

Analyzing the Dynamics of HIV/AIDS Phobia with Stability and Awareness Strategies

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Abstract:

One of the risks to global health is sexually transmitted diseases (STDs). Usually, diseases do not cause death, but 50% of deaths are caused by fear of disease. This causes policymakers and experts studying diseases to become increasingly concerned. In this section we discuss about HIV/AIDS phobia, a particular type of nosophobia is an excessive and illogical dread of contracting HIV/AIDS. It is still a major global health concern due to its high death rate, and in most African nations as well as other places, it is the main source of HIV/AIDS anxiety. We develop a five-component deterministic model to examine how phobia affects HIV/AIDS dynamics within a particular population. The system's HIV/AIDS-free equilibrium is considered asymptotically stable when the effective reproduction number $\mathfrak{R}_0 < 1$, and unstable in other cases. Additionally, we used to investigate the endemic equilibrium's stability by using the Lyapunov function, positivity, boundedness, Lipschitz condition, and the conditions for the existence of uniqueness are discussed. The outcome demonstrates that everyone must have awareness about HIV/AIDS. Unfortunately, those affected with HIV/AIDS don't fear; take healthy food, and medicines; and exercise regularly; the counselling about HIV/AIDS will prolong life and reduce the phobia of HIV/AIDS in the population. The HIV/AIDS phobia epidemic model is analyzed by way of Matrix Laboratory.

Keywords: Epidemic Model, Sexually transmitted diseases (STDs), HIV/AIDS phobia, HIV free equilibrium (HFE), HIV endemic equilibrium (HEE), Stability, Lipschitz and Lyapunov.

1. Introduction:

Many people are afraid of "cancer" and "AIDS." Even if those worries could be quite reasonable, what would happen if they started to rule your life? "HIV phobia is an extreme fear of contracting HIV." especially the anxiety of getting infected with the virus, even if your risk is relatively minimal. Put differently, the worry is excessive and unfounded. Before very effective antiretroviral medication was available, in the 1980s and 1990s, there was a lot of documentation and description of AIDS phobia. These kinds of phobias are not well known in human behaviour. According to some mental health

professionals, the reason could be inherited—a predisposition to acquire phobias as a result of your inherited composition. HIV-phobias frequently believe they have the virus, and even negative tests cannot make them feel any less afraid. Some people think that traumatic experiences and incidents in a person's life are the cause of phobias. For instance, knowing someone who drowned may cause someone to develop a phobia of the water. Similarly, knowing those who have succumbed to the illness or been critically ill could also cause someone to develop an HIV fear. Some people, despite their obviously irrational behaviour, will stop at nothing to prevent contracting HIV. African countries account for eight of the top 10 countries with the highest number of AIDS-related deaths worldwide. It makes reasonable that someone with severe anxiety problems would mistake early HIV infection symptoms for other; more frequent illnesses because they are so similar to them. These symptoms exacerbate the worry because they are common to many ailments. Furthermore, even if they are not physically ill, their dread has the ability to cause them to become so consumed by their worry about infection that they are able to physically manifest these symptoms in their minds. Lerman refers to this as "Pseudo-AIDS." An HIV-phobic individual assumes they have HIV whenever they have comparable symptoms. HIV phobia typically presents as physical symptoms, according to Phillips. These symptoms—which include nausea, fever, rapid weight loss, night sweats, exhaustion, diarrhea, mouth or vaginal sores, and headaches—can resemble those that some people have experienced after recently becoming infected with HIV. An HIV phobia is frequently greatly influenced by culture.

Phobias stem from biological and social elements as well as experiences. The subject of how HIV fear arises has no universally accepted explanation. After engaging in one condom-free sexual encounter, an individual may start to fear that they might be infected with a STD. At least one HIV test should be taken by anyone between the ages of 13 and 64, according to the CDC [Centers for Disease Control and Prevention]. This may often be done at your yearly physical examination. Ask your doctor about it if you haven't had the test. Tests should be performed more frequently if you are at a higher risk; ideally, every three or six months. However, Pantalone notes that a further factor contributing to the lack of testing is people's misconception that the high risk of the illness "fits within an identity" when in fact it's a virus spread by normal human conduct, such as having sex. You need to get tested for HIV if you've ever engaged in condom-free sexual activity. Pantalone advises against doing anything infrequently, even if it seems little risk.

According to the CDC, you have an increased chance of contracting HIV if you can say "yes" to any of the following questions:

2. Have you had sex with an HIV-positive person, either anally or vaginally?
3. Since your last HIV test, have you dated anyone more than once?
4. Have you given anyone access to needles, injectable medications, or other injection supplies?
5. Have you ever traded drugs or cash for sex?
6. Have you ever engaged in sexual activity with someone whose past you are unaware of?
7. Have you received medical attention or a diagnosis for any other STD?
8. Do you identify as a male who has slept with another man?

We note that health behaviour theory needs to address the mechanisms relating many levels of influence on behaviour and offer useful direction for multi-level behaviour modification treatments tailored to particular settings. We develop a mathematical model to examine the causes of the sexually

transmitted disease (STD) outbreaks and the best ways to prevent or lessen them. A model that represents the effect of phobia on HIV/AIDS interaction with disease transmission in the population is considered in this research. We start by describing some basic properties of the system, including equilibrium and its fundamental reproduction value. Additionally, we investigate if non-negative solutions to the given process exist and are unique. Mathematically, the concepts of difference equations are used to develop the epidemic model of HIV/AIDS. The properties of the qualitative HIV/AIDS epidemic model are analyzed. The definite function of Lyapunov is derived from the equation of difference to stabilize the system of the HIV/AIDS epidemic model.

