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More on Contra B-Open Mappings in a Quadripartitioned NeutrosophicTopological Spaces

[1]Mohanarao Navuluri, [2]V Sathishkumar

[1]Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu, India.(Deputed to Government college of Engineering , Theni, Tamilnadu, India.)

[2]Department of Mathematics, Rajalakshmi institute of technology (Autonomous), Chennai;Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu, India.

mohanaraonavuluri@gmail.com[1], vsathishkumar2020@gmail.com[2]

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Abstract

In this article, we introduce the concept of a quadripartitioned neutrosophic contra β-continuous, quadripartitioned neutrosophic contra β-open and a quadripartitioned neutrosophic contra β-closed mappings in a quadri-partitioned neutrosophic topological spaces and studied some of their related properties. Further the work is extended to a quadripartitioned neutrosophic contra β-homeomorphism and a quadripartitioned neutrosophic contra β-Completely homeomorphism in a quadripartitioned neutrosophic topological spaces and establishes some of their related properties.

Keywords: Quadripartitioned neutrosophic β-open set, Quadripartitioned neutrosophic contra β-continuous map, Quadripartitioned neutrosophic contra β-open map, Quadripartitioned neutrosophic contra β-closed map, Quadripartitioned neutrosophic contra β-homeomorphism, Quadripartitioned neutrosophic contra β- completely homeomorphism.

1 Introduction

In mathematics, Zadeh[25] was first presented a idea of fuzzy set between the intervals in order of logic and sethypothesis. The fuzzy set was attempted in general topology by Chang[2] as fuzzy topological space. The intuitionistic fuzzy set which contains a membership and non-membership values was introduced by Atanassov[1] in 1983. Coker[4] made intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological spaces. Theideas of neutrosophy and neutrosophic set was presented by Smarandache[16,17] toward the start of 20*th* century. Salama and Alblowi[14,15] in 2012, originated neutrosophic set and neutrosophic crisp set in a neutrosophic topological space. In the year 2016, Chatterjee et al.[3] grounded the idea of quadripartitioned neutrosophic set and defined several similarity measures between two quadripartitioned neutrosophic sets. Iswaraya and Bageerathi[9] studied the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets. Pushpalatha and Nandhini[12]grounded the idea of neutrosophic generalized closed sets in NTS's. The notion of neutrosophic b-open sets in NTS's was presented by Ebenanjar et al.[8] Rao and Srinivasa[13] grounded the concept of pre open set and pre closed set via neutrosophic topological spaces. Thereafter, Maheswari et al.[10] studied the neutrosophic generalized b-closed sets in NTS's. In the year 2019, Mohammed Ali Jaffer and Ramesh[11] studied the concept of neutrosophic generalized pre-regular closed sets. The generalized neutrosophic b-open sets in NTS's was introduced by Das and

Pramanik.[6] Das and Pramanik[7] also defined the neutrosophic Φ-open sets and neutrosophic Φ-continuous mappings via *NTS*'s. Vadivel and Sundar defined γ open sets,[18] γ continuous maps,[20,21] β-open sets[19] and β continuous maps[22–24] in N -neutrosophic crisp topological spaces.

In this paper, we develop the concept of quadripartitioned neutrosophic contra β-continuous maps, quadri- partitioned neutrosophic contra β-open maps and quadripartitioned neutrosophic contra β-closed maps in a quadripartitioned neutrosophic topological spaces and also specialized some of their basic properties with examples. Also, we discuss about quadripartitioned neutrosophic contra β-homeomorphism and quadripar- titioned neutrosophic contra β-completely homeomorphism in a quadripartitioned neutrosophic topological spaces and also specialized some of their basic properties with examples.

2 Preliminaries

The needful basic definitions & properties are discussed in this section.

Definition 2.1. [3] Let Z be a fixed set. Then, a quadripartitioned neutrosophic set (in-short, Q-$N_s s$) U over Z is defined by $U = \{(u, T_U(u), C_U(u), I_U(u), F_U(u)) : u \in Z\}$ where T_U, C_U, I_U and F_U ($\in [0, 1]$) are the truth, contradiction, ignorance, and falsity membership values of $u \in Z$. So, $0 \leq T_U(u) + C_U(u) + I_U(u) + F_U(u) \leq 4$.

Definition 2.2. [3] Let Z be a non-empty set & the Q-$N_S s$'s U & U_0 in the form $U = \{(u, T_U(u), C_U(u), I_U(u), F_U(u)) : u \in Z\}$, $U = \{(u, T_U\ (u), C_U\ (u), I_U\ (u), F_U\ ) : u \in Z\}$, then

(i) $0_{QNS} = (u, 0, 0, 1, 1)$ and $1_{QNS} = (u, 1, 1, 0, 0)$,

(ii) $U \subseteq U_0$ iff $T_U(u) \leq T_{U0}(u)$, $C_U(u) \leq C_{U0}(u)$, $I_U(u) \geq I_{U0}(u)$ & $F_U(u) \geq F_{U0}(u) : u \in Z$,

(iii) $1_{QNS} - U = \{(u, F_U(u), I_U(u), C_U(u), T_U(u)) : u \in Z\} = U^c$,

(iv) $U \cup U_0 = \{(u, \max(T_U(u), T_{U0}(u)), \max(C_U(u), C_{U0}(u)), \min(I_U(u), I_U\ (u)), \min(F_U(u), F_{U0}(u))) : u \in Z\}$,

(v) $U \cap U\ = \{(u, \min(T_U(u), T_{U0}u)), \min(C_U(u), C_{U0}(u)), \max(I_U(u), I_{U0}(u)), \max(F_U(u), F_{U0}(u))) : u \in Z\}$.

Definition 2.3. [5] Let Z be a fixed set. A collection Γ_Q of some Q-$N_s s$'s over Z is called a quadripartitionedneutrosophic topology (in-short, Q-N_{st}) on Z, if the following conditions holds:

(i) $0_N, 1_N \in \Gamma_Q$.

(ii) $G_\phi \cap G_\varphi \in \Gamma_Q$ for any $G_\phi, G_\varphi \in \Gamma_Q$.

