfts's. A mapping $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy continuous (briefly, pfCts) if the inverse image of every pfos in (X_2, Ψ_P) is a pfos.

3 Pythagorean fuzzy δ-open mapping

Definition 3.1 Let (X, τ) be an pfts and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the δ-interior and the δ-closure of A are denoted by pfδint(A) and pfδcl(A) and are defined as follows. pfδcl(A) $= \cap$ {K|K is an pfrcs and $A \subseteq K$}, (pfδint(A) $= \cup$ {G|G is an pfros and $G \subseteq A$}.

Definition 3.2 Let (X, τ) be an pfts and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. A set A is said to be pf

1. δ-open set (briefly, pfδos) if A = pfδint(A),
2. δ-pre open set (briefly, pfδ$\mathcal{P}$os) if A ⊆ pfint(pfδcl(A)).
3. δ-semi open set (briefly, pfδ$\mathcal{S}$os) if A ⊆ pfcl(pfδint(A)).
4. δ-α open set or a-open set (briefly, pfδαos or pfaos) if A ⊆ pfint(pfcl(pfδint(A))).
5. δ-β open set or e^*-open set (briefly, pfδβos or pfe^*os) if A ⊆ pfcl(pfint(pfδcl(A))).
6. δ (resp. δ-pre, δ-semi, δ-α and δ-β) dense if pfδcl(A) (resp. pfδpcl(A), pfδ$\mathcal{S}$cl(A), pfδαcl(A) and pfδβcl(A)) = X.

The complement of an pfδos (resp. pfδ$\mathcal{P}$os, pfδ$\mathcal{S}$os, pfδαos and pfδβos) is called an pfδ (resp. pfδ$\mathcal{P}$, pfδ$\mathcal{S}$, pfδα and pfδβ) closed set (briefly, pfδcs (resp. pfδ$\mathcal{P}$cs, pfδ$\mathcal{S}$cs, pfδαcs and pfδβcs in X.

The family of all pfδos (resp. pfδcs, pfδ$\mathcal{P}$os, pfδ$\mathcal{P}$cs, pfδ$\mathcal{S}$os, pfδ$\mathcal{S}$cs, pfδαos, pfδαcs, pfδβos and pfδβcs) of X is denoted by pfδOS(X), (resp. pfδCS(X), pfδ$\mathcal{P}$OS(X), pfδ$\mathcal{P}$CS(X), pfδ$\mathcal{S}$OS(X), pfδ$\mathcal{S}$CS(X), pfδαOS(X), pfδαCS(X), pfδβOS(X) and pfδβCS(X)).

Definition 3.3 Let (X, τ) be an pfts and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an pfs in X. Then the pfδ-pre (resp. pfδ-semi, pfδα and pfδβ)-interior and the pfδ-pre (resp. pfδ-semi, pfδα and pfδβ)-closure of A are denoted by pfδ$\mathcal{P}$int(A) (resp. pfδ$\mathcal{S}$int(A), pfδαint(A) and pfδβint(A)) and the pfδ$\mathcal{P}$cl(A) (resp. pfδ$\mathcal{S}$cl(A), pfδαcl(A) and pfδβcl(A) and are defined as follows:

pfδ$\mathcal{P}$int(A) (resp. pfδ$\mathcal{S}$int(A), pfδαint(A) and pfδβint(A) $= \cup$ {G|G in a pfδ$\mathcal{P}$os (resp. pfδ$\mathcal{S}$os, pfδαos and pfδβos)

and G ⊆ A} and pfδ$\mathcal{P}$cl(A) (resp. pfδ$\mathcal{S}$cl(A), pfδαcl(A) and pfδβcl(A) $= \cap$ {K|K is an pfδ$\mathcal{P}$cs (resp. pfδ$\mathcal{S}$cs, pfδαcs, pfδβcs) and A ⊆ K}.

Definition 3.4 Let (X_1, Γ_P) & (X_2, Ψ_P) be a pfts's. A mapping $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy δ (resp. δα, δ$\mathcal{S}$, δ$\mathcal{P}$ & δβ or e^*)-continuous (briefly, pfδCts (resp. pfδαCts, pfδ$\mathcal{S}$Cts, pfδ$\mathcal{P}$Cts & pfδβCts or pfe^*Cts)) if the inverse image of every pfos in (X_2, Ψ_P) is a pfδos (resp. pfδαos, pfδ$\mathcal{S}$os, pfδ$\mathcal{P}$os & pfδβos or pfe^*os) in (X_1, Γ_P).

Definition 3.5 Let (X_1, Γ_P) & (X_2, Ψ_P) be a pfts's. A mapping $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy (resp. δ, δα, δ$\mathcal{S}$, δ$\mathcal{P}$ & δβ or e*)-open map (briefly, pfO (resp. pfδO, pfδαO, pfδ$\mathcal{S}$O, pfδ$\mathcal{P}$O & pfδβO or pfe*O)) if the image of every pfos in (X_1, Γ_P) is a pfos (resp. pfδos, pfδαos, pfδ$\mathcal{S}$os, pfδ$\mathcal{P}$os & pfδβos or pfe*os) in (X_2, Ψ_P).

Theorem 3.1 Let (X_1, Γ_P) & (X_2, Ψ_P) be a pfts's. Let $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ be a mapping. Then the following statements are hold for pfts, but not conversely.