HIV and HSV, the two main virus-caused STDs, are incurable and can have detrimental long-term effects on one's health. Lifelong HSV infection is also characterized by recurrent outbreaks at the sites of infection. HSV type 1 primarily affects the mouth and lips (cold sores), while HSV type 2 primarily affects the genital area, albeit genital HSV type 1 is on the rise. Despite the lack of a vaccine or cure, antiviral drugs can lessen the symptoms of HSV. There is a significant chance that a newborn will die or become severely disabled if HSV is spread during or after childbirth. HIV/AIDS risk is also increased by HSV type 2 infections. Since they were first used centuries ago to prevent STDs, male condoms have gained popularity as a key component of HIV/AIDS preventive strategies. Male condoms are a very efficient STD prevention tool that can cut the risk of infection by 80% when used correctly and consistently. In particular in Sub-Saharan Africa, the usage of female condoms as a method of female managed prevention has grown in popularity. Despite their effectiveness, female condoms are still not widely accepted by both women and their sexual partners.

Safe sexual behaviour has numerous benefits for both your physical and emotional well-being. You might be able to safeguard your mental health by taking precautions against sexually transmitted diseases (STDs), such as HIV. Because mental health and HIV seem to be related issues, that can be challenging. In both HIV-positive and at-risk populations, mental health issues are more common. HIV risk is positively correlated with mental health. As a result of having HIV, some people may experience mental health issues, but others may develop them because they are afraid of contracting the virus. The diagnosis of a sexually transmitted infection (STI), particularly HIV, can also set off symptoms of other mental health conditions. One of the most prevalent mental health conditions affecting people living with HIV, for instance, is depression. Individuals living with HIV have twice the chance of developing depression compared to those who are at risk for HIV but do not yet have the virus. You might not be able to get tested, get results, seek or receive care, or continue your care if you have a mental health condition. By taking precautions to avoid contracting or spreading HIV, you may enhance your general mental well-being.

Psychotherapy and medicine are two possible treatments for people with a crippling fear of HIV. Investigating the underlying causes of the anxieties may be more crucial than seeking all the information about the illness from a doctor or counsellor, even though doing so may be helpful. Many times, there will be no connection at all between the phobia and HIV. It typically helps to sit with a qualified mental health expert. Family counselling, group therapy, and individual therapy are all possible forms of treatment. Prescription medications such as Zoloft and Lexapro may be helpful for people who have been diagnosed with anxiety disorders. One's life need not be dictated by HIV fear. Returning to your best self, you can enjoy a life of fulfilment.

9. **Epidemic model of HIV/AIDS:** The HIV/AIDS epidemic model is mathematically portrayed by the system of equations with the following suppositions.

ϖ =Existing populace

H_s =No. of susceptible

H_i =No. of infected

H_c =No. of HIV patients attended Counselling

H_n =No. of HIV patients didn't attend the Counselling

H_a = Final stage of HIV/AIDS

ϖ_1 =Rate of phobia about HIV

ϖ_2 = Proper counselling about HIV and social phobia

ϖ_3 =Not proper counselling about HIV and social phobia

ϖ_4 =The rate of patient's relief from HIV phobia and their lifetime extend

ϖ_5 =The rate of patient's not relieved from HIV phobia and their lifetime is easily reduced.

v =Normal death rate at all stages

$v + a$ =The population death rate of infected HIV.

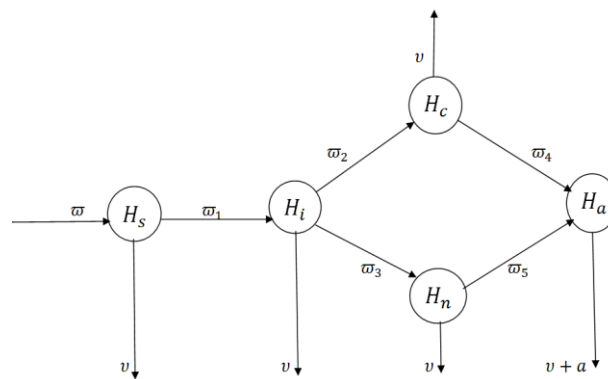


Fig 1: HIV/AIDS phobia epidemic model of flow diagram

Form epidemic model of HIV/AIDS is allowed by the accompanying arrangement of D.E.

$$\frac{dH_s}{dt} = \varpi - (\varpi_1 + v) H_s \quad (1)$$

$$\frac{dH_i}{dt} = \varpi_1 H_s - (\varpi_2 + \varpi_3 + v) H_i \quad (2)$$

$$\frac{dH_c}{dt} = \varpi_2 H_i - (\varpi_4 + v) H_c \quad (3)$$

$$\frac{dH_n}{dt} = \varpi_3 H_i - (\varpi_5 + v) H_n \quad (4)$$

$$\frac{dH_a}{dt} = \varpi_4 H_c + \varpi_5 H_n - (v + a) H_a \quad (5)$$

Where $H_s(0) > 0, H_i(0) > 0, H_c(0) > 0, H_n(0) > 0,$ and $H_a(0) > 0$ are initial condition

10. Determination of Fixed Points

To find the equilibrium points $(\widetilde{H}_s, \widetilde{H}_i, \widetilde{H}_c, \widetilde{H}_n, \widetilde{H}_a)$ of the system Equations (1-5), we set the derivatives equal to zero. So, at equilibrium states, we get

$$\left\{ \begin{array}{l} \varpi - (\varpi_1 + \nu)\widetilde{H}_s = 0 \\ \varpi_1\widetilde{H}_s - (\varpi_2 + \varpi_3 + \nu)\widetilde{H}_i = 0 \\ \varpi_2\widetilde{H}_i - (\varpi_4 + \nu)\widetilde{H}_c = 0 \\ \varpi_3\widetilde{H}_i - (\varpi_5 + \nu)\widetilde{H}_n = 0 \\ \varpi_4\widetilde{H}_c + \varpi_5\widetilde{H}_n - (\nu + a)\widetilde{H}_a = 0 \end{array} \right. \quad (6)$$

4. Equilibrium states

4.1 HIV free equilibrium (HFE) state

The HIV free equilibrium for the HFE, we replace the variables as $\mathfrak{E}^0 = (\widetilde{H}_s, \widetilde{H}_i, \widetilde{H}_c, \widetilde{H}_n, \widetilde{H}_a) = (H_s^0, H_i^0, H_c^0, H_n^0, H_a^0)$ is characterized as the place where no disease is available in the populace. Every one of the contaminated classes will be equivalent to zero.

