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Inverse Independent Outer Connected Domination Number of Few Classes of Graphs

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Abstract

In a graph $G = (V, E)$, a vertex set $D \subseteq V$ such that for every vertex $v \notin D$ the vertex v is adjacent to atleast one vertex in D and no vertex in D is adjacent to each other, then ,D is the independent dominating set.when the vertex set $V - D \subseteq V$ contains a independent dominating set, say D', then D' is called the inverse independent domiating set. If the subgraph induced by $V - D'$ is connected then D' is called the inverse independent outer connected dominating set. The minimum cardinality of such set of vertices is called the *Inverse Independent Outer Connected Domination Number* denoted by $\tilde{\gamma}^{-}_{IOC}(G)$. In this paper we will be discussing $\tilde{\gamma}^{-}_{IOC}S(n,k)$, $\tilde{\gamma}^{-}_{IOC}S^{+}(n,k)$, $\tilde{\gamma}^{-}_{IOC}S^{++}(n,k)$, $\tilde{\gamma}^{-}_{IOC}(S_n)$, $\tilde{\gamma}^{-}_{IOC}(G_1 \circ G_2)$ where $S(n,k)$, $S^{+}(n,k)$, $S^{++}(n,k)$, S_n are Sierpinski Graph, Extended Sierpinski Graphs and Sierpinski Gasket Graph respectively and G_1 and G_2 are two standard graphs.

Keywords Inverse Independent domination, Outer Connected, Inverse Independent Outer Connected Domination Number, Sierpinski Graph, Sierpinski-like Graphs, Corona of Graphs

1 Introduction

Let $G = (V, E)$ be a simple, finite, undirected graph, with $V(G)$ being the vertex set and $E(G)$ being the edge set of the graph. We we will be using V instead of $V(G)$ and E instead of $E(G)$ for simplicity. As the Independent Inverse Outer Connected Domination Number does not require a graph with self-loops and that with multiple edges, we have chosen only the simple graphs. For any vertex $v \in V$ and set $S \subseteq V$, the open neighborhood of v in S is the set $N_s(v) = u \in S | uv \in E$. The closed neighbourhood of v in S is $N_s[v]$ $N_s(v) \cup v$. If $S = V$, then we simply write $N(v)$ and $N[V]$ rather than $N_v(V)$.

For a graph G , a set $D \subseteq V$ is a dominating set if every $v \notin D$ is adjacent to atleast one element in D. The minimum domination number is denoted by $\tilde{\gamma}(G)$. If no, $v \in D$, is adjacent to each other, then D is called the Independent Dominating Set. The minimum cardinality of the Independent Dominating Set is called the Independent Domination Number , denoted by $\tilde{\gamma}_I(G)$. If $V - D$ contains an Independent dominating set say $D' \subseteq V$,then D' is called an Inverse Independent Dominating set of G with respect to D. The number of vertices in the smallest Inverse Independent Dominating Set is called the Inverse Independent Domination Number, denoted by $\tilde{\gamma}_I^{-1}(G)$.

If the set $V - D'$ induces a connected subgraph, where D' is an Inverse Independent Dominating set of the graph G, then D' is called the Inverse Independent Outer Connected Dominating Set , abbreviated as IIOCD-set. The cardinality of the minimum IIOCD-set is the IIOCD Number which is denoted by $\tilde{\gamma}_{IOC}^{-1}(G)$.