(iii) $\cup G_\phi \in \Gamma_Q, \forall\ \{G_\phi : \phi \in Z\} \subseteq \Gamma_Q$.

Then (Z, Γ_Q) is called a quadripartitioned neutrosophic topological space (in-short, Q-$N_s ts$) in Z. Every element of Γ_Q are called a quadripartitioned neutrosophic open sets (in-short, Q-$N_s o$ set). If $C \in \Gamma_Q$, then C^c is called a quadripartitioned neutrosophic closed sets (in-short, Q-$N_s c$ set).

Definition 2.4. [5] Let (Z, Γ_Q) be $Q\text{-}N_sts$ on Z and U be an $Q\text{-}N_ss$ on Z, then a quadripartitioned neutrosophicinterior (resp. closure) of U (in-short, $Q\text{-}N_sint(U)$ (resp. $Q\text{-}N_scl(U)$)) are defined as

$Q\text{-}N_sint(U) = \cup\{U_0 : U_0 \subseteq U \ \&\ U_0 \text{ is a } Q\text{-}N_so \text{ in } Z\}$,

$Q\text{-}N_scl(U) = \quad \cap\{U_0 : U \subseteq U_0 \ \&\ U_0 \text{ is a } Q\text{-}N_sc \text{ in } Z\}$,

Definition 2.5. [5] Let (Z, Γ_Q) be $Q\text{-}N_sts$ on Z and U be an $Q\text{-}N_ss$ on Z. Then U is said to be a quadripartitioned neutrosophic pre (resp. semi, α & b) open set (in-short, $Q\text{-}N_s\, po$ set (resp. $Q\text{-}N_s\, \alpha o$ set, $Q\text{-}N_s\alpha o$ set &$Q\text{-}N_sbo$ set)) if $U \subseteq Q\text{-}N_sint(Q\text{-}N_scl(U))$ (resp. $U \subseteq Q\text{-}N_scl(Q\text{-}N_sint(U))$, $U \subseteq Q\text{-}N_sint(Q\text{-}N_scl(Q\text{-}N_sint(U)))$ & $U \subseteq Q\text{-}N_scl(Q\text{-}N_sint(U)) \cup Q\text{-}N_sint(Q\text{-}N_scl(U))$).

The complement of an $Q\text{-}N_s\, o$ set (resp. $Q\text{-}N_s\, o$ set, $Q\text{-}N_s\alpha o$ set & $Q\text{-}N_sbo$ set) is called a quadripartitioned neutrosophic pre (resp. semi, α & b) closed set (in-short, $Q\text{-}N_s\, c$ set (resp. $Q\text{-}N_s\, pc$ set, $Q\text{-}N_s\alpha c$ set& $Q\text{-}N_sbc$ set)) in Z.

The family of all $Q\text{-}N_s\mathrm{P}o$ set (resp. $Q\text{-}N_s\mathrm{P}c$ set, $Q\text{-}N_s\mathrm{S}o$ set, $Q\text{-}N_s\mathrm{S}c$ set, $Q\text{-}N_s\alpha o$ set, $Q\text{-}N_s\alpha c$ set, $Q\text{-}N_sbo$ set & $Q\text{-}N_sbc$ set) of Z is denoted by $Q\text{-}N_s\mathrm{P}OS(Z)$ (resp. $Q\text{-}N_s\mathrm{P}CS(Z)$, $Q\text{-}N_sSOS(Z)$, $Q\text{-}\ N_sSCS(Z)$, $Q\text{-}N_s\alpha OS(Z)$, $Q\text{-}N_s\alpha CS(Z)$, $Q\text{-}N_sbOS(Z)$ & $Q\text{-}N_sbCS(Z)$).

Definition 2.6. Let (Z, Γ_Q) be $Q\text{-}N_sts$ on Z and U be an $Q\text{-}N_ss$ on Z. Then U is said to be a quadriparti-tioned neutrosophic β open set (in-short, $Q\text{-}N_s\beta o$) set if $U \subseteq Q\text{-}N_scl(Q\text{-}N_sint(Q\text{-}N_scl(U)))$.

The complement of an $Q\text{-}N_s\beta o$ set is called a quadripartitioned neutrosophic β closed set (in-short, $Q\text{-}N_s\beta c$ set in Z.

The family of all $Q\text{-}N_s\beta o$ set (resp. $Q\text{-}N_s\beta c$ set) of Z is denoted by $Q\text{-}N_s\beta OS(Z)$ (resp. $Q\text{-}N_s\beta CS(Z)$).

Definition 2.7. The $Q\text{-}N_s\beta$ interior of U (briefly, $Q\text{-}N_s\beta int(U)$) and $Q\text{-}N_s\beta$ closure of U (briefly, $Q\text{-}N_s\beta cl(U)$) are defined as

(i) $Q\text{-}N_s\beta int(U) = \cup\{U_0 : U_0 \subseteq U \ \&\ U_0 \text{ is a } Q\text{-}N_s\beta o \text{ set in } Z\}$.

(ii) $Q\text{-}N_s\beta cl(U) = \cap\{U_0 : U \subseteq U_0 \ \&\ U_0 \text{ is a } Q\text{-}N_s\beta c \text{ set in } Z\}$.

Definition 2.8. Let (Z_1, Γ_Q) and (Z_2, σ_Q) be any two $Q\text{-}N_sts$'s. A map $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is said tobe quadripartitioned neutrosophic (resp. semi, pre, b & β) continuous (briefly, $Q\text{-}N_sCts$ (resp. $Q\text{-}N_s\mathrm{S}Cts$, $Q\text{-}N_s\mathrm{P}Cts$, $Q\text{-}N_sbCts$ & $Q\text{-}N_s\beta Cts$)) if the inverse image of every $Q\text{-}N_so$ set in (Z_2, σ_Q) is a $Q\text{-}N_so$ set (resp. $Q\text{-}N_s\mathrm{S}o$ set, $Q\text{-}N_s\mathrm{P}o$ set, $Q\text{-}N_sbo$ set & $Q\text{-}N_s\beta o$ set) in (Z_1, Γ_Q).