1. Every pfδO mapping is a pfO mapping.
2. Every pfδO mapping is a pfδ$\mathcal{S}$O mapping.
3. Every pfδO mapping is a pfδ$\mathcal{P}$O mapping.
4. Every pfδ$\mathcal{S}$O mapping is a pfδβO mapping.
5. Every pfδ$\mathcal{P}$O mapping is a pfδβO mapping.
6. Every pfδαO mapping is a pfδ$\mathcal{S}$O mapping.
7. Every pfδαO mapping is a pfδ$\mathcal{P}$O mapping.

Proof. (i) Let M be a pfos in X_1. Since h_P is pfδO map, $h_P(M)$ is a pfδos in X_2. Since every pfδos is a pfos, $h_P(M)$ is a pfos in X_2. Hence h_P is a pfO.

(ii) Let M be a pfos in X_1. Since h_P is pfO map, $h_P(M)$ is a pfos in X_2. Since every pfos is a pfδ$\mathcal{S}$os, $h_P(M)$ is a pfδ$\mathcal{S}$os in X_2. Hence h_P is a pfδ$\mathcal{S}$O.

(iii) Let M be a pfos in X_1. Since h_P is pfO map, $h_P(M)$ is a pfos in X_2. Since every pfos is a pfδ$\mathcal{P}$os, $h_P(M)$ is a pfδ$\mathcal{P}$os in X_2. Hence h_P is a pfδ$\mathcal{P}$O.

(iv) Let M be a pfos in X_1. Since h_P is pfδ$\mathcal{S}$O map, $h_P(M)$ is a pfδ$\mathcal{S}$os in X_2. Since every pfδ$\mathcal{S}$os is a pfδβos, $h_P(M)$ is a pfδβos in X_2. Hence h_P is a pfδβO.

(v) Let M be a pfos in X_1. Since h_P is pfδ$\mathcal{P}$O map, $h_P(M)$ is a pfδ$\mathcal{P}$os in X_2. Since every pfδ$\mathcal{P}$os is a pfδβos, $h_P(M)$ is a pfδβos in X_2. Hence h_P is a pfδβO.

(vi) Let M be a pfos in X_1. Since h_P is pfδαO map, $h_P(M)$ is a pfδαos in X_2. Since every pfδαos is a pfδ$\mathcal{S}$os, $h_P(M)$ is a pfδ$\mathcal{S}$os in X_2. Hence h_P is a pfδ$\mathcal{S}$O.

(vii) Let M be a pfos in X_1. Since h_P is pfδαO map, $h_P(M)$ is a pfδαos in X_2. Since every pfδαos is a pfδ$\mathcal{P}$os, $h_P(M)$ is a pfδ$\mathcal{P}$os in X_2. Hence h_P is a pfδ$\mathcal{P}$O.

Remark 3.1 We obtain the following diagram from the results we discussed above.

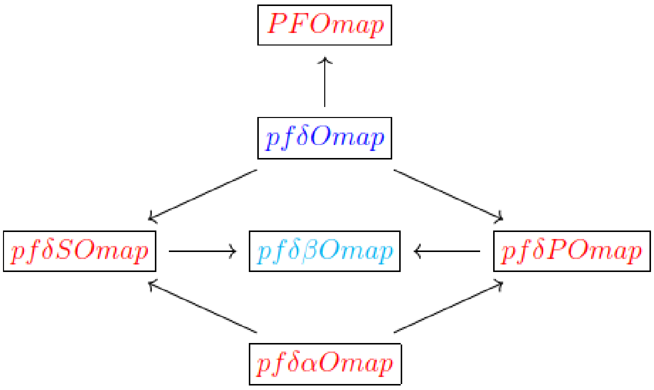

Fig. 1 : $pf\delta O$ mappings in $pfts$.

Note: $A \rightarrow B$ denotes A implies B. But not conversely.

Example 3.1 Let $X = X_1 = X_2 = X_3 = X_4 = X_5 = \{x_1, x_2\}$ and the pfs's A_1, A_2 and A_3 are defined as $A_1 = \{< x_1, 0.020, 0.040 >, < x_2, 0.050, 0.050 >\}$

$$A_2 = \{< x_1, 0.010, 0.040 >, < x_2, 0.050, 0.050 >\}$$

$$A_3 = \{< x_1, 0.020, 0.030 >, < x_2, 0.050, 0.050 >\}$$

Here we have $\tau_1 = \{0_{X_1}, 1_{X_1}, A_1, A_2\}$, $\tau_2 = \{0_{X_2}, 1_{X_2}, A_2\}$, $\tau_3 = \{0_{X_3}, 1_{X_3}, A_1^c\}$, $\tau_4 = \{0_{X_4}, 1_{X_4}, A_2^c\}$ and $\tau_5 = \{0_{X_5}, 1_{X_5}, A_3\}$ be a pfts's on X. Let $h1_P: (X_2, \tau_2) \rightarrow (X_1, \tau_1)$, $h2_P: (X_3, \tau_3) \rightarrow (X_1, \tau_1)$, $h3_P: (X_4, \tau_4) \rightarrow (X_1, \tau_1)$, $h4_P: (X_5, \tau_5) \rightarrow (X_1, \tau_1)$ be an identity mapping. Then