S. Klavzar and U. Milutinovic were the first to introduce the concept of Sierpinski graphs in [7]. the Sierpinski graphs are denoted as $S(n, k)$. As per the definition in [8], the vertex set of $S(n, k)$ consists of all n-tuples of integers 1,2,...,k , for integers $n \geq 1$ and $k \geq 3$. The cardinality of $V(G)$ of a sierpinski graph will be k^n. As in [1], [6] the Sierpinski graphs S(n,k), $n \geq 1$ are defined in the following way:
$V(S(n, k)) = 1, 2, ..., k^n$, two different vertices $u = (u_1, ..., u_n)$ and $v = (v_1, ..., v_n)$ being adjacent if and only if there exists an $h \in 1, ..., n$ such that
(i) $u_t = v_t$, for $t = 1, ..., h - 1$;
(ii) $u_h \neq v_h$; and
(iii) $u_t = v_h$ and $v_t = u_h$ for t = h+1,...,n.
For the labelling of a sieprinski graph each n-tuple is labelled in such a way that the first $'n - 1'$ members of the n-tuple denotes the level of the subgraph in which the vertex is chosen. For instance, we can say that the first member denotes the subgraph $S(n - 1, k)$ and the $(n - 1)^{th}$ member denotes the subgraph $S(1, k)$ in which the vertex is chosen. The n^{th} member denotes the position of the vertex in the graph. The extended Sierpinski graph $S^+(n, k)$ for $n \geq 1$ and $k \geq 3$ is derived from the Sierpinski graph $S(n, k)$ by the addition of an extra vertex say, $'s'$ and the vertex $'s'$ is adjacent to the end vertices of the Sierpinski graph $S(n, k)$. The extended Sierpinski graph $S^{++}(n, k)$ for $n \geq 2$ and $k \geq 3$ is derived from the Sierpinski graph $S(n, k)$ by the addition of an extra copy of the Sierpinski graph, $S(n - 1, k)$ and the end vertices of this $S(n - 1, k)$ is adjacent to the end vertices of the Sierpinski graph $S(n, k)$. Now the Sierpinski gasket graphs are obtained from the Sierpinski graph $S(n, 3)$ where the edges joining each copy is removed. A Sierpinski gasket graph is denoted as S_n.

In the year 1970 Frucht and Harary in [5] had introduced the concept of corona product of two graphs G_1 and G_2 denoted by $G_1 \circ G_2$. The corona product is obtained by taking one copy of center graph G_1 and $|V(G_1)|$ copies of the outer graph G_2, where the i^{th} vertex of G_1 is adjacent to every vertex of the i^{th} copy of G_2. In [9] S.Nada, A.Elrokh, E.A.Elsakhawi, D.E.Sabrafollows had inferred from the definition of the corona that $G_1 \circ G_2$ has $n_1 + n_1 n_2$ vertices. It is easy to see that $G_1 \circ G_2$ is not in general isomorphic to $G_2 \circ G_1$. If G_1 and G_2

are two graphs, where G_1 has n vertices, the labeling of the corona $G_1 \circ G_2$ is often denoted by $[A : B_1, B_2, \ldots, B_n]$, where A is the labeling of the n vertices of G_1, and B_i, $1 \leq i \leq n$ is the labeling of the vertices of the copy of G_2 that is connected to the i^{th} vertex of G_1. The corona product is not commutative as well as associative. Joanna Cymann in [2] has observed the outer connected domination number of the cycle graphs and the complete graphs. In [4] Renario G. Hinampas, Jr. Jocecar Lomarda-Hinampas, Analyn Dahunan have found out the independent outer connected domination number of corona product of two connected non trivial graphs of order $m \geq 2$ and $n \geq 2$. Taking the corona product of the complete graphs and the cycle graphs in different combinations and imposing two more conditions (Inverse and Independent) on them we get few new results. We will be discussing the Inverse Independent Outer Connected Domination Number of the corona graph when G_1 and G_2 both are complete graphs with different number of vertices ; Inverse Independent Outer Connected Domination Number of the corona graph when G_1 is a cycle graph and G_2 is a complete graph with different number of vertices ; Inverse Independent Outer Connected Domination Number of the corona graph when G_1 is a complete graph and G_2 is a cycle graph with different number of vertices .