$$\begin{aligned} \frac{dH_s}{dt} &= \varpi - (\varpi_1 + \nu) H_s = 0 \\ \varpi - \varpi_1 H_s - \nu H_s &= 0 \end{aligned}$$

But, $\varpi_1 = 0$

$$\begin{aligned} \varpi - \nu H_s &= 0 \\ H_s &= \frac{\varpi}{\nu} \end{aligned}$$

Thus, the HIV free equilibrium satisfies

$$\mathfrak{E}^0 = (H_s^0, H_i^0, H_c^0, H_n^0, H_a^0) = \left(\frac{\varpi}{\nu}, 0, 0, 0, 0\right)$$

4.2 HIV endemic equilibrium (HEE) state

For the HEE, we replace the variables as $\mathfrak{E}^1 = (\widetilde{H}_s, \widetilde{H}_i, \widetilde{H}_c, \widetilde{H}_n, \widetilde{H}_a) = (H_s^1, H_i^1, H_c^1, H_n^1, H_a^1)$ is defined as areas where the population has HIV. In this case, all affected classes do not equal zero. The point at which the disease exists within the susceptible population referred to as the endemic equilibrium. We set the equations resulting from our model is zero.

From $\frac{dH_s}{dt} = \varpi - (\varpi_1 + \nu) H_s$

Making H_s the subject of the equation we get;

$$\varpi - (\varpi_1 + \nu) H_s = 0$$

$$H_s^1 = \frac{\varpi}{(\varpi_1 + \nu)}$$

From $\frac{dH_i}{dt} = \varpi_1 H_s - (\varpi_2 + \varpi_3 + \nu) H_i$

Making H_i the subject of the equation we get;

$$\varpi_1 H_s - (\varpi_2 + \varpi_3 + \nu) H_i = 0$$

$$H_i^1 = \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} H_s^1$$

From $\frac{dH_c}{dt} = \varpi_2 H_i - (\varpi_4 + \nu) H_c$

Making H_c the subject of the equation we get;

$$\varpi_2 H_i - (\varpi_4 + \nu) H_c = 0$$

$$H_c^1 = \frac{\varpi_2}{(\varpi_4 + \nu)} H_i^1$$

From $\frac{dH_n}{dt} = \varpi_3 H_i - (\varpi_5 + \nu) H_n$

Making H_n the subject of the equation we get;

$$\varpi_3 H_i - (\varpi_5 + \nu) H_n = 0$$

$$H_n^1 = \frac{\varpi_3}{(\varpi_5 + \nu)} H_i^1$$

From $\frac{dH_a}{dt} = \varpi_4 H_c + \varpi_5 H_n - (\nu + a) H_a$

Making H_a the subject of the equation we get;

$$\varpi_4 H_c + \varpi_5 H_n - (\nu + a) H_a = 0$$

$$H_a^1 = \frac{\varpi_4}{(\nu + a)} H_c^1 + \frac{\varpi_5}{(\nu + a)} H_n^1$$

By substituting the value of H_s^1 in H_i^1 we getting,

$$H_i^1 = \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} \frac{\varpi}{(\varpi_1 + \nu)} = \frac{\varpi \varpi_1}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)}$$

By putting the value of H_i^1 in H_c^1 we have,

$$H_c^1 = \frac{\varpi_2}{(\varpi_4 + \nu)} \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} \frac{\varpi}{(\varpi_1 + \nu)} = \frac{\varpi \varpi_1 \varpi_2}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_4 + \nu)}$$

By substituting the value of H_i^1 in H_n^1 we getting,

$$H_n^1 = \frac{\varpi_3}{(\varpi_5 + \nu)} \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} \frac{\varpi}{(\varpi_1 + \nu)} = \frac{\varpi \varpi_1 \varpi_3}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_5 + \nu)}$$

By substituting the values of H_c^1 and H_n^1 in H_a^1 we have,

$$H_a^1 = \frac{\varpi_4}{(\nu + a)} \frac{\varpi_2}{(\varpi_4 + \nu)} \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} \frac{\varpi}{(\varpi_1 + \nu)} + \frac{\varpi_5}{(\nu + a)} \frac{\varpi_3}{(\varpi_5 + \nu)} \frac{\varpi_1}{(\varpi_2 + \varpi_3 + \nu)} \frac{\varpi}{(\varpi_1 + \nu)}$$

$$H_a^1 = \frac{\varpi \varpi_1 \varpi_2 \varpi_4 (\varpi_5 + \nu) + \varpi \varpi_1 \varpi_2 \varpi_5 (\varpi_4 + \nu)}{(\nu + a)(\varpi_4 + \nu)(\varpi_5 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_1 + \nu)}$$

Hence, the HIV Endemic equilibrium satisfies

$$\mathcal{E}^1 = (H_s^1, H_i^1, H_c^1, H_n^1, H_a^1) = \left(\frac{\varpi}{(\varpi_1 + \nu)}, \frac{\varpi\varpi_1}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)}, \frac{\varpi\varpi_1\varpi_2}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_4 + \nu)}, \frac{\varpi\varpi_1\varpi_3}{(\varpi_1 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_5 + \nu)}, \frac{\varpi\varpi_1\varpi_2\varpi_4(\varpi_5 + \nu) + \varpi\varpi_1\varpi_2\varpi_5(\varpi_4 + \nu)}{(\nu + a)(\varpi_4 + \nu)(\varpi_5 + \nu)(\varpi_2 + \varpi_3 + \nu)(\varpi_1 + \nu)} \right)$$

5. Basic reproduction number (\mathcal{R}_0)

In this segment, we get the basic reproduction number (\mathcal{R}_0) which is defined as the quantity of secondary infection created by a single infected person when introduced into a totally susceptible populace. To get the basic reproduction number (\mathcal{R}_0), we take on the same methodology utilized in where the authors utilized \mathcal{F} and \mathcal{V} of two matrices, representing the remaining terms of transfer as \mathcal{V} and the new infection terms as \mathcal{F} .