1. $h1_P$ is pfO (resp. pfδβO and pfδ$\mathcal{P}$O) but not pfδO (resp. pfδ$\mathcal{S}$O and pfδαO), because the set A_2 is a pfos in X_2 but $h1_P(A_2) = A_2$ is not pfδos (resp. pfδ$\mathcal{S}$os and pfδαos) in X_1.
2. $h2_P$ is pfδ$\mathcal{S}$O but not pfδO, because the set A_1^c is a pfos X_3 but $h2_P(A_1^c) = A_1^c$ is not pfδos in X_1.
3. $h3_P$ is pfδ$\mathcal{P}$O but not pfδO, because the set A_2^c is a pfos X_4 but $h3_P(A_2^c) = A_2^c$ is not pfδ$\mathcal{P}$os in X_1.
4. $h4_P$ is pfδβO (resp. pfδ$\mathcal{S}$O) but not pfδ$\mathcal{P}$O (resp. pfδαO), because the set A_3 is a pfos X_5 but $h4(A_3) = A_3$ is not pfδ$\mathcal{P}$os (resp. pfδαos) in X_1.

Theorem 3.2 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. A mapping $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ is pfδβO iff for every pfs M of (X_1, Γ_P), $h_P(pfint(M)) \subseteq pf\delta\beta int(h_P(M))$.

Necessity: Let h_P be a pfδβO and M be a pfos in (X_1, Γ_P). Now, $pfint(M) \subseteq M$ implies $h_P(pfint(M)) \subseteq h_P(M)$. Since h_P is a pfδβO, $h_P(pfint(M))$ is pfδβos in (X_2, Ψ_P) such that $h_P(pfint(M)) \subseteq h_P(M)$ therefore $h_P(pfint(M)) \subseteq pf\delta\beta int(h_P(M))$.

Sufficiency: Assume M is a pfos of (X_1, Γ_P). Then $h_P(M) = h_P(pfint(M)) \subseteq pf\delta\beta int(h_P(M))$. But $pf\delta\beta int\ (h_P(M)) \subseteq h_P(M)$. So $h_P(M) = pf\delta\beta int(M)$ which implies $h_P(M)$ is a pfδβos of (X_2, Ψ_P) and hence h_P is a pfδβO.

Theorem 3.3 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. If $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is a pfδβO, then $\text{pfint}(h_P^{-1}(M)) \subseteq h_P^{-1}(\text{pf}\delta\beta\text{int}(M))$ for every pfs M of (X_2, Ψ_P).

Proof. Let M be a pfs of (X_2, Ψ_P). Then $\text{pfint}(h_P^{-1}(M))$ is a pfos in (X_1, Γ_P). Since h_P is pfδβO, $h_P(\text{pfint } (h_P^{-1}(M))$ is pfδβo in (X_2, Ψ_P) and hence $h_P(\text{pfint}(h_P^{-1}(\ \lambda))) \subseteq \text{pf}\delta\beta\text{int}(h_P(h_P^{-1}(M))) \subseteq \text{pf}\delta\beta\text{int}(M)$. Thus $\text{pfint}(h_P^{-1}(M)) \subseteq h_P^{-1}(\text{pf}\delta\beta\text{int}(M))$.

Theorem 3.4 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfδβO iff for each pfs μ_1 of (X_2, Ψ_P) and for each pfs μ_2 of (X_1, Γ_P) containing $h_P^{-1}(\mu)$ there is an pfδβcs ν of (X_2, Ψ_P) such that $\mu_1 \subseteq \mu_2$ and $h_P^{-1}(\nu) \subseteq \mu_2$.

Necessity: Assume h_P is a pfδβO. Let μ_1 be the pfcs of (X_2, Ψ_P) and μ_2 is a pfcs of (X_1, Γ_P) such that $h_P^{-1}(\mu_1) \subseteq \mu_2$. Then $\nu = (h_P^{-1}(\mu_2^c))^c$ is pfδβcs of (X_2, Ψ_P) such that $h_P^{-1}(\nu) \subseteq \mu_2$.

Sufficiency: Assume ω is a pfos of (X_1, Γ_P). Then $h_P^{-1}((h_P(\omega))^c \subseteq \omega^c$ and ω^c is pfcs in (X_1, Γ_P). By hypothesis there is a pfδβcs ν of (X_2, Ψ_P) such that $(h_P(\omega))^c \subseteq \nu$ and $h_P^{-1}(\nu) \subseteq \omega^c$. Therefore $\omega \subseteq (h_P^{-1}(\nu))^c$. Hence $\nu^c \subseteq h_P(\omega) \subseteq h_P((h_P^{-1}(\nu))^c) \subseteq \nu^c$ which implies $h_P(\omega) = \nu^c$. Since ν^c is pfδβos of (X_2, Ψ_P). Hence $h_P(\omega)$ is pfδβo in (X_2, Ψ_P) and thus h_P is pfδβO.

Theorem 3.5 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfδβO iff $h_P^{-1}(\text{pf}\delta\beta\text{cl}(M) \subseteq \text{pfcl}(h_P^{-1}(M))$ for every pfs M of (X_2, Ψ_P).