2 Inverse Independent Outer Connected Domination Number of the Sierpinski Graphs

Theorem 2.1. *Let $S(n, k)$ be a Sierpinski graph, consisting of k copies of $S(n-1, k)$ for $n > 1$, k is the number of vertices in the complete graph, then $\tilde{\gamma}_{IOC}^{-1}(S(n, k)) = k^{n-1}$ where $k \geq 5$, and $n \geq 1$*

Proof. Apply induction on $'n'$. For the initial value $n = 1$ $\tilde{\gamma}_{IOC}^{-1}(S(1, k)) = k^0 = 1$
Here all the $k's$ are the number of vertices in a complete graph. We know the IIOCD Number of a complete graph is 1.
The Outer Connectedness property : After the removal of one vertex from a complete graph with k vertices , the graph will still be a connected graph .
Hence $\tilde{\gamma}_{IOC}^{-1}(S(1, k)) = k^0$, $k \geq 5$, $n = 1$ holds true.

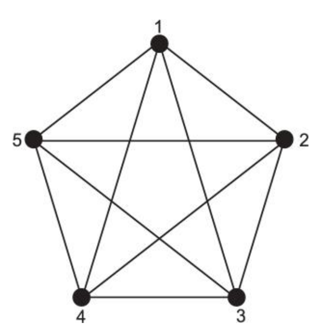

Figure 1: $(S(1, 5))$

We make an assumption that for $n = i$ the result holds true.
$\tilde{\gamma}_{IOC}^{-1}(S(i, k)) = k^{i-1}$, $k \geq 5$, $n = i$.

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To be proved that for $n = i + 1$, $\tilde{\gamma}^{-1}_{IOC}(S(i+1,k)) = k^{(i+1)-1}$, $\quad k \geq 5$, $\quad n = i + 1$

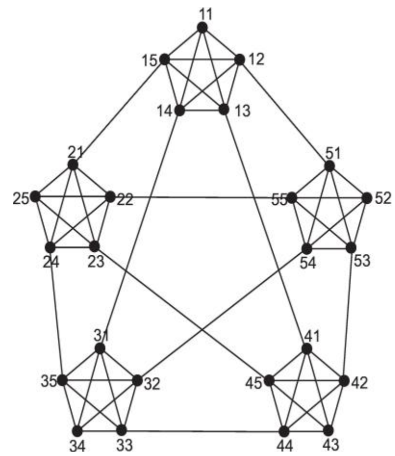

Figure 2: $(S(2,5))$

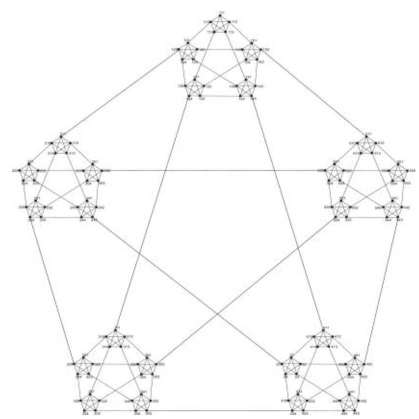

Figure 3: $(S(3,5))$

By the definition of Sierpinski graphs, there will be $'k'$ copies of $S(n,k)$ incorporated inside $S(n+1,k)$ and the unit graphs will always be complete graphs and hence we take k^{n-1} number of vertices from each copy. Figure 2 and Figure 3 show the Sierpinski graphs for k=5 and n=2 and n=3 respectively. Finally we will be left with $k \times k^{n-1}$ number of vertices that dominates all the edges of $S(n+1,k)$.

Therefore, $\tilde{\gamma}^{-1}_{IOC}(S(n,k)) = k \times k^{i-1} = k^{i} = k^{(i+1)-1}$, $\quad k \geq 5$, $\quad n = i + 1$

The Outer Connectedness property : To ensure the outer connectedness, vertex chosen must be such that , they dont engulf one unit and isolate it after the vertex deletion as the induced subgraph must be connected.

Hence the result

$\tilde{\gamma}^{-1}_{IOC}(S(n,k)) = k^{n-1}$, $\quad k \geq 5$, $\quad n \geq 1$.