Definition 2.9. A map $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is called a quadripartitioned neutrosophic β-irresolute (briefly, $Q\text{-}N_s\beta Irr$) map if $\mathrm{K}^{-1}(\lambda)$ is a $Q\text{-}N_s\beta o$ set in (Z_1, Γ_Q) for every $Q\text{-}N_s\beta o$ set λ of (Z_2, σ_Q).

Definition 2.10. A $Q\text{-}N_sts$ (Z, Γ_Q) is said to be an quadripartitioned neutrosophic $\beta\ U_{1/2}$ (in short $Q\text{-}N_s\beta U_{1/2}$)-space, if every $Q\text{-}N_s\beta o$ set in Z is a $Q\text{-}N_so$ set in Z.

Definition 2.11. Let (Z_1, Γ_Q) and (Z_2, σ_Q) be any two $Q\text{-}N_sts$'s. A map $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is said to be quadripartitioned neutrosophic (resp. semi, pre, b & β) open map (briefly, $Q\text{-}N_sO$

(resp. Q-N_sSO, Q-N_sPO, Q-N_sbO & Q-$N_s\beta O$)) if the inverse image of every Q-N_so set in (Z_1, Γ_Q) is a Q-N_so set (resp. Q-N_sSo set, Q-N_sPo set, Q-N_sbo set & Q-$N_s\beta o$ set) in (Z_2, σ_Q).

Definition 2.12. Let (Z_1, Γ_Q) and (Z_2, σ_Q) be any two Q-N_sts's. A map K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is said to be quadripartitioned neutrosophic (resp. semi, pre, b & β) closed map (briefly, Q-N_sC (resp. Q-N_sSC, Q-N_sPC, Q-N_sbC & Q-$N_s\beta C$)) if the inverse image of every Q-N_sc set in (Z_1, Γ_Q) is a Q-N_sc set (resp. Q-N_sSc set, Q-N_sPc set, Q-N_sbc set & Q-$N_s\beta c$ set) in (Z_2, σ_Q).

Definition 2.13. A bijection K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is called a

(i) quadripartitioned neutrosophic homeomorphism (briefly Q-N_sHom) if K and K^{-1} are Q-N_sCts.

(ii) quadripartitioned neutrosophic β-homeomorphism (briefly Q-$N_s\beta Hom$) if K and K^{-1} are Q-$N_s\beta Cts$.

Definition 2.14. A bijection K : (Z_1, Γ_Q) (Z_2, σ_Q) is called a quadripartitioned neutrosophic β-Completely homeomorphism (briefly, Q-$N_s\beta CHom$) if K and K^{-1} are Q-$N_s\beta Irr$ mappings.

Definition 2.15. A Q-N_sts (Z, Γ_Q) is said to be a quadripartitioned neutrosophic $\beta_{½}$ space (*briefly*, Q -$N_s\beta T_{1/2}$)-space if every Q-$N_s\beta cs$ is Q-N_sc in (Z, Γ_Q).

3 Quadripartitioned neutrosophic contra β-continuous maps

In this section, quadripartitioned neutrosophic contra β-continuous maps are introduced and some of its properties are discussed.

Definition 3.1. A mapping K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is said to be a quadripartitioned neutrosophic contra (resp. semi, pre, b & β) continuous (in short, Q-NsɕCts (resp. Q-N_s ɕ$SCts$, Q-N_sɕ$pCts$, Q-N_sɕ$bCts$ & Q-N_sɕβCts)) if the inverse image of each Q-N_so set of (Z_2, σ_Q) is Q-N_sc (resp. Q-N_s Sc, Q-N_s ρc, Q-N_sbc & Q-$N_s\beta c$) set in (Z_1, Γ_Q).

Example 3.2. Let $V = \{V_a, V_b, V_c\} = W$ and define Q-N_sS's V_1, V_2 & V_3 in V and W_1 in W are

$V_1 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.3, 0.5, 0.5, 0.7), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_2 = \{(V_a, 0.1, 0.5, 0.5, 0.9), (V_b, 0.1, 0.5, 0.5, 0.9), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_3 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.4, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q = \{ 0_{QNS}, V_1, V_2, 1_{QNS} \}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$.

Let K : $(V, \Gamma_Q)(W, \sigma_Q)$ be anidentity mapping, then K is Q-N_sɕβCts function.

Proposition 3.3. A map K : (Z_1, Γ_Q) (Z_2, σ_Q), then the statements are hold but the converse does nottrue. Every

(i) Q-N_sɕCts is a Q-N_sɕ$SCts$.

(ii) Q-N_sɕCts is a Q-N_sɕ$PCts$.

(iii) Q-N_sɕ$SCts$ is a Q-N_sɕ$bCts$.

(iv) Q-N_sɕ$PCts$ is a Q-N_sɕ$bCts$.

(v) Q-N_sɕ$bCts$ is a Q-N_sɕβCts.

Proof. (i) Let η be a Q-$N_s o$ set in Z_2. Since K is Q-N_sɕCts, $K^{-1}(\eta)$ is a Q-$N_s c$ set in Z_1. Since every Q-$N_s c$ set is a Q-N_sSc set, $K^{-1}(\eta)$ is a Q-N_sSc set in Z_1. Hence K is a Q-N_sɕSCts.

(ii) Let η be a Q-$N_s o$ set in Z_2. Since K is Q-N_sɕCts, $K^{-1}(\eta)$ is a Q-$N_s c$ set in Z_1. Since every Q-$N_s c$ set is a Q-N_sPc set, $K^{-1}(\eta)$ is a Q-N_sPc set in Z_1. Hence K is a Q-N_sCPCts.

(iii) Let η be a Q-$N_s o$ set in Z_2. Since K is Q-N_sɕSCts, $K^{-1}(\eta)$ is a Q-N_sSc set in Z_1. Since every Q-N_sSc set is a Q-$N_s bc$ set, $K^{-1}(\eta)$ is a Q-$N_s bc$ set in Z_1. Hence K is a Q-N_sC$bCts$.