Proof. Necessity: Assume h_P is a pfδβO. For any pfs M of (X_2, Ψ_P), $h_P^{-1}(M) \subseteq \text{pfcl}(h_P^{-1}(M))$. Therefore by Theorem 3.4, there exists a pfδβcs μ in (X_2, Ψ_P) such that $\lambda \subseteq \mu$ and $h_P^{-1}(\mu) \subseteq \text{pfcl}(h_P^{-1}(M))$. Therefore we obtain that $h_P^{-1}(\text{pf}\delta\beta\text{cl}(M)) \subseteq h_P^{-1}(\mu) \subseteq \text{pfcl}(h_P^{-1}(M))$.

Sufficiency: Assume M is a pfs of (X_2, Ψ_P) and μ is a pfos of (X_1, Γ_P) containing $h_P^{-1}(M)$. Put $\zeta = \text{cl}(M)$, then $M \subseteq \zeta$ and ζ is pfδβc and $h_P^{-1}(\zeta) \subsetneq \text{cl}(h_P^{-1}(M)) \subseteq \mu$. Then by Theorem 3.4, h_P is pfδβO map.

Theorem 3.6 Let (X_1, Γ_P), (X_2, Ψ_P) & (X_3, Φ_P) be any pfts's. If $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ and $g_p: (X_2, \Psi_P) \to (X_3, \Phi_P)$ are mappings and $g_p \circ h_P: (X_1, \Gamma_P) \to (X_3, \Phi_P)$ is pfδβO. If $g_p: (X_2, \Psi_P) \to (X_3, \Phi_P)$ is pfδβIrr then $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfδβO.

Proof. Let ν be a pfos in (X_1, Γ_P). Then $g_p \circ h_P(\nu)$ is pfδβos of (X_3, Φ_P) because $g_p \circ h_P$ is pfδβO. Since g_p is pfδβIrr and $g_p \circ h_P(\psi)$ is pfδβos of (X_3, Φ_P), $g_p^{-1}(g_p \circ h_P(\psi)) = h_P(\psi)$ is pfδβos in (X_2, Ψ_P). Hence h_P is pfδβO.

Theorem 3.7 Let (X_1, Γ_P), (X_2, Ψ_P) & (X_3, Φ_P) be any pfts's. If $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfO and $g_p: (X_2, \Psi_P) \to (X_3, \Phi_P)$ is pfδβO, then $g_p \circ h_P: (X_1, \Gamma_P) \to (X_3, \Phi_P)$ is pfδβO.

Proof. Let ψ be a pfos in (X_1, Γ_P). Then $h_P(\psi)$ is a pfos of (X_2, Ψ_P) because h_P is a pfO map. Since g_p is pfδβO, $g_p(h_P(\psi)) = (g_p \circ h)(\psi)$ is pfδβos of (X_3, Φ_P). Hence $g_p \circ h_P$ is pfδβO map.

Remark 3.2 Theorems 3.2 to 3.7 is also holds for pfO (resp. pfδO, pfδ$\mathcal{S}$O and pfδ$\mathcal{P}$O) mappings.

4 Pythagorean fuzzy δ-closed mappings

In this section, Pythagorean fuzzy δ-closed mappings are introduced and studied their properties.

Definition 4.1 Let (X_1, Γ_P) & (X_2, Ψ_P) be any two pfts's. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is said to be Pythagorean fuzzy (resp. δ, $\delta\mathcal{S}$, $\delta\mathcal{P}$ and δβ) closed map (briefly, pfC (resp. pfδC, pf$\delta\mathcal{S}$C, pf$\delta\mathcal{P}$C and pfδβC)) if the image of every pfcs in (X_1, Γ_P) is a pfcs (resp. pfδcs, pf$\delta\mathcal{S}$cs, pf$\delta\mathcal{P}$cs and pfδβcs) in (X_2, Ψ_P).

Theorem 4.1 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. Then the following statements are hold.

1. Every pfδC map is a pfC map.
2. Every pfC map is a pf$\delta\mathcal{S}$C map.
3. Every pfC map is a pf$\delta\mathcal{P}$C map.
4. Every pf$\delta\mathcal{S}$C map is a pfδβC map.
5. Every pf$\delta\mathcal{P}$C map is a pfδβC map.
6. Every pfδαC map is a pf$\delta\mathcal{S}$C map.
7. Every pfδαC map is a pf$\delta\mathcal{P}$C map.

Proof. (i) Let M be a pfcs in X_1. Since h_P is pfδC map, $h_P(M)$ is a pfδcs in X_2. Since every pfδcs is a pfcs, $h_P(M)$ is a pfcs in X_2. Hence h_P is a pfC.

(ii) Let M be a pfcs in X_1. Since h_P is pfC map, $h_P(M)$ is a pfcs in X_2. Since every pfcs is a pf$\delta\mathcal{S}$cs, $h_P(M)$ is a pf$\delta\mathcal{S}$cs in X_2. Hence h_P is a pf$\delta\mathcal{S}$C.

(iii) Let M be a pfcs in X_1. Since h_P is pfC map, $h_P(M)$ is a pfcs in X_2. Since every pfcs is a pf$\delta\mathcal{P}$cs, $h_P(M)$ is a pf$\delta\mathcal{P}$cs in X_2. Hence h_P is a pf$\delta\mathcal{P}$C.