□

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3 Inverse Independent Outer Connected Domination Number of Extended Sierpinski Graph $S^+(n,k)$

Theorem 3.1. *Let $S^+(n,k)$ be a Sierpinski-like graph, where the end vertices of the Sierpinski graph $S(n,k)$ is connected to an extra vertex then,*
$\tilde{\gamma}^{-1}_{IOC}(S^+(n,k)) = k^{n-1}$*, where* $k \geq 5$*, and* $n \geq 1$

Proof. Apply induction on $'n'$. For the initial value $n = 1$ $\tilde{\gamma}^{-1}_{IOC}(S^+(1,k)) = k^0 = 1$
Here all the $k's$ are the number of vertices in a complete graph. We know the IIOCD Number of a complete graph is 1,and also the end vertices of $S(1,k)$ are connected to an extra vertex 's'.
The Outer Connectedness property : After The removal of one vertex from a complete graph with k vertices , the graph will still be a connected graph . Hence $\tilde{\gamma}^{-1}_{IOC}(S^+(1,k)) = k^0$, $k \geq 5$, $n = 1$ holds true.

We make an assumption that for $n = i$ the result holds good.
$\tilde{\gamma}^{-1}_{IOC}(S^+(i,k)) = k^{i-1}$, $k \geq 5$, $n = i$.
To be proved that for $n = i + 1$ $\tilde{\gamma}^{-1}_{IOC}(S^+(i+1,k)) = k^{(i+1)-1}$, $k \geq 5$, $n = i + 1$

By the definition of the Extended Sierpinski graphs $S^+(n,k)$ is obtained from $S(n,k)$ by adding an extra vertex $'s'$,and edges joining s to all extreme vertices of $S(n,k)$. The unit graphs will always be complete graphs and hence we take k^{n-1} number of vertices from each copy and any one of these vertices will be an end vertex and hence these vertices will dominate all the vertices in the graph. Finally we will be left with $k \times k^{n-1}$ number of vertices that dominates all the edges of $S^+(n+1,k)$.
$\tilde{\gamma}^{-1}_{IOC}(S^+(n,k)) = k \times k^{i-1} = k^i = k^{(i+1)-1}$, $k \geq 5$, $n = i + 1$
The Outer Connectedness property : To ensure the outer connectedness, vertex chosen must be such that , they do not engulf one unit and isolate it after the vertex deletion as the induced subgraph must be connected.
Hence the result
$\tilde{\gamma}^{-1}_{IOC}(S(n,k)) = k^{n-1}$, $k \geq 5$, $n \geq 1$ holds good.

□

4 Inverse Independent Outer Connected Domination Number of Extended Sierpinski Graph $S^{++}(n,k)$

Theorem 4.1. *Let $S^{++}(n,k)$ be a Sierpinski-like graph obtained from $S(n,k)$ by adding a new copy of $S(n-1,k)$, denoted by $S_{k+1}(n-1,k)$ and joining the j^{th} extreme vertices of $S(n,k)$ and $S_{k+1}(n-1,k)$ with an edge, then,*
$\tilde{\gamma}^{-1}_{IOC}(S^{++}(n,k)) = k^{n-1} + k^{n-2}$*, where* $k \geq 5$*, and* $n \geq 2$

Proof. Apply induction on $'n'$. For The initial value $n = 1$; $\tilde{\gamma}^{-1}_{IOC}(S^+(1,k)) = k^0 = 1$
Here all the $k's$ are the number of vertices in a complete graph. We know the IIOCD Number of a complete graph is 1,and also the end vertices of $S(1,k)$ are connected to an extra vertex 's'.

The Outer Connectedness property : After The removal of one vertex from a complete graph with k vertices , the graph will still be a connected graph .
Hence $\tilde{\gamma}^{-1}_{IOC}(S^{+}(1, k)) = k^{0}, \quad k \geq 5, \quad n = 1$ holds true.