(iv) Let η be a Q-$N_s o$ set in Z_2. Since K is Q-N_sɕPCts, $K^{-1}(\eta)$ is a Q-N_sPc set in Z_1. Since every Q-N_sPc set is a Q-$N_s bc$ set, $K^{-1}(\eta)$ is a Q-$N_s bc$ set in Z_1. Hence K is a Q-N_sC$bCts$.

(v) Let η be a Q-$N_s o$ set in Z_2. Since K is Q-N_sɕ$bCts$, $K^{-1}(\eta)$ is a Q-$N_s bc$ set in Z_1. Since every Q-$N_s bc$ set is a Q-$N_s\beta c$ set, $K^{-1}(\eta)$ is a Q-$N_s\beta c$ set in Z_1. Hence K is a Q-N_sɕβCts.

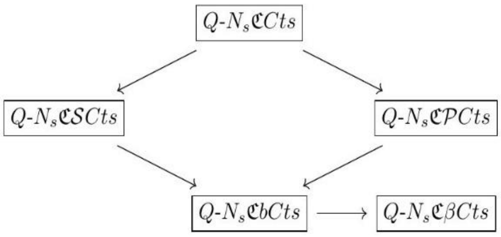

Figure 1: Q-N_sɕβCts maps in Q-$N_s ts$

Example 3.4. In Example 3.2, K is Q-N_sɕ$bCts$ but not Q-N_sCPCts, the set $K^{-1}(W_1) = V_3{}^c$ is a Q-$N_s bc$ set but not Q-N_sPc set.

Example 3.5. Let $V = \{V_a, V_b, V_c\} = W$ and define Q-$N_s s$'s V_1, V_2, V_3 & V_4 in V and W_1 in W are

$V_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.5, 0.5, 0.5, 0.5)\}$,

$V_2 = \{(V_a, 0.4, 0.5, 0.5, 0.6), (V_b, 0.2, 0.5, 0.5, 0.8), (V_c, 0.6, 0.5, 0.5, 0.4)\}$,

$V_3 = \{(V_a, 0.4, 0.5, 0.5, 0.6), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.6, 0.5, 0.5, 0.4)\}$,

$V_4 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.4, 0.5, 0.5, 0.6)\}$

$W_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.4, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q=\{0_{QNS}, V_1, V_2, V_3, V_1 \cap V_2, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$. Let $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ be an identity mapping, then K is Q-N_sC$bCts$ but not Q-N_sɕSCts, the set $K^{-1}(W_1) = V^c$ is a Q-$N_s bc$ set but not Q-N_sSc set.

Example 3.6. Let $V = \{V_a, V_b\} = W$ and define Q-$N_s s$'s V_1 & V_2 in V and W_1 in W are

$V_1 = \{(V_a, 0.3, 0.5, 0.5, 0.5), (V_b, 0.2, 0.5, 0.5, 0.5)\}$,

$V_2 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$. Let $K : (Z_1, \Gamma_Q)(Z_2, \sigma_Q)$ be anidentity mapping, then K is $Q\text{-}N_s\mathfrak{g}\beta Cts$ but not $Q\text{-}N_s\mathfrak{g}bCts$, the set $K^{-1}(W_1) = V_2{}^c$ is a $Q\text{-}N_s\beta c$ set but not $Q\text{-}N_sbc$ set.

Theorem 3.7. A map $K : (Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is $Q\text{-}N_s\mathfrak{g}\beta Cts$ iff the inverse image of every $Q\text{-}N_scs$ in Z_2 is $Q\text{-}N_s\beta os$ in Z_1.

Proof. Consider a $Q\text{-}N_scs$ $\tilde{\Psi}$ in Z_2. Then $\tilde{\Psi}^C$ is $Q\text{-}N_sos$ in Z_2. As K is $Q\text{-}N_s\mathfrak{g}\beta Cts$, $K^{-1}(\tilde{\Psi}^C)$ is $Q\text{-}N_s\beta cs$ in Z_1. As $K^{-1}(\tilde{\Psi}^C) = (K^{-1}(\tilde{\Psi}))^c$, $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta os$ in Z_1.

Conversely, consider a $Q\text{-}N_scs$ $\tilde{\Psi}$ in Z_2. So $\tilde{\Psi}^C$ is a $Q\text{-}N_sos$ in Z_2. By presumption, $K^{-1}(\tilde{\Psi}^C)$ is $Q\text{-}N_s\beta cs$ in Z_1. As $K^{-1}(\tilde{\Psi}^C) = (K^{-1}(\tilde{\Psi}))^c$, $(K^{-1}(\tilde{\Psi}))^c$ is a $Q\text{-}N_s\beta cs$ in Z_1. Hence $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta os$ in Z_1. Thus K is $Q\text{-}N_s\mathfrak{g}\beta Cts$.

Theorem 3.8. Let $K : (Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be $Q\text{-}N_sC\beta Cts$. If Z_1 is a $Q\text{-}N_s\beta U_{1/2}$-space, then K is a $Q\text{-}N_s\mathfrak{g}Cts$.

Proof. Consider a $Q\text{-}N_sos$ $\tilde{\Psi}$ in Z_2. So $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in Z_1, by presumption. As Z_1 is a $Q\text{-}N_s\beta U_{1/2}$-space, $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_scs$ in Z_1. Thus K is a $Q\text{-}N_s\mathfrak{g}Cts$.

Theorem 3.9. Let $K : (Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be a $Q\text{-}N_s\mathfrak{g}\beta Cts$ map and $G : (Z_2, \sigma_Q) \rightarrow (Z_3, \rho_Q)$ be a $Q\text{-}N_sCts$, then $G \circ K : (Z_1, \Gamma_Q) \rightarrow (Z_3, \rho_Q)$ is a $Q\text{-}N_s\mathfrak{g}\beta Cts$.