(iv) Let M be a pfcs in X_1. Since h_P is pf$\delta\mathcal{S}$C map, $h_P(M)$ is a pf$\delta\mathcal{S}$cs in X_2. Since every pf$\delta\mathcal{S}$cs is a pfδβcs, $h_P(M)$ is a pfδβcs in X_2. Hence h_P is a pfδβC.

(v) Let M be a pfcs in X_1. Since h_P is pf$\delta\mathcal{P}$C map, $h_P(M)$ is a pf$\delta\mathcal{P}$cs in X_2. Since every pf$\delta\mathcal{P}$cs is a pfδβcs, $h_P(M)$ is a pfδβcs in X_2. Hence h_P is a pfδβC.

(vi) Let M be a pfcs in X_1. Since h_P is pfδαC map, $h_P(M)$ is a pfδαcs in X_2. Since every pfδαcs is a pf$\delta\mathcal{S}$cs, $h_P(M)$ is a pf$\delta\mathcal{S}$cs in X_2. Hence h_P is a pf$\delta\mathcal{S}$C.

(vii) Let M be a pfcs in X_1. Since h_P is pfδαC map, $h_P(M)$ is a pfδαcs in X_2. Since every pfδαcs is a pf$\delta\mathcal{P}$cs, $h_P(M)$ is a pf$\delta\mathcal{P}$cs in X_2. Hence h_P is a pf$\delta\mathcal{P}$C.

Example 4.1 Let $X = X_1 = X_2 = X_3 = X_4 = X_5 = \{x_1, x_2\}$ and the pfs's A_1, A_2 and A_3 are defined as

$$A_1 = \{< x_1, 0.020, 0.040 >, < x_2, 0.050, 0.050 >\}$$

$$A_2 = \{< x_1, 0.010, 0.040 >, < x_2, 0.050, 0.050 >\}$$

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$$A_3 = \{< x_1, 0.020, 0.030 >, < x_2, 0.050, 0.050 >\}$$

Here we have $\tau_1 = \{0_{X_1}, 1_{X_1}, A_1, A_2\}$, $\tau_2 = \{0_{X_2}, 1_{X_2}, A_2\}$, $\tau_3 = \{0_{X_3}, 1_{X_3}, A_1^c\}$, $\tau_4 = \{0_{X_4}, 1_{X_4}, A_2^c\}$ and $\tau_5 = \{0_{X_5}, 1_{X_5}, A_3\}$ be a pfts's on X. Let $h1_P: (X_2, \tau_2) \to (X_1, \tau_1)$, $h2_P: (X_3, \tau_3) \to (X_1, \tau_1)$, $h3_P: (X_4, \tau_4) \to (X_1, \tau_1)$, $h4_P: (X_5, \tau_5) \to (X_1, \tau_1)$ be an identity mapping. Then [(i)]

1. $h1_P$ is pfC (resp. pfδβC and pfδ$\mathcal{P}$C) but not pfδC (resp. pfδ$\mathcal{S}$C and pfδαC), because the set A_2^c is a pfcs in X_2 but $h1_P(A_2^c) = A_2^c$ is not pfδcs (resp. pfδ$\mathcal{S}$cs and pfδαcs) in X_1.
2. $h2_P$ is pfδ$\mathcal{S}$C but not pfδC, because the set A_1 is a pfcs X_3 but $h2_P(A_1) = A_1$ is not pfδcs in X_1.
3. $h3_P$ is pfδ$\mathcal{P}$C but not pfδC, because the set A_2 is a pfcs X_4 but $h3_P(A_2) = A_2$ is not pfδ$\mathcal{P}$cs in X_1.
4. $h4_P$ is pfδβC (resp. pfδ$\mathcal{S}$C) but not pfδ$\mathcal{P}$C (resp. pfδαC), because the set A_3^c is a pfcs in X_5 but $h4(A_3^c) = A_3^c$ is not pfδ$\mathcal{P}$cs (resp. pfδαcs) in X_1.

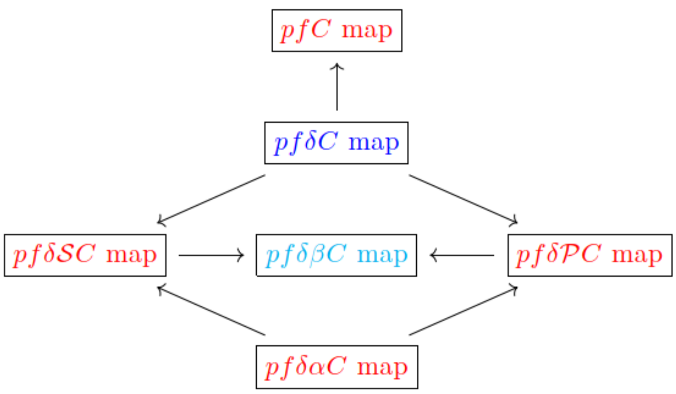

Fig. 2: $pf\delta C$ mappings in $pfts$.

Theorem 4.2 Let (X_1, Γ_P) & (X_2, Ψ_P) be any pfts's. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfδβC iff for each pfs μ of (X_2, Ψ_P) and for each pfos M of (X_1, Γ_P) containing $h_P^{-1}(\mu)$ there is an pfδβos ψ of (X_2, Ψ_P) such that $\mu \subseteq \psi$ and $h_P^{-1}(\psi) \subseteq M$.