Our model has only five compartments with only one infective compartment.

$$\text{i. e., } \frac{dH_i}{dt} = \varpi_1 H_s - (\varpi_2 + \varpi_3 + \nu) H_i$$

We have that $\mathcal{F} = [\varpi_1 H_s]$ and $\mathcal{V} = [(\varpi_2 + \varpi_3 + \nu) H_i]$

At the equilibrium of disease free, the matrix \mathcal{F} becomes

$$\mathcal{F} = \begin{bmatrix} \varpi_1 & \varpi \\ & \nu \end{bmatrix}$$

At the equilibrium of disease free, the matrix \mathcal{V} becomes

$$\mathcal{V} = (\varpi_2 + \varpi_3 + \nu)$$

The inverse matrix of \mathcal{V} becomes

$$\mathcal{V}^{-1} = \frac{1}{(\varpi_2 + \varpi_3 + \nu)}$$

Let $\mathcal{R}_0 = \mathcal{F}\mathcal{V}^{-1}$

$$\text{Then } \mathcal{R}_0 = \begin{bmatrix} \varpi_1 & \varpi \\ & \nu \end{bmatrix} \begin{bmatrix} \frac{1}{(\varpi_2 + \varpi_3 + \nu)} \\ \end{bmatrix}$$

$$\mathcal{R}_0 = \frac{\varpi_1\varpi}{\nu(\varpi_2 + \varpi_3 + \nu)}$$

Using the characteristic equation $|\mathcal{F}\mathcal{V}^{-1} - \Lambda I| = 0$. From equation of \mathcal{R}_0 we compute the eigenvalues as follows

$|\mathcal{F}\mathcal{V}^{-1} - \Lambda I| = 0$ implies $|\mathcal{R}_0 - \Lambda I| = 0$, where I is an identity matrix.

$$\left| \frac{\varpi_1\varpi}{\nu(\varpi_2 + \varpi_3 + \nu)} - \Lambda \right| = 0$$

$$\frac{\varpi_1\varpi}{\nu(\varpi_2 + \varpi_3 + \nu)} - \Lambda = 0$$

$$\Lambda = \frac{\varpi_1\varpi}{\nu(\varpi_2 + \varpi_3 + \nu)}$$

Hence, \mathfrak{R}_0 of basic reproduction number is given as

$$\mathfrak{R}_0 = \frac{\varpi_1 \varpi}{\nu(\varpi_2 + \varpi_3 + \nu)} < 1.$$

Epidemiologically, for widely utilized infection models when:

$\mathfrak{R}_0 < 1$; this implies that the pathogen will be prevented from entering the population.

$\mathfrak{R}_0 = 1$; this implies that the pathogen will be constant.

$\mathfrak{R}_0 > 1$; this implies that the infection can begin to spread throughout the populace

Considering these criticisms of our model, we have

If $\varpi_1 \varpi < \nu(\varpi_2 + \varpi_3 + \nu)$, the occurrence of the infection and phobia will decrease.

If $\varpi_1 \varpi = \nu(\varpi_2 + \varpi_3 + \nu)$, the infection and phobia occurrence will be constant.

If $\varpi_1 \varpi > \nu(\varpi_2 + \varpi_3 + \nu)$, the frequency of infection and fear will rise..

As a result, we also came to the following results.

6. Uniqueness and Existence

We can write the system (1–5) like this:

$$\frac{d\mathfrak{X}}{dt} = \mathfrak{F}(\mathfrak{X}), \mathfrak{X} = \mathfrak{X}_0, \quad (7)$$

In the case where $\mathfrak{X} = (H_s, H_i, H_c, H_n, H_a)^T$ the state variables are contained in a column vector in \mathcal{R}^5 , which also defines a mapping from $[0, +\infty)$ to \mathcal{R}^5 . But still,

$\mathfrak{F}(\mathfrak{X}) = (\mathfrak{F}_1(\mathfrak{X}), \mathfrak{F}_2(\mathfrak{X}), \mathfrak{F}_3(\mathfrak{X}), \mathfrak{F}_4(\mathfrak{X}), \mathfrak{F}_5(\mathfrak{X}))^T \in \mathcal{R}^5$, is a component-based vector valued function from \mathcal{R}^5 to \mathcal{R}^5 .

$$\mathfrak{F}_1(\mathfrak{X}) = \varpi - (\varpi_1 + \nu) H_s$$

$$\mathfrak{F}_2(\mathfrak{X}) = \varpi_1 H_s - (\varpi_2 + \varpi_3 + \nu) H_i$$

$$\mathfrak{F}_3(\mathfrak{X}) = \varpi_2 H_i - (\varpi_4 + \nu) H_c$$

$$\mathfrak{F}_4(\mathfrak{X}) = \varpi_3 H_i - (\varpi_5 + \nu) H_n$$

$$\mathfrak{F}_5(\mathfrak{X}) = \varpi_4 H_c + \varpi_5 H_n - (\nu + a) H_a$$

Lipschitz continuous in \mathfrak{X} is the function \mathfrak{F} in (7). Therefore, the nonlinear system (7) has a unique solution according to the existence and uniqueness theorem. If this solution is bounded and non-negative, it will have greater biological significance and be more grounded in reality. In the next Theorems 1 and 2, we illustrate these essential characteristics.

7. Positivity and boundedness

Theorem 1. Let $H_s(0) = H_{s0}$, $H_i(0) = H_{i0}$, $H_c(0) = H_{c0}$, $H_n(0) = H_{n0}$ and $H_a(0) = H_{a0}$ be the initial values of the state variables. If H_{s0} , H_{i0} , H_{c0} , H_{n0} and H_{a0} are positive then it implies that $H_s(t)$, $H_i(t)$, $H_c(t)$, $H_n(t)$, and $H_a(t)$ are positive for all time $t > 0$.