Proof. Let $\tilde{A}$ be a $Q\text{-}N_sos$ in Z_3. By presumption, $G^{-1}(\tilde{A})$ is a $Q\text{-}N_sos$ in Z_2. As K is a $Q\text{-}N_s\mathfrak{g}\beta Cts$ map, $K^{-1}(G^{-1}(\tilde{A}))$ is a $Q\text{-}N_s\beta cs$ in Z_1. Thus $G \circ K$ is a $Q\text{-}N_sC\beta Cts$ map.

Theorem 3.10. Let $K : (Z_1, \Gamma_Q) \quad (Z_2, \sigma_Q)$ be a $Q\text{-}N_s\mathfrak{g}\beta Cts$ map. Then, the succeeding conditions aretrue.

(i) $K(Q\text{-}N_s\beta cl(\tilde{\Psi})) \supseteq Q\text{-}N_sint(K(\tilde{\Psi})), \forall (\tilde{\Psi})$ in Z_1.

(ii) $Q\text{-}N_s\beta cl(K^{-1}(\tilde{\Phi})) \supseteq K^{-1}(Q\text{-}N_sint(\tilde{\Phi})), \forall (\tilde{\Phi})$ in Z_2.

Proof. (i) As $Q\text{-}N_s\beta cl(K(\tilde{\Psi}))$ is a $Q\text{-}N_s\beta cs$ in Z_2 and K is $Q\text{-}N_s\mathfrak{g}\beta Cts$, $K^{-1}(Q\text{-}N_s\beta cl(K(\tilde{\Psi})))$ is $Q\text{-}N_s\beta o$ in Z_1. Now, as $(\tilde{\Psi}) \supseteq K^{-1}(Q\text{-}N_sint(K(\tilde{\Psi})))$, $Q\text{-}N_s\beta cl(\tilde{\Psi}) \supseteq K^{-1}(Q\text{-}N_sint(K(\tilde{\Psi})))$.

Therefore, $K(Q\text{-}N_s\beta cl(\tilde{\Psi})) \quad Q\text{-}N_sint(K(\tilde{\Psi}))$.

(ii) By replacing $(\tilde{\Psi})$ with $(\tilde{\Phi})$ in (i), we get $K(Q\text{-}N_s\beta cl(K^{-1}(\tilde{\Phi}))) \supseteq Q\text{-}N_sint(K(K^{-1}(\tilde{\Phi}))) \supseteq Q\text{-}N_sint(\tilde{\Phi})$.

Hence, $Q\text{-}N_s\beta cl(K^{-1}(\tilde{\Phi})) \supseteq K^{-1}(Q\text{-}N_sint(\tilde{\Phi}))$.

4 Quadripartitioned neutrosophic contra β-irresolute maps

The quadripartitioned neutrosophic contra β-irresolute maps are introduced and some of its properties arediscussed in this section.

Definition 4.1. A map K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is known as a quadripartitioned neutrosophic contra β-irresolute (in short, $Q\text{-}N_s C\beta Irr$) map if $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in (Z_1, Γ_Q) for each $Q\text{-}N_s\beta os$ $\tilde{\Psi}$ of (Z_2, σ_Q).

Theorem 4.2. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be a $Q\text{-}N_s C\beta Irr$ map. Then K is $Q\text{-}N_s C\beta Cts$. But the converseneed not be true.

Proof. Assume K is a $Q\text{-}N_s C\beta Irr$ map. Consider a $Q\text{-}N_s os$ $\tilde{\Psi}$ in Z_2. As each $Q\text{-}N_s os$ is a $Q\text{-}N_s\beta os$, $\tilde{\Psi}$ is a

$Q\text{-}N_s\beta os$ in Z_2. By presumption, $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in Z_1. Thus K is a $Q\text{-}N_s g\beta Cts$ map.

Example 4.3. Let $V = \{V_a, V_b, V_c\} = W$ and define $Q\text{-}N_s S$'s V_1, V_2 & V_3 in V and W_1 & W_2 in W are

$V_1 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.3, 0.5, 0.5, 0.7), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_2 = \{(V_a, 0.1, 0.5, 0.5, 0.9), (V_b, 0.1, 0.5, 0.5, 0.9), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_3 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(V_a, 0.1, 0.5, 0.5, 0.9), (V_b, 0.1, 0.5, 0.5, 0.9), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_2 = \{(V_a, 0.1, 0.5, 0.5, 0.9), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.5, 0.5, 0.5, 0.5)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, V_2, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be an identity mapping, then K is $Q\text{-}N_s g\beta Cts$ but not $Q\text{-}N_s g\beta Irr$, the set W_2 is a $Q\text{-}N_s\beta c$ set in W but $K^{-1}(W_2)$ is not $Q\text{-}N_s\beta c$ set in V.

Theorem 4.4. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be a $Q\text{-}N_s g\beta Irr$. If Z_1 is a $Q\text{-}N_s\beta U_{1/2}$-space, then K is a $Q\text{-}N_s g Cts$ map.

Proof. Consider a $Q\text{-}N_s os$ $\tilde{\Psi}$ in Z_2. Then $\tilde{\Psi}$ is a $Q\text{-}N_s\beta os$ in Z_2. Hence $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in Z_1. As Z_1 is a $Q\text{-}N_s\beta U_{1/2}$-space, $K^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s cs$ in Z_1. Thus K is a $Q\text{-}N_s g Cts$ map.

Theorem 4.5. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ be a $Q\text{-}N_s C\beta Irr$ map and G :$(Z_2, \sigma_Q) \rightarrow (Z_3, \rho_Q)$ be $Q\text{-}N_s\beta Cts$ map. Then G ∘ K : $(Z_1, \Gamma_Q) \rightarrow (Z_3, \rho_Q)$ is a $Q\text{-}N_s g\beta Cts$ map.

Proof. Consider a $Q\text{-}N_s os$ $\tilde{A}$ in Z_3. Then $G^{-1}(\tilde{A})$ is a $Q\text{-}N_s\beta os$ in Z_2. As K is a $Q\text{-}N_s g\beta Irr$, $K^{-1}(G^{-1}(\tilde{A}))$ is a $Q\text{-}N_s\beta cs$ in Z_1. Thus G ∘ K is a $Q\text{-}N_s g\beta Cts$ map.