Proof. Necessity: Assume h_P is a pfδβC. Let μ be the pfcs of (X_2, Ψ_P) and M is a pfos of (X_1, Γ_P) such that $h_P^{-1}(\mu) \subseteq M$. Then $\psi = Y - h_P^{-1}(M^c)$ is pfδβos of (X_2, Ψ_P) such that $h_P^{-1}(\psi) \subseteq M$.

Sufficiency: Assume ψ is a pfcs of (X_1, Γ_P). Then $(h_P(\psi))^c$ is a pfs of (X_2, Ψ_P) and ψ^c is pfos in (X_1, Γ_P) such that $h_P^{-1}((h_P(\psi))^c) \subseteq \psi^c$. By hypothesis there is a pfδβos ψ of (X_2, Ψ_P) such that $(h_P(\psi))^c \subseteq \psi$ and $h_P^{-1}(\psi) \subseteq \psi^c$. Therefore $\psi \subseteq (h_P^{-1}(\psi))^c$. Hence $\psi^c \subseteq h_P(\psi) \subseteq h_P((h_P^{-1}(\psi))^c) \subseteq \psi^c$ which implies $h_P(\psi) = \psi^c$. Since ψ^c is pfδβcs of (X_2, Ψ_P). Hence $h_P(\psi)$ is pfδβc in (X_2, Ψ_P) and thus h_P is pfδβC.

Theorem 4.3 Let (X_1, Γ_P), (X_2, Ψ_P) & (X_3, Φ_P) be any pfts's. If $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is pfC and $g_p: (X_2, \Psi_P) \to (X_3, \Phi_P)$ is pfδβC, then $g_p \circ h_P: (X_1, \Gamma_P) \to (X_3, \Phi_P)$ is pfδβC.

Proof. Let ψ be a pfcs in (X_1, Γ_P). Then $h_P(\psi)$ is pfcs of (X_2, Ψ_P) because h_P is pfC. Now $(g_p \circ h_P)(\psi) = g_p(h_P(\psi))$ is pfδβcs in (X_3, Φ_P) because g_p is pfδβC. Thus $g_p \circ h_P$ is pfδβC.

Theorem 4.4 Let (X_1,Γ_P) & (X_2,Ψ_P) be any pfts's. If $h_P:(X_1,\Gamma_P) \to (X_2,\Psi_P)$ is pfδβC, then $pf\delta\beta cl(h_P(\psi)) \subsetneq h_P(pfcl(\psi))$.

Proof. Necessity: Let h_P be a pfδβC and K be a pfcs in (X_1,Γ_P). Now, $K \subseteq pfcl(K)$ implies $h_P(K) \subseteq h_P(pfcl(K))$. Since h_P is a pfδβC, $(pf\delta\beta cl(h_P(K))$ is pfδβcs in (X_2,Ψ_P) such that $h_P(K) \subseteq pf\delta\beta cl(h_P(K))$ therefore $pf\delta\beta cl(h_P(K)) \subseteq h_P(pfcl(K))$.

Sufficiency: Assume K is a pfcs of (X_1,Γ_P). Then $h_P(K) = pf\delta\beta cl(h_P(K)) \subseteq h_P(pfcl(K))$. But $h_P(K) \subseteq pf\delta\beta cl(h_P(K))$. So $h_P(K) = pf\delta\beta cl(K)$ which implies $h_P(K)$ is a pfδβcs of (X_2,Ψ_P) and hence h_P is a pfδβC.

Theorem 4.5 Let $h_P:(X_1,\Gamma_P) \to (X_2,\Psi_P)$ and $g_p:(X_2,\Psi_P) \to (X_3,\Phi_P)$ be pfδβC mappings. If every pfδβcs of (X_2,Ψ_P) is pfc then, $g_p \circ h_P:(X_1,\Gamma_P) \to (X_3,\Phi_P)$ is pfδβC.

Proof. Let ψ be a pfcs in (X_1,Γ_P). Then $h_P(\psi)$ is pfδβcs of (X_2,Ψ_P) because h_P is pfδβC. By hypothesis $h_P(\psi)$ is pfcs of (X_2,Ψ_P). Now $g_p(h_P(\psi)) = (g_p \circ h)(\psi)$ is pfδβcs in (X_3,Φ_P) because g_p is pfδβC. Thus $g_p \circ h_P$ is pfδβC.

Theorem 4.6 Let (X_1,Γ_P) & (X_2,Ψ_P) be any pfts's. Let $h_P:(X_1,\Gamma_P) \to (X_2,\Psi_P)$ be a map, then the following statements are equivalent:

1. h_P is a pfδβO.
2. h_P is a pfδβC.
3. h_P^{-1} is pfδβCts.

Proof. (i) ⇒ (ii): Let us assume that h_P is a pfδβO.

By definition, ψ is a pfos in (X_1,Γ_P), then $h_P(\psi)$ is a pfδβos in (X_2,Ψ_P).

Here, ψ is pfcs in (X_1,Γ_P), then $X - \psi$ is a pfos in (X_1,Γ_P).

By assumption, $h_P(X - \psi)$ is a pfδβos in (X_2,Ψ_P).

Hence, $Y - h_P(X - \psi)$ is a pfδβcs in (X_2,Ψ_P).