We make an assumption that for $n = i$ the result holds good.
$\tilde{\gamma}^{-1}_{IOC}(S^{+}(i, k)) = k^{i-1}, \quad k \geq 5, \quad n = i.$
To be proved that for $n = i + 1$ $\tilde{\gamma}^{-1}_{IOC}(S^{+}(i + 1, k)) = k^{(i+1)-1}, \quad k \geq 5, \quad n = i + 1$

By the definition of the Extended Sierpinski graphs $S^{+}(n, k)$ is obtained from $S(n, k)$ by adding an extra vertex 's',and edges joining s to all extreme vertices of $S(n, k)$. The unit graphs will always be complete graphs and hence we take k^{n-1} number of vertices from each copy and any one of these vertices will be an end vertex and hence these vertices will dominate all the vertices in the graph. Finally we will be left with $k \times k^{n-1}$ number of vertices that dominates all the edges of $S^{+}(n + 1, k)$.
$\tilde{\gamma}^{-1}_{IOC}(S^{+}(n, k)) = k \times k^{i-1} = k^{i} = k^{(i+1)-1}, \quad k \geq 5, \quad n = i + 1$
The Outer Connectedness property : To ensure the outer connectedness, vertex chosen must be such that , they do not engulf one unit and isolate it after the vertex deletion as the induced subgraph must be connected.
Hence the result
$\tilde{\gamma}^{-1}_{IOC}(S(n, k)) = k^{n-1}, \quad k \geq 5, \quad n \geq 1$ holds good.

□

5 Inverse Independent Outer Connected Domination Number of Sierpinki Gasket Graph

Theorem 5.1. *Let S_n be the Sierpinski Gasket graph, a variant of the Sierpinski graph $S(n, 3)$, then*

$$\gamma^{-1}_{IOC}(S_n) = \begin{cases} 3^{n-2}, (n \geq 3) \\ n, \quad n = 1, 2 \end{cases}$$

Proof. The graph S_n can be obtained from $S(n, 3)$ by contracting every edge of $S(n, 3)$ that lies in no triangle.
An induction on 'n' is applied.
For $n = 1$ the Sierpinski gasket graph will be a complete graph with 3 vertices , which is a triangle. Inverse Independent Domination Number of a complete graph with 3 vertices will be one.
The Outer Connectedness Property: If that one vertex is deleted the subgraph induced by it still remains connected.
$\tilde{\gamma}^{-1}_{IOC}(S_1) = 1$
In (S_2), see figure 4, the Inverse Independent Dominating Sets are chosen by selecting the vertex with maximum degree. Vertex labelled $\{p, q\}$ is chosen since the degree of such vertex is maximum. This vertex dominates 5 out of 6 vertices. Now the left out end vertex $\{rr\}$ is chosen, where $p, q, r \in \{1, 2, 3\}$ respectively. Hence the Inverse Independent Domination Number is 2.

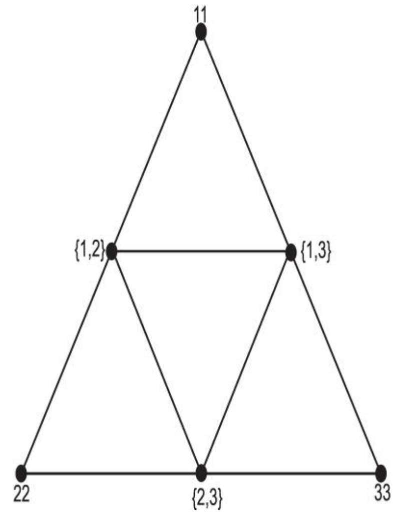

Figure 4: S_2

The graph induced by the remaining vertices is a connected graph.
$\tilde{\gamma}^{-1}_{IOC}(S_2) = 2$
Now we use induction hypothesis. For $n = 3$, to choose the Inverse Independent Dominating Set , from the first copy of (S_2) one vertex is chosen which has the maximum degree. Maximum degree in a Sierpinski gasket graph is 4. There are 15 vertices in (S_3). If we choose one vertex from each copy of (S_2) we will get three vertices that will dominate the rest of the twelve vertices. Refer Figure 5 for S_3
Therefore , $\tilde{\gamma}^{-1}_{IOC}(S_3) = 3$.

The Outer Connectedness Property: (S_2) is Inverse Outer Connected. Three copies of (S_2) is incorporated in (S_3). We have chosen one vertex less in (S_3) than the number of vertices chosen in (S_2) to satisfy the Inverse Independent Domination. Hence the graph induced by the vertices excluding the Inverse Independent Dominating Set is Connected. Figure 6 shows that S_3 is inverse outer connected.