Theorem 4.6. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ and G : $(Z_2, \sigma_Q) \rightarrow (Z_3, \rho_Q)$ be mappings. Then G ∘ K :$(Z_1, \Gamma_Q) \rightarrow (Z_3, \rho_Q)$ is:

(i) $Q\text{-}N_s g\beta Cts$ if K is $Q\text{-}N_s\beta Irr$ and G is $Q\text{-}N_s g\beta Cts$.

(ii) $Q\text{-}N_s g\beta Irr$ if K is $Q\text{-}N_s g\beta Irr$ (resp. $Q\text{-}N_s\beta Irr$) and G is $Q\text{-}N_s\beta Irr$ (resp.$Q\text{-}N_s g\beta Irr$).

Proof. (i) Let $\tilde{A}$ be a $Q\text{-}N_s os$ in Z_3. Then $G^{-1}(\tilde{A})$ is a $Q\text{-}N_s\beta cs$ in Z_2. As K is a $Q\text{-}N_s\beta Irr$, $K^{-1}(G^{-1}(\tilde{A}))$is a $Q\text{-}N_s\beta cs$ in Z_1. Thus G K is a $Q\text{-}N_s g\beta Cts$ map.

The other cases are similar.

Theorem 4.7. Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ be a mapping.

(i) If (Z_1, Γ_Q) is Q-$N_s\beta U_{1/2}$ -space, then the concepts of Q-$N_s\mathfrak{g}Cts$ and Q-$N_s\mathfrak{g}\beta Cts$ are equivalent.

(ii) If (Z_2, σ_Q) is Q-$N_s\beta U_{1/2}$ -space, then the concepts of Q-$N_s\mathfrak{g}\beta Cts$ and Q-$N_s\mathfrak{g}\beta Irr$ are equivalent.

(iii) If (Z_1, Γ_Q) and (Z_2, σ_Q) are Q-$N_s\beta U_{1/2}$ -spaces, then the concepts of Q-$N_s\mathfrak{g}Cts$, Q-$N_s\mathfrak{g}\beta Cts$ and$Q$-$N_s\mathfrak{g}\beta Irr$ are equivalent.

Proof. (i) Let $\tilde{\Psi}$ be a Q-N_scs in Z_2. Then $G^{-1}(\tilde{\Psi})$ is a Q-$N_s\beta os$ in Z_1 if K is Q-$N_s\mathfrak{g}\beta Cts$. As (Z_1, Γ_Q) is a Q-$N_s\beta U_{1/2}$ -space, $G^{-1}(\tilde{\Psi})$ is a Q-N_sos in Z_1. Hence K is also Q-$N_s\mathfrak{g}Cts$ map.

The other cases are similar.

Theorem 4.8. Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ and G : $(Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ be Q-$N_s\mathfrak{g}\beta Cts$ mappings and (Z_2, σ_Q) be a Q-$N_s\beta U_{1/2}$ -space. Then $G \circ K : (Z_1, \Gamma_Q) \to (Z_3, \rho_Q)$ is Q-$N_s\beta Cts$.

Proof. Let $\tilde{A}$ be a Q-N_scs in Z_3. Then $G^{-1}(\tilde{A})$ is a Q-$N_s\beta os$ in Z_2, since G is Q-$N_s\mathfrak{g}\beta Cts$. As (Z_2, σ_Q) is a Q-$N_s\beta U_{1/2}$ -space, $G^{-1}(\tilde{A})$ is a Q-N_sos in Z_2. Then, $K(G^{-1}(\tilde{A}))$ is Q-$N_s\beta cs$ in Z_1 because K is Q-$N_s\mathfrak{g}\beta Cts$. Hence, $G \circ K$ is a Q-$N_s\beta Cts$ map.

Theorem 4.9. Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ be a map from a Q-N_st Z_1 into a Q-N_st Z_2. If Z_1 and Z_2 are Q-$N_s\beta U_{1/2}$ -spaces, then the following are equivalent:

(i) K is a Q-$N_s\mathfrak{g}\beta Irr$ map.

(ii) $K^{-1}(\tilde{\Psi})$ is a Q-$N_s\beta os$ in Z_1 for every Q-$N_s\beta cs$ $\tilde{\Psi}$ in Z_2.

(iii) Q-$N_scl(K^{-1}(\tilde{\Psi})) \supseteq K^{-1}(Q$-$N_sint(\tilde{\Psi}))$ for each Q-N_ss $\tilde{\Psi}$ of Z_2.

Proof. (i) → (ii): Consider a Q-$N_s\beta cs$ $\tilde{\Psi}$ in Z_2. So $\tilde{\Psi}^C$ is a Q-$N_s\beta os$ in Z_2. As K is Q-$N_s\mathfrak{g}\beta Irr$, $K^{-1}(\tilde{\Psi}^C)$is a Q-$N_s\beta cs$ in Z_1. We know that, $K^{-1}(\tilde{\Psi}^C) = (K^{-1}(\tilde{\Psi}))^c$. Thus $K^{-1}(\tilde{\Psi})$ is a Q-$N_s\beta os$ in Z_1.

(ii) → (iii): Consider a Q-N_ss $\tilde{\Psi}$ in Z_2 and Q-$N_sint(\tilde{\Psi}) \subseteq (\tilde{\Psi})$. Then $K^{-1}(Q$-$N_sint(\tilde{\Psi})) \subseteq K^{-1}(\tilde{\Psi})$. As Q-$N_sint(\tilde{\Psi})$ is a Q-N_sos in Z_2, Q-$N_sint(\tilde{\Psi})$ is a Q-$N_s\beta os$ in Z_2. Therefore $(Q$-$N_sint(\tilde{\Psi}))^c$ is a Q-$N_s\beta cs$ in Z_2. By presumption, $K^{-1}((Q$-$N_sint\ (\tilde{\Psi}))^c)$ is a Q-$N_s\beta os$ in Z_1. As $K^{-1}((Q$-$N_sint(\tilde{\Psi}))^c) = (K^{-1}(Q$- $N_sint(\tilde{\Psi})))^c$, $K^{-1}(Q$-$N_sint(\tilde{\Psi}))$ is a Q-$N_s\beta os$ in Z_1. As Z_1 is Q-$N_s\beta U_{1/2}$-space, $K^{-1}(Q$-$N_sint(\tilde{\Psi}))$ is a Q-N_sos in Z_1.