Therefore, h_P is a pfδβC.

(ii) ⇒ (iii): Let ψ be a pfcs in (X_1,Γ_P) By (ii), $h_P(\psi)$ is a pfδβcs in (X_2,Ψ_P). Hence, $h_P(\psi) = (h_P^{-1})^{-1}(\psi)$, so h_P^{-1} is a pfδβcs in (X_2,Ψ_P). Hence, h_P^{-1} is pfδβCts.

(iii) ⇒ (i): Let ψ be a pfos in (X_1,Γ_P). By (iii), $(h_P^{-1})^{-1}(\psi) = h_P(\psi)$ is a pfδβO.

5 Application

In current scenario people with symptom of Covid-19 like fever, cough, sneezing, sore throat, loss of taste and smell etc., were panic about the disease, and the diagnosis of Covid-19 takes many hours and people cannot go for the test frequently. Some other diseases like flu, pneumonia, cold etc., also has the same symptoms.

Each patients has unique experience of that particular symptom and some time they may not experience that symptom even though they were affected by the Covid-19.

Here we tried to diagnosis Covid-19 with the help of Pythagorean fuzzy sets (in short pfs's) which helps to record all symptoms in prÃ©cised manner.

5.1 Algorithm and flow chart

This section includes the algorithm based on the computation of the Hamming distance, Normalized Hamming distance, Euclidean distance and Normalized Euclidean distance between the pfs's.

Step:1 Identify the universe set with most common symptoms of the Covid-19 patients.

Step:2 Formulates the pfs of each patient based on their experience of each symptom of universe set.

Step:3 Formulates the ideal pfs from the patients who affected by Covid-19 based on their experience of each symptom of covid-19.

Step:4 Compute the various distances between the ideal pfs of Covid-19 affected patients and the pfs of the patient who experiences the symptoms of Covid-19.

Step:5 Compare the distance between the pfs sets and also between the various distances.

Step:6 Conclude, the patient with minimum distance from the pfs of Covid-19 patient has the huge chance to affected by Covid-19 virus.

5.2 Flow chart

Identify universe set of Covid-19 symptoms

↓

Formulates the pfs of each patient

↓

Formulates the ideal pfs from the covid-19 patient

↓

Compute distance between the ideal pfs the pfs of the patient

↓

Conclude, huge chance for affected by covid-19 by who has minimum distance.

5.3 Example

Let A, B, C denote the patients who has the symptoms of Covid-19. Now their symptoms can be represented by the pfs and their members are taken from the universe set X which includes the all possible symptoms of Covid-19 are f denotes fever, t denotes tiredness, d denotes dry cough, s denotes shortness of breath, p denotes body pain / chest pain, a denotes diarrhea, l denotes loss of taste or smell, st denotes sore throat, b denotes difficulty in breathing and r denotes rhinorrhea.

We convert the frequency of experience of the symptoms of the patients from last three days as Pythagorean fuzzy set by considering the severe symptom as the degree of positive membership, mild symptom as the degree of nutral membership and no experience of that symptom as the degree of negative membership.

Now, the ideal Pythagorean fuzzy set $I = \{(x, \mu_i(x), \nu_i(x))/x \in X\}$ denotes the model set for the Covid-19 patient which is framed by the data collected from the hospital resources. The membership values of the elements of I are

$(f, \mu_i(f), \nu_i(f))$, where $\mu_i(f) \geq 0.8, \nu_i(f) \geq 0.1$

$(t, \mu_i(t), \nu_i(t))$, where $\mu_i(t) \geq 0.5, \nu_i(t) \geq 0.02$

$(d, \mu_i(d), \nu_i(d))$, where $\mu_i(d) \geq 0.7, \nu_i(d) \geq 0.02$

$(s, \mu_i(s), \nu_i(s))$, where $\mu_i(s) \geq 0.6, \nu_i(s) \geq 0.04$

$(p, \mu_i(p), \nu_i(p))$, where $\mu_i(p) \geq 0.4, \nu_i(p) \geq 0.05$

$(a, \mu_i(a), \nu_i(a))$, where $\mu_i(a) \geq 0.2, \nu_i(a) \geq 0.4$

$(l, \mu_i(l), \nu_i(l))$, where $\mu_i(l) \geq 0.6, \nu_i(l) \geq 0.06$

$(st, \mu_i(st), \nu_i(st))$, where $\mu_i(st) \geq 0.7, \nu_i(st) \geq 0.02$

$(b, \mu_i(b), \nu_i(b))$, where $\mu_i(b) \geq 0.5, \nu_i(b) \geq 0.16$

$(r, \mu_i(r), \nu_i(r))$, where $\mu_i(r) \geq 0.78, \nu_i(r) \geq 0.01$

$\mu_i(x)$ - severe symptom,

$\nu_i(x)$ - no symptom and

$0 \leq (\mu_i(x))^2 + (\nu_i(x))^2 \leq 1$.

The decision can be made, which patient have the more chances to have the Covid-19 by finding the distance between the ideal pfs and the pfs's of the patients A, B, C.