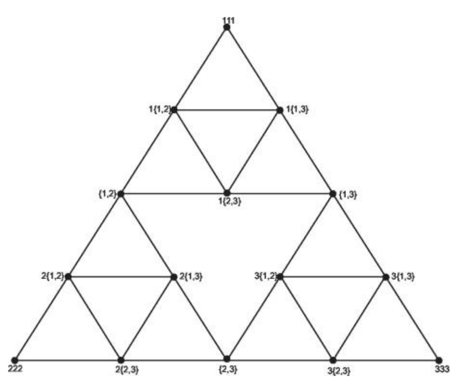

Figure 5: (S_3)

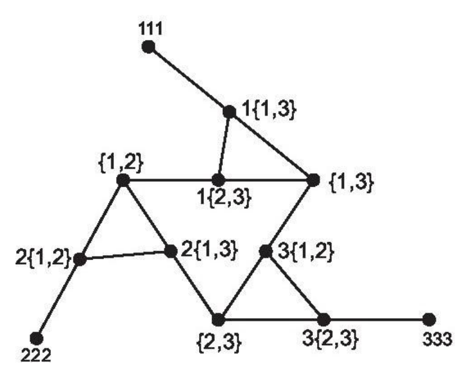

Figure 6: S_3 is outer connected

We make the assumption that the result is true for any $n = i$ where $i \geq 3$, then $\tilde{\gamma}^{-1}_{IOC}(S_i) = 3^{i-2}$
For $n = i + 1$, The Sierpinski Gasket Graph S_{i+1} is obtained by taking three copies of S_i. We take 3^{i-2} vertices in each j^{th} copy of S_i of S_{i+1}. Then $\tilde{\gamma}^{-1}_{IOC}(S_{i+1}) = 3 \times 3^{i-2} = 3^{(i+1)-2}$
The Outer Connectedness Property: (S_i) is Inverse Independent Outer Connected and hence (S_{i+1}) is also Inverse Independent Outer Connected as (S_1) is contained inside (S_{i+1}). To uphold this property we have to restrain ourselves from choosing vertices labelled $\{p, q\}$, $\{q, r\}$, $\{p, r\}$ where $\{p, q, r\} = \{1, 2, 3\}$ respectively.
Therefore $\tilde{\gamma}^{-1}_{IOC}(S_n) = 3^{n-2}, \quad n \geq 3$

□

6 Inverse Independent Outer Connected Domination Number of Corona of a finite simple undirected connected Graph and Complete Graph

Theorem 6.1. *G_1 is a complete graph with 'n' vertices and G_2 is a complete graph with 'm' vertices, then*
$\gamma^{-1}_{IOC}(K_n \circ K_m) = n$ *where* $n \geq 2$ *and* $m \geq 2$

Proof. The Inverse Independent Dominating set for such corona graphs can be chosen from G_2. Since G_1 and G_2 are complete graphs, choosing one vertex from the i^{th} copy of G_2 will dominate all the vertex in the i^{th} copy of G_2 and the i^{th} vertex of G_1. Hence we choose one vertex each from the G_2 graphs. The total number of G_2 graphs is equal to the number of vertices in G_1.
Therefore $\gamma^{-1}_{IOC}(K_n \circ K_m) = n$
A $K_6 \circ K_4$ graph is shown in figure 7 and the $\gamma^{-1}_{IOC}(K_6 \circ K_4) = 6$

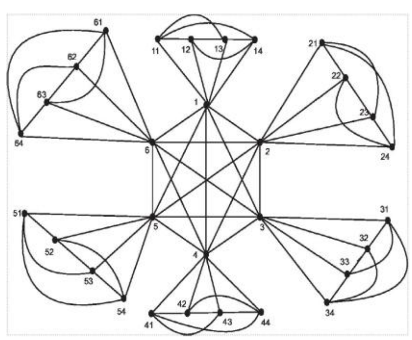

Figure 7: $\gamma^{-1}_{IOC}(K_6 \circ K_4)$

The Outer Connectedness Property: Removing one vertex from a complete graph will not affect the connectedness of the graph since all the vertices in G_2 are connected to all other vertices in G_2 and also it is connected to G_1 in one way or the other. Choosing a vertex fromthe graph G_1 must be restricted. □