Thus, Q-$N_scl(K^{-1}(\tilde{\Psi})) \supseteq$ Q-$N_scl(K^{-1}(Q$-$N_sint(\tilde{\Psi}))) = K^{-1}(Q$-$N_sint(\tilde{\Psi}))$. That is Q-$N_scl(K^{-1}(\tilde{\Psi})) \supseteq K^{-1}(Q$-$N_sint(\tilde{\Psi}))$.

(iii) → (i): Consider a Q- in Z_2. As Z_2 is Q-$N_s\beta U_{1/2}$ -space, $\tilde{\Psi}$is a Q-N_scs in

Z_2 and Q -$N_s cl(\tilde{\Psi}) = (\tilde{\Psi})$. Hence $K^{-1}(\tilde{\Psi}) = K^{-1}(Q\text{-}N_s cl(\tilde{\Psi}))$ Q-$N_s int(K^{-1}(\tilde{\Psi}))$. But clearly $K^{-1}(\tilde{\Psi})$ Q- $N_s int(K^{-1}(\tilde{\Psi}))$. Therefore Q-$N_s int(K^{-1}(\tilde{\Psi})) = K^{-1}(\tilde{\Psi})$. So, $K^{-1}(\tilde{\Psi})$ is a Q-$N_s os$ and hence it is a Q- $N_s \beta os$ in Z_1. Thus K is a Q-N_sɕβIrr map.

5 Quadripartitioned Neutrosophic contra β-open mapping

The quadripartitioned neutrosophic contra β-open maps are introduced in this section and some of their characteristics are analyzed.

Definition 5.1. A mapping K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ is quadripartitioned neutrosophic contra (resp. semi, pre, b & β) open (in short, Q-N_sɕO (resp. Q-N_sɕSO, Q-N_sɕPO, Q-N_sɕbO & Q-N_sɕβO)) if the image of each Q-$N_s o$ set of (Z_1, Γ_Q) is Q-$N_s c$ (resp. Q-N_sSc, Q-N_sPc, Q-$N_s bc$ & Q-$N_s \beta c$) set in (Z_2, σ_Q).

Proposition 5.2. A map K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$, then the statements are hold but the converse does nottrue. Every

(i) Q-N_sɕO is a Q-N_sɕSO.

(ii) Q-N_sɕO is a Q-N_sɕPO.

(iii) Q-N_sɕSO is a Q-N_sɕbO.

(iv) Q-N_sɕPO is a Q-N_sɕbO.

(v) Q-N_sɕbO is a Q-N_sɕβO.

Proof. (i) Let η be a Q-$N_s o$ set in Z_1. Since K is Q-N_sɕO, K(η) is a Q-$N_s c$ set in Z_2. Since every Q-$N_s c$ set is a Q-N_sSc set, K(η) is a Q-N_sSc set in Z_2. Hence K is a Q-N_sɕSO.

(ii) Let η be a Q-$N_s o$ set in Z_1. Since K is Q-N_sɕO, K(η) is a Q-$N_s c$ set in Z_2. Since every Q-$N_s c$ set is a Q-N_sPc set, K(η) is a Q-N_sPc set in Z_2. Hence K is a Q-N_sɕPO.

(iii) Let η be a Q-$N_s o$ set in Z_1. Since K is Q-N_sɕSO, K(η) is a Q-N_s Sc set in Z_2. Since every Q-N_s Sc set is a Q-$N_s bc$ set, K(η) is a Q-$N_s bc$ set in Z_2. Hence K is a Q-N_sɕbO.

(iv) Let η be a Q-$N_s o$ set in Z_1. Since K is Q-N_sɕ ρO, K(η) is a Q-N_s ρc set in Z_2. Since every Q-N_s bc set is a Q-$N_s bc$ set, K(η) is a Q-$N_s bc$ set in Z_2. Hence K is a Q-N_sɕbO.

(v) Let η be a Q-$N_s o$ set in Z_1. Since K is Q-N_sɕbO, K(η) is a Q-$N_s bc$ set in Z_2. Since every Q-$N_s bc$ setis a Q-$N_s \beta c$ set, K(η) is a Q-$N_s \beta c$ set in Z_2. Hence K is a Q-N_sɕβO.

Example 5.3. Let $V = \{V_a, V_b, V_c\} = W$ and define Q-$N_s s$'s V_1 in V and W_1, W_2 & W_3 in W are

$V_1 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.3, 0.5, 0.5, 0.7), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_2 = \{(V_a, 0.1, 0.5, 0.5, 0.9), (V_b, 0.1, 0.5, 0.5, 0.9), (V_c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_3 = \{(V_a, 0.2, 0.5, 0.5, 0.8), (V_b, 0.4, 0.5, 0.5, 0.6), (V_c, 0.4, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, W_2, 1_{QNS}\}$. Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ bean identity mapping, then K is Q-N_sCbO but not Q-N_sC𝒫O, the set K$(V_1) = W^c$ is a Q-N_sbc set but not Q-N_sPc set.

Example 5.4. Let $V = \{V_a, V_b, V_c\} = W$ and define Q-N_ss's V_1 in V and W_1, W_2, W_3 & W_4 in W are

$$V_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.4, 0.5, 0.5, 0.6)\},$$

$$W_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.5, 0.5, 0.5, 0.5)\},$$

$$W_2 = \{(V_a, 0.4, 0.5, 0.5, 0.6), (V_b, 0.2, 0.5, 0.5, 0.8), (V_c, 0.6, 0.5, 0.5, 0.4)\},$$

$$W_3 = \{(V_a, 0.4, 0.5, 0.5, 0.6), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.6, 0.5, 0.5, 0.4)\},$$

$$W_4 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.5), (V_c, 0.4, 0.5, 0.5, 0.6)\}.$$

Then we have $\Gamma = \{0_{QNS}, V_1, 1_{QNS}\}$ and $\sigma_Q = 0_{QNS}, W_1, W_2, W_3, W_1 \cap W_2, 1_{QNS}\}$. Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ be an identity mapping, then K is Q-$N_s\mathfrak{c}bO$ but not Q-$N_s\mathfrak{c}$SO, the set K(V_1) = $W_3{}^c$ is a Q-N_sbc set but not Q-N_sSc set.