Proof: Let $H_{s0}, H_{i0}, H_{c0}, H_{n0}$, and H_{a0} and be positive, it is our goal to demonstrate the positivity of the state variables. According to system (1), we have:

$$\frac{dH_s}{dt} = \varpi - (\varpi_1 + \nu) H_s$$

Then, $\frac{dH_s}{dt} + (\varpi_1 + \nu) H_s = \varpi$

Since $\varpi \geq 0$ it follows that,

$$\frac{dH_s}{dt} + (\varpi_1 + \nu)H_s \geq 0$$

Now, we have:

$$\frac{dH_s}{H_s} \geq -(\varpi_1 + \nu)dt \quad (8)$$

By integrating equation (8) we have:

$$\ln(H_s) \geq - \int (\varpi_1 + \nu) dt + c$$

Let $A(t) = - \int (\varpi_1 + \nu) dt$, it suggests that

$$\ln(H_s(t)) \geq A(t) + c \quad (9)$$

at $t = 0$ we have

$$\ln(H_s(0)) \geq A(0) + c \quad (10)$$

We subtract equations (10) from (9) to have $\ln\left(\frac{H_s(t)}{H_s(0)}\right) \geq A(t) - A(0)$

By calculating the exponential of either side, we get:

$$H_s(t) \geq H_s(0)e^{A(t)-A(0)} \geq H_s(0) \text{ for } t \geq 0$$

Since $H_s(0) = H_{s0}$ is positive, that is, $H_{s0} > 0$ for $t > 0$ it suggests that

$$H_s(t) \geq H_s(0) > 0 \text{ for } t > 0$$

We have $H_s(t) > 0$ for all $t > 0$ and we determine that $H_s(t)$ is non-negative for all $t > 0$.

Therefore, $H_s > 0$. The other state variables also hold true in this manner.

This demonstrates that $H_s(t), H_i(t), H_c(t), H_n(t)$ and $H_a(t)$ are positive for all time $t > 0$.

Theorem 2. Within the invariant region, the nonlinear system (1–5) has bounded solutions.

$$\Phi = \left\{ (H_s, H_i, H_c, H_n, H_a) \in \mathcal{R}_+^5 : 0 < \mathcal{N}_H(t) \leq \frac{\varpi}{\nu}; H_s, H_i, H_c, H_n, H_a > 0 \right\}$$

Proof: The total population for human is given by

$$\mathcal{N}_H(t) = H_s(t) + H_i(t) + H_c(t) + H_n(t) + H_a(t)$$

Such that

$$\frac{d\mathcal{N}_H(t)}{dt} = \frac{dH_s(t)}{dt} + \frac{dH_i(t)}{dt} + \frac{dH_c(t)}{dt} + \frac{dH_n(t)}{dt} + \frac{dH_a(t)}{dt}$$

By simplification we have

$$\frac{d\mathcal{N}_H(t)}{dt} = \varpi - \nu(H_s + H_i + H_c + H_n + H_a) - aH_a$$

But $\mathcal{N}_H = H_s + H_i + H_c + H_n + H_a$

So, we have

$$\frac{d\mathcal{N}_H(t)}{dt} = \varpi - \nu\mathcal{N}_H - aH_a$$

$\leq \varpi - \nu\mathcal{N}_H$ In the absence of HIV death in the population ($a = 0$)

$$\frac{d\mathcal{N}_H(t)}{\varpi - \nu\mathcal{N}_H} \leq dt$$

Integrate the two forms $t = 0$ and $t = t_\alpha$, we have

$$\int_0^{t_\alpha} \frac{d\mathcal{N}_H(t)}{\varpi - \nu\mathcal{N}_H} \leq \int_0^{t_\alpha} dt$$

we have

$$\left[-\frac{1}{\nu} \ln(\varpi - \nu\mathcal{N}_H) \right]_0^{t_\alpha} \leq [t]_0^{t_\alpha}$$

This gives us

$$\ln(\varpi - \nu\mathcal{N}_H(t_\alpha)) - \ln(\varpi - \nu\mathcal{N}_H(0)) \geq -\nu t_\alpha$$

Then

$$\ln\left(\frac{\varpi - \nu\mathcal{N}_H(t_\alpha)}{\varpi - \nu\mathcal{N}_H(0)}\right) \geq -\nu t_\alpha$$

Taking exponential of both sides we have:

$$\frac{\varpi - \nu\mathcal{N}_H(t_\alpha)}{\varpi - \nu\mathcal{N}_H(0)} \geq e^{-\nu t_\alpha}$$

$$\varpi - \nu\mathcal{N}_H(t_\alpha) \geq (\varpi - \nu\mathcal{N}_H(0))e^{-\nu t_\alpha}$$

which gives

$$\nu\mathcal{N}_H(t_\alpha) \leq \varpi - (\varpi - \nu\mathcal{N}_H(0))e^{-\nu t_\alpha}$$

$$\mathcal{N}_H(t_\alpha) \leq \frac{\varpi}{\nu} - \frac{(\varpi - \nu\mathcal{N}_H(0))}{\nu} e^{-\nu t_\alpha} \quad (11)$$

Taking the inequality (11) limit as $t_\alpha \rightarrow \infty$, we get

$$\mathcal{N}_H(t_\alpha) \leq \frac{\varpi}{\nu}$$

Thus, we have $\mathcal{N}_H(t) \leq \frac{\varpi}{\nu}$.

It follows that $0 < \mathcal{N}_H(t) \leq \frac{\varpi}{\nu}$. As a result, all the solutions are bounded in a feasible region Φ . If $\mathfrak{X}_0 = 0$, the answer for the simple case stays $\mathfrak{X} = 0$ for all $t > 0$. Furthermore, for any $t > 0$, every solution trajectory with an initial condition in \mathcal{R}_+^5 will remain in the region Φ . Thus, all of the solutions in \mathcal{R}_+^5 are drawn to the region Φ . As a potential solution set of state variables for our system (1–5), it follows that the closed set Φ is positively invariant in \mathcal{R}_+^5 . Therefore, the system (1–5), or equivalently (7), is well-posed mathematically and epidemiologically.

Theorem 3. The HIV free equilibrium point, \mathfrak{E}^0 , is stable if $\mathfrak{R}_0 < 1$, whereas \mathfrak{E}^0 is unstable if $\mathfrak{R}_0 > 1$.