Theorem 6.2. *G_1 is a cycle graph with 'n' vertices and G_2 is a complete graph with 'm' vertices, then*
$\gamma^{-1}_{IOC}(C_n \circ K_m) = n$ *where* $n \geq 3$ *and* $m \geq 2$

Proof. The first graph is a cycle graph and the second graph is a complete graph. We choose one vertex from each copy of G_2 which will dominate all vertices of that particular graph and also one vertex each of the graph G_1. Since we choose one vertex from different copies the vertices will be independent.
Therefore $$\gamma^{-1}_{IOC}(C_n \circ K_m) = n$$

The Outer Connectedness Property: Choosing a vertex from G_1 must be restricted. It will be inverse independent outer connected as the complete graphs after removing a vertex will be still connected and all the vertices of each G_2 are connected to G_1.

□

Note:
From Theorem 6.1 and Theorem 6.2, whenever G_1 is a finite simple undirected connected graph and G_2 is a complete graph, then the Inverse Independent Outer Connected Domination Number of the corona of these two graphs will be equal to the number of vertices in G_1. Since we are choosing the vertices from the complete graph G_2 which is connected to G_1 always. The type of the graph G_1 need not be considered except that it is connected.
The Outer Connectedness Property: Choosing a vertex from G_1 must be restricted.

7 Inverse Independent Outer Connected Domination Number of Corona of Complete Graph and Cycle Graph

Theorem 7.1. *G_1 is a complete graph with 'm' vertices and G_2 is a cycle graph with 'n' vertices, then*
$\gamma^{-1}_{IOC}(K_m \circ C_n) = m\gamma^{-1}_{I}(C_n)$ *where* $m \geq 2$ *and* $n \geq 3$

Proof. G_1 is a complete graph and G_2 is a cycle graph, the Inverse Independent Outer Connected Dominating Set of the corona of two graphs can be found out by choosing the Inverse Independent Dominating Set of the cycle graph. In the corona graphs the Inverse Independent Outer Connected Dominating Set is chosen from the second graph as it will dominate the vertices in G_2 as well as in G_1. Inverse Independent Dominating Set in each copy will be the same as the number of copies of G_2, that is same as the number of vertices in G_1.
Therefore $$\gamma^{-1}_{IOC}(K_m \circ C_n) = m\gamma_I^{-1}(C_n)$$

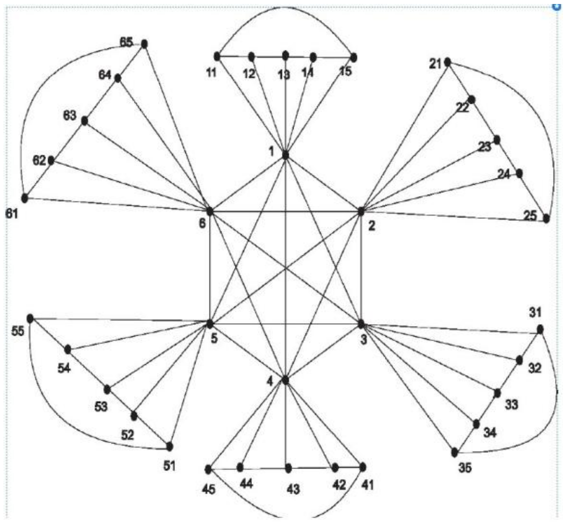

Figure 8: $\gamma_{IOC}^{-1}(K_6 \circ C_5)$

The $K_6 \circ C_5$ graph is shown in figure 8 and the $\gamma_{OC}^{-1}(K_6 \circ C_5) = 6 \times 2 = 12$
The Outer Connected Property: All the vertices in G_2 are connected to G_1 and G_1 is a complete graph. After deleting the Inverse Independent Dominating Set from the graph, the subgraph will again be a connected graph. A vertex from G_1 must be restricted to be chosen.

□

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