Example 5.5. Let $V = \{V_a, V_b\} = W$ and define Q-N_ss's V_1 in V and W_1 & W_2 in W are

$V_1 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(V_a, 0.3, 0.5, 0.5, 0.5), (V_b, 0.2, 0.5, 0.5, 0.5)\}$,

$W_2 = \{(V_a, 0.3, 0.5, 0.5, 0.7), (V_b, 0.5, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, 1_{QNS}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$.Let K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ bean identity mapping, then K is Q-$N_s\mathfrak{c}\beta O$ but not Q-$N_s\mathfrak{c}bO$, the set K$(V_1) = W^c$ is a Q-$N_s\beta c$ set but not Q-N_sbc set.

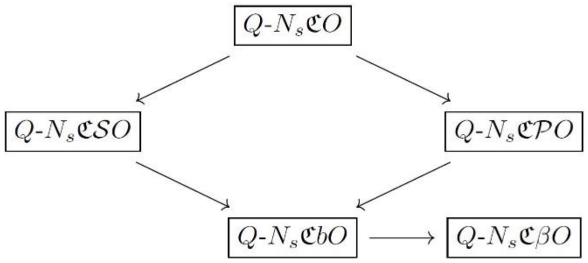

Figure 2: Q-N_sCβO maps in Q-N_sts

Theorem 5.6. A mapping K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is Q-N_sCβO iff for every Q-N_ss $(\tilde{\Psi})$ of (Z_1, Γ_Q), K$(Q$-$N_sint(\tilde{\Psi})) \supseteq Q$-$N_s\beta cl(K(\tilde{\Psi}))$.

Proof. Necessity: Assume K is a Q-$N_s\mathfrak{c}\beta O$ mapping and $(\tilde{\Psi})$ is a Q-N_sos in (Z_1, Γ_Q). Now, Q-$N_sint(\tilde{\Psi}) \subseteq (\tilde{\Psi})$ implies K$(Q$-$N_sint(\tilde{\Psi})) \subseteq$ K$(\tilde{\Psi})$. Since K is a Q-$N_s\mathfrak{c}\beta O$ mapping, K$(Q$-$N_sint(\tilde{\Psi}))$ is Q-$N_s\beta cs$ in (Z_2, σ_Q) such that K$(Q$-$N_sint(\tilde{\Psi})) \supseteq$ K$(\tilde{\Psi})$. Therefore K$(Q$-$N_sint(\tilde{\Psi}))$ $\supseteq Q$-$N_s\beta cl($K$(\tilde{\Psi}))$.

Sufficiency: Assume $(\tilde{\Psi})$ is a $Q\text{-}N_s os$ of (Z_1, Γ_Q). Then we have $\mathrm{K}(\tilde{\Psi}) = \mathrm{K}(Q\text{-}N_s int(\tilde{\Psi})) \supseteq Q\text{-}\, N_s\beta cl(\mathrm{K}(\tilde{\Psi}))$. But $Q\text{-}N_s\beta cl(\mathrm{K}(\tilde{\Psi})) \supseteq \mathrm{K}(\tilde{\Psi})$. So, $\mathrm{K}(\tilde{\Psi}) = Q\text{-}N_s\beta cl(\tilde{\Psi})$ which implies $\mathrm{K}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$

of (Z_2, σ_Q) and hence K is a $Q\text{-}N_s C\beta O$.

Theorem 5.7. If $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is a $Q\text{-}N_s$ɕβO mapping, then $Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi}))$ for every $Q\text{-}N_s s$ $(\tilde{\Psi})$ of (Z_2, σ_Q).

Proof. Consider a $Q\text{-}N_s s$ $(\tilde{\Psi})$ in (Z_2, σ_Q). We know that $Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi}))$ is a $Q\text{-}N_s os$ in (Z_1, Γ_Q). Since K is $Q\text{-}N_s\beta O$, $\mathrm{K}(Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi})))$ is $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q) and hence $\mathrm{K}(Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi}))) \subseteq Q\text{-}\, N_s\beta cl(\mathrm{K}(\mathrm{K}^{-1}(\tilde{\Psi}))) \subseteq Q\text{-}N_s\beta cl(\tilde{\Psi})$. Thus $Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi}))$.

Theorem 5.8. A mapping $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is $Q\text{-}N_s C\beta O$ iff for each $Q\text{-}N_s s$ $\tilde{\Psi}$ of (Z_2, σ_Q) and for each $Q\text{-}N_s os$ $(\tilde{\Psi})$ of (Z_1, Γ_Q) containing $\mathrm{K}^{-1}(\tilde{\Psi})$, there is a $Q\text{-}N_s\beta os$ $\tilde{A}$ of (Z_2, σ_Q) such that $(\tilde{\Psi}) \subseteq (\tilde{A})$ and $\mathrm{K}^{-1}(\tilde{A}) \subseteq (\tilde{\Psi})$.

Proof. Necessity: Let K be a $Q\text{-}N_s$ɕβO mapping. Consider a $Q\text{-}N_s cs$ $\tilde{\Psi}$ in (Z_2, σ_Q) and a $Q\text{-}N_s os$ $(\tilde{\Psi})$ in (Z_1, Γ_Q) such that $\mathrm{K}^{-1}(\tilde{\Psi}) \subseteq (\tilde{\Psi})$. Then $(\tilde{A}) = (\mathrm{K}(\tilde{\Psi})^c)^c$ is $Q\text{-}N_s\beta os$ of (Z_2, σ_Q) such that $\mathrm{K