Proof. The Jacobian matrix associated to the system (1-5) at a given point $(H_s, H_i, H_c, H_n, H_a)$ is calculated as follows:

$$J(\mathfrak{E}^0) = \begin{pmatrix} -(\varpi_1 + \nu) & 0 & 0 & 0 & 0 \\ \varpi_1 & -(\varpi_2 + \varpi_3 + \nu) & 0 & 0 & 0 \\ 0 & \varpi_2 & -(\varpi_4 + \nu) & 0 & 0 \\ 0 & \varpi_3 & 0 & -(\varpi_5 + \nu) & 0 \\ 0 & 0 & \varpi_4 & \varpi_5 & -(\nu + a) \end{pmatrix}$$

and the characteristic polynomial is $|J(\mathfrak{E}^0) - \lambda I| = 0$. Solving this polynomial, the Eigen values are given

$$\text{by } |J(\mathfrak{E}^0) - \lambda I| = \begin{pmatrix} -(\varpi_1 + \nu) - \lambda_1 & 0 & 0 & 0 & 0 \\ \varpi_1 & -(\varpi_2 + \varpi_3 + \nu) - \lambda_2 & 0 & 0 & 0 \\ 0 & \varpi_2 & -(\varpi_4 + \nu) - \lambda_3 & 0 & 0 \\ 0 & \varpi_3 & 0 & -(\varpi_5 + \nu) - \lambda_4 & 0 \\ 0 & 0 & \varpi_4 & \varpi_5 & -(\nu + a) - \lambda_5 \end{pmatrix} = 0$$

We get $\lambda_1 = -(\varpi_1 + \nu) < 0$

$$\lambda_2 = -(\varpi_2 + \varpi_3 + \nu) < 0$$

$$\lambda_3 = -(\varpi_4 + \nu) < 0$$

$$\lambda_4 = -(\varpi_5 + \nu) < 0$$

$$\lambda_5 = -(\nu + a) < 0$$

All eigenvalues are negative. Hence, \mathfrak{E}^0 is stable. Thus biologically it reduces HIV phobia.

Theorem 4. If $\mathfrak{R}_0 > 1$, then the system (1-5) has a unique HIV point of endemic equilibrium.

Proof. Using the Lyapunov function, the endemic equilibrium point's locally stable analysis is examined. For this, the following is defined:

$$\mathfrak{L}(H_s, H_i, H_c, H_n, H_a) = \frac{1}{2} (H_s^2 + H_i^2 + H_c^2 + H_n^2 + H_a^2)$$

At the endemic equilibrium point, the function \mathfrak{L} equals zero and is bigger than zero. When we differentiate the function in relation to time, we get

$$\frac{d\mathcal{Q}}{dt} = \frac{1}{2} \left(2H_s \frac{dH_s}{dt} + 2H_i \frac{dH_i}{dt} + 2H_c \frac{dH_c}{dt} + 2H_n \frac{dH_n}{dt} + 2H_a \frac{dH_a}{dt} \right)$$

Applying model (1-5) to the derivative values, we obtain

$$\begin{aligned} \frac{d\mathcal{Q}}{dt} = & H_s(\varpi - (\varpi_1 + \nu)H_s) + H_i(\varpi_1H_s - (\varpi_2 + \varpi_3 + \nu)H_i) \\ & + H_c(\varpi_2H_i - (\varpi_4 + \nu)H_c) + H_n(\varpi_3H_i - (\varpi_5 + \nu)H_n) \\ & + H_a(\varpi_4H_c + \varpi_5H_n - (\nu + a)H_a) \end{aligned}$$

By solving this, we get

$$\begin{aligned} \frac{d\mathcal{Q}}{dt} = & -([\varpi_1 + \nu]H_s^2 + [\varpi_2 + \varpi_3 + \nu]H_i^2 + [\varpi_4 + \nu]H_c^2 + [\varpi_5 + \nu]H_n^2 + [\nu + a]H_a^2) \\ & + [[\varpi + \varpi_1H_i]H_s + [\varpi_2H_c + \varpi_3H_n]H_i + [\varpi_4H_c + \varpi_5H_n]H_a] \end{aligned}$$

From model (1-5), we know that $\varpi, \varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \nu$, and a are all non-negative, and the values of H_s, H_i, H_c, H_n , and H_a are all non-negative. As a result, it is clear that every term enclosed in a closed bracket is non-negative, and every term enclosed in an open bracket is non-positive. Thus, we have

$$\begin{aligned} -([\varpi_1 + \nu]H_s^2 + [\varpi_2 + \varpi_3 + \nu]H_i^2 + [\varpi_4 + \nu]H_c^2 + [\varpi_5 + \nu]H_n^2 + [\nu + a]H_a^2) & \leq 0 \\ [[\varpi + \varpi_1H_i]H_s + [\varpi_2H_c + \varpi_3H_n]H_i + [\varpi_4H_c + \varpi_5H_n]H_a] & \geq 0 \end{aligned}$$

Thus, we can say that $\frac{d\mathcal{Q}}{dt} \leq 0$ for all non-zero vectors H_s, H_i, H_c, H_n , and H_a and non-negative parameters that satisfy the model (1-5). The endemic equilibrium point is locally asymptotically stable since the derivative of the Lyapunov function \mathcal{Q} is non-positive. This finding suggests, epidemiologically, that HIV phobia will persist in the human population for a very long time.

An explanation of the model

We briefly provide basic property which will be useful in the next theorem. We define the Banach's space $B = \mathbb{B} = \mathcal{C} \times \mathcal{C} \times \mathcal{C} \times \dots \times \mathcal{C}$ and $\mathcal{C} = \mathcal{C}[0, \mathcal{T}]$, with the norm

$$\|\mathcal{W}\| = \|(\omega_1, \omega_2, \dots, \omega_n)\| = \max_{t \in [0, \mathcal{T}]} \{|\omega_1(t)| + |\omega_2(t)| + \dots + |\omega_n(t)|\},$$

$\omega_i \in \mathcal{C}[0, \mathcal{T}]$ for $i = 1, 2, \dots, n$.