In the following table symptom, membership values, Ideal pfs, Patient A, Patient B, Patient C, Hamming Distance (I, A), Hamming Distance (I, B), Hamming Distance (I, C), Euclidean distance (I, A), Euclidean distance (I, B) and Euclidean distance (I, C) are briefly denoted as sym, mval, $IPFS$, PA, PB, PC, $HD(I, A)$, $HD(I, B)$, $HD(I, C)$, $ED(I, A)$, $ED(I, B)$ and $ED(I, C)$.

sym	mval	IPFS	PA	PB	PC	HD(I,A)	HD(I,B)	HD(I,C)	ED(I,A)	ED(I,B)	ED(I,C)
f	$\mu_i(f)$	0.08	0.7	0.59	0.5	0.10	0.21	0.09	0.01	0.04	0.09
f	$\nu_i(f)$	0.10	0.2	0.09	0.18	0.10	0.01	0.09	0.01	0.0	0.01
t	$\mu_i(t)$	0.5	0.7	0.84	0.03	0.2	0.34	0.47	0.04	0.12	0.22
t	$\nu_i(t)$	0.02	0.03	0.02	0.7	0.01	0.0	0.18	0.0	0.0	0.46
d	$\mu_i(d)$	0.7	0.6	0.62	0.4	0.1	0.08	0.3	0.01	0.01	0.09
d	$\nu_i(d)$	0.02	0.01	0.05	0.40	0.01	0.03	0.38	0.0	0.0	0.14
s	$\mu_i(s)$	0.6	0.02	0.29	0.2	0.58	0.31	0.4	0.34	0.1	0.16
s	$\nu_i(s)$	0.04	0.01	0.1	0.6	0.03	0.06	0.56	0.0	0.0	0.31
p	$\mu_i(p)$	0.4	0.03	0.3	0.16	0.37	0.1	0.24	0.14	0.01	0.06
p	$\nu_i(p)$	0.05	0.07	0.2	0.5	0.02	0.15	0.45	0.0	0.02	0.20
a	$\mu_i(a)$	0.2	0.09	0.72	0.4	0.11	0.52	0.2	0.01	0.27	0.04
a	$\nu_i(a)$	0.4	0.26	0.02	0.2	0.14	0.38	0.2	0.02	0.14	0.04
l	$\mu_i(l)$	0.6	0.57	0.44	0.3	0.03	0.16	0.30	0.00	0.03	0.09
l	$\nu_i(l)$	0.06	0.08	0.19	0.63	0.02	0.13	0.57	0.00	0.02	0.32
st	$\mu_i(st)$	0.7	0.72	0.41	0.5	0.02	0.29	0.2	0.0	0.08	0.04
st	$\nu_i(st)$	0.02	0.04	0.8	0.24	0.02	0.06	0.22	0.0	0.0	0.05
b	$\mu_i(b)$	0.5	0.44	0.6	0.21	0.06	0.1	0.89	0.0	0.01	0.08
b	$\nu_i(b)$	0.16	0.19	0.2	0.3	0.03	0.04	0.14	0.0	0.0	0.02
r	$\mu_i(r)$	0.78	0.65	0.6	0.51	0.13	0.18	0.27	0.02	0.03	0.07
r	$\nu_i(r)$	0.01	0.08	0.18	0.20	0.07	0.17	0.19	0.0	0.03	0.04

Calculation:

Hamming distance:

$d_{HD}(I, A) = 1.075$

$d_{HD}(I, B) = 1.660$

$d_{HD}(I, C) = 3.220$

Normalized Hamming distance:

$d_{NHD}(I, A) = 0.108$

$d_{NHD}(I, B) = 0.166$

$d_{NHD}(I, C) = 0.322$

Euclidean distance:

$ED(I, A) = 0.550$

$ED(I, B) = 0.677$

$ED(I, C) = 1.128$

Normalized Euclidean distance:

$NED(I, A) = 0.174$

$NED(I, B) = 0.214$

$NED(I, C) = 0.357$

Threfore, from the above table we observe that

$$d_{HD}(I, A) < d_{HD}(I, B) < d_{HD}(I, C)$$

$$d_{NHD}(I,A) < d_{NHD}(I,B) < d_{NHD}(I,C)$$
$$ED(I,A) < ED(I,B) < ED(I,C) \text{ and}$$
$$NED(I,A) < NED(I,B) < NED(I,C)$$

with this evidence we may conclude that the patient A have the more chance to affected by the Covid-19 among these three patients.

6 Conclusion

In this paper, Some new notions of strongly Pythagorean fuzzy open (closed) maps called Pythagorean fuzzy δ-open and Pythagorean fuzzy δ-closed maps are introduced and discussed their relationship between their near mappings with examples. Also, we have tried to diagnosis Covid-19 with the help of Pythagorean fuzzy sets which helps to record all symptoms in prÃ©cised manner. In future, researchers can extend this model to other extensions of fuzzy sets such as rough sets and utilize the interdependency among the various evaluation criteria for better judgement.

$$\sqrt{\left(\frac{a}{b}+\frac{c}{d}\right)^2+\left(\frac{e}{f}+\frac{g}{h}\right)^2+\left(\frac{i}{j}+\frac{k}{l}\right)^2} \quad (1)$$

$$\sqrt{\left(\frac{a}{b}+\frac{c}{d}\right)^2+\left(\frac{e}{f}+\frac{g}{h}\right)^2+\left(\frac{i}{j}+\frac{k}{l}\right)^2} \quad (2)$$

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