Theorem 5. For each $i \in \{i = 1, \dots, 5\}$, the kernels $g_i(i = 1, \dots, 5)$ are \mathcal{L}_i -Lipschitz continuous. Furthermore, if $\mathcal{L}_i < 1$, for all $i \in \{i = 1, \dots, 5\}$, then the kernels define a contraction in $\mathcal{C}[0, \mathcal{T}]$.

Proof. Let $\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \nu \in \mathcal{C}[0, \mathcal{T}]$, and consider

$$\|\varpi_1\| < a, \|\varpi_2\| < b, \|\varpi_3\| < c, \|\varpi_4\| < d, \|\varpi_5\| < e, \|\nu\| < f.$$

Let H_s and H_{s^*} be given, then it holds that

$$\begin{aligned} \|g_1(t, H_s) - g_1(t, H_{s^*})\| & = \|\varpi - \varpi_1(t)H_s(t) - \nu(t)H_s(t) - [\varpi - \varpi_1(t)H_{s^*}(t) - \nu(t)H_{s^*}(t)]\| \\ & = \|[\varpi_1(t) + \nu(t)][H_{s^*}(t) - H_s(t)]\| \\ & \leq \|\varpi_1(t) + \nu(t)\| \|H_{s^*}(t) - H_s(t)\| \end{aligned}$$

$$\leq [\alpha + f] \|H_s^* - H_s\|$$

Consequently,

$\|g_1(t, H_s) - g_1(t, H_s^*)\| \leq \mathcal{L}_1 \|H_s^* - H_s\|$, where $\mathcal{L}_1 = \alpha + f$. For g_1 , it is evident that the Lipschitz condition is met. Additionally, g_1 defined a contraction if $0 \leq \mathcal{L}_1 < 1$. In the same way, we can demonstrate that g_2, g_3, g_4 , and g_5 have the Lipschitz and contraction properties. Hence,

$$\|g_2(t, H_i) - g_2(t, H_i^*)\| \leq \mathcal{L}_2 \|H_i^* - H_i\|$$

$$\|g_3(t, H_c) - g_3(t, H_c^*)\| \leq \mathcal{L}_3 \|H_c^* - H_c\|$$

$$\|g_4(t, H_n) - g_4(t, H_n^*)\| \leq \mathcal{L}_4 \|H_n^* - H_n\|$$

$$\|g_5(t, H_a) - g_5(t, H_a^*)\| \leq \mathcal{L}_5 \|H_a^* - H_a\|$$

Where

$$\mathcal{L}_2 = b + c + f, \mathcal{L}_3 = d + f, \mathcal{L}_4 = e + f, \mathcal{L}_5 = f + a$$

11. Simulation of Numerical

To solve the model equations using numerical methods, we utilized the MATLAB ODE algorithm for the constructed model. We generated graphs depicting each compartment of the model as a function of time, where the time interval extended from 0 to 365 days.

Now we need to investigate the significance of each model parameter of the disease phobia. These analyses help us identify which parameters have the greatest influence on the phobia of the HIV/AIDS disease. By pinpointing the most sensitive parameters those with higher sensitivity indices we gain valuable insights into which factors play the most significant role in driving the phobia of the disease. Specifically, our sensitivity analyses focused on parameters $\varpi = 0.4$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

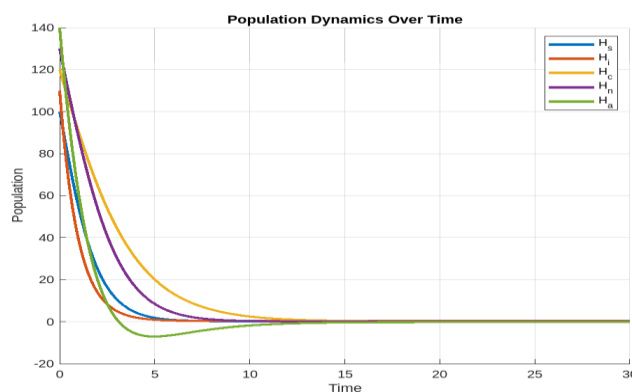


Fig. (2): Time series of Population Dynamics of all stages

The parameters values of Fig. (3) as follows: $\varpi = 20$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

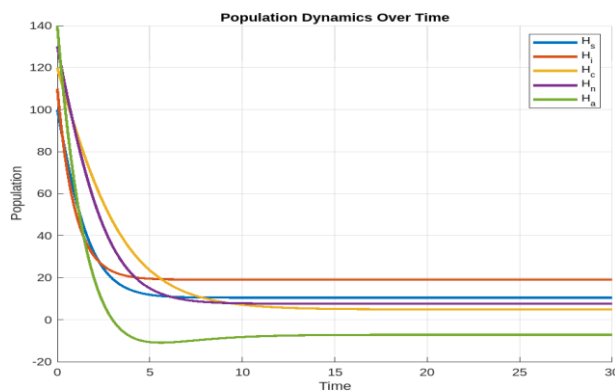


Fig. (3): Time series of Population Dynamics in the stages of H_s, H_i, H_c, H_n, H_a

The parameters values of Fig. (4) as follows: $\varpi = 100$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

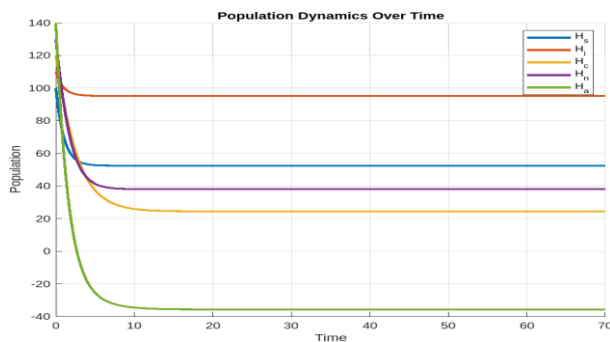


Fig. (4): Time series of Population Dynamics

The parameters values of Fig. (5) as follows: $\varpi = 1000$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

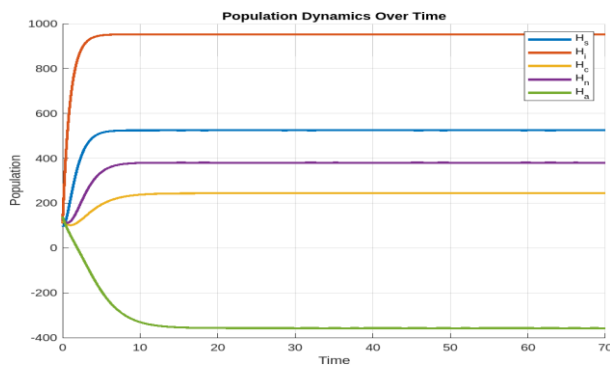


Fig. (5): Time series of Population Dynamics

The parameters values of Fig. (6a) and Fig. (6b) as follows: $\varpi = 0.5$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

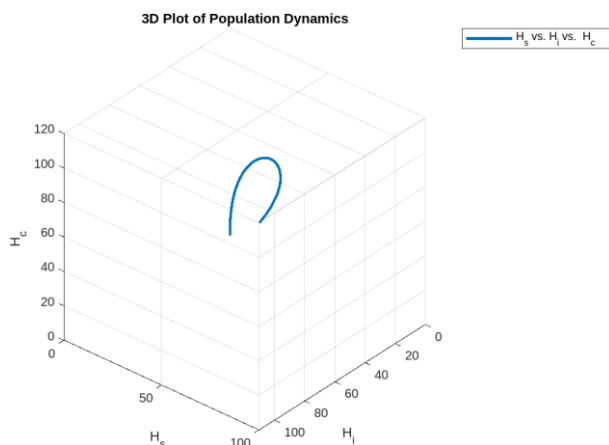


Fig. (6a): Comparison of the stages H_s vs H_n vs H_a of 3D Plot of Population Dynamics

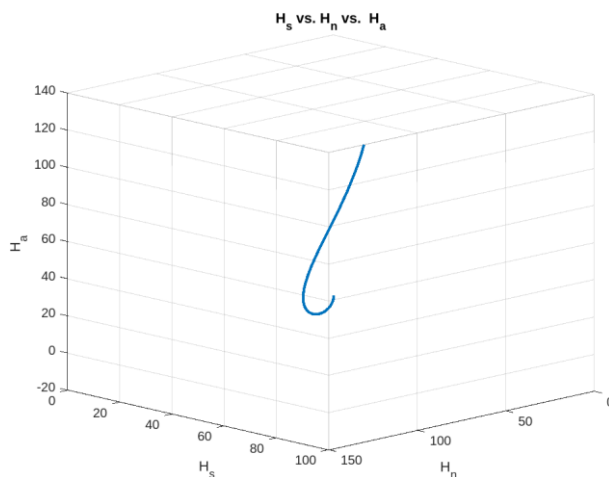


Fig. (6b): Comparison of the stages H_s vs H_n vs H_a

The parameters values of Fig. (7) as follows: $\varpi = 0.5$, $\varpi_1 = 0.65$, $\varpi_2 = 0.2$, $\varpi_3 = 0.58$, $\varpi_4 = 0.03$, $\varpi_5 = 0.4$, $v = 0.4$, $a = 0.005$.

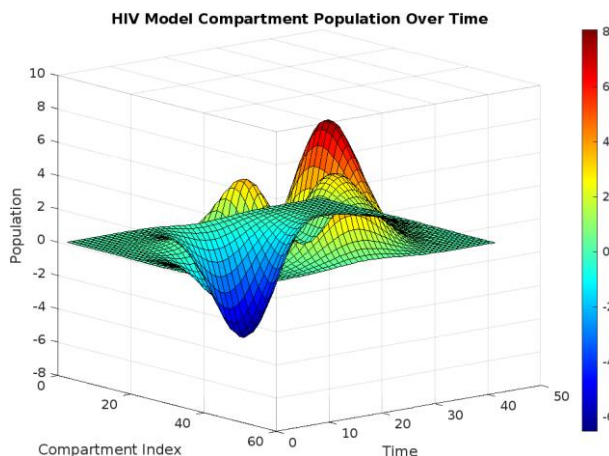


Fig. (7): HIV Model Compartment Index.

The reason for this is that while there is no cure for HIV infection, it can be effectively managed. The graphs indicate that the prevalence of HIV phobia decreases as the percentage of

diagnosed individuals who are treated rises. From a biological perspective, this implies that the HIV/AIDS phobia spreads evenly throughout the community.

12. Conclusion

HIV phobia can be diagnosed by a mental health professional, and standard treatment entails therapy in the form of exposure therapy or cognitive behavioural therapy (CBT). Dealing with HIV phobia can also be accomplished through therapeutic and care interventions, such as stress management training sessions. These therapies can enhance one's quality of life and mental health. To prevent HIV phobia, use condoms, avoid sexual activity, not share sterile needles, or think about taking a PrEP or PEP medication. Your mind may be at ease if you are certain of your safety. Seek advice from a medical practitioner or contact a local AIDS organization for recommendations to specialized providers if you or a loved one suffers from HIV phobia. Or, through your community HIV clinic or the 24-hour AIDS hotline that's available in most states, you might be able to connect with a local support group. Whenever researching the HIV/AIDS disease phenomenon, human safety is the most important consideration. Certain scholars have directed their attention on creating a model that takes preventive measures for the human population into account. In the human population, this preventive measure lowers the fear and infection risk.

The HIV/AIDS is demonstrated beneath the idea of D.E. The model of the stability is assessed in view of the circumstances of the Lyapunov. The planning of the function of Lyapunov is advancement in the difference equation idea. Additionally investigated the Lipschitz condition, and the existence of uniqueness of the system of equations. For this HIV model, the boundedness, non-negativity, continuity results have been examined. Also, disease free equilibrium and equilibrium of endemic are analyzed. The D.E. point is asymptotically stable when the reproduction number $\mathcal{R}_0 < 1$ and unstable when $\mathcal{R}_0 > 1$. Subsequently, they can anticipate the results of HIV/AIDS. Also, the review will assist with the help to fight against HIV/AIDS-phobia by policy-makers, NGOs, and other concerned organizations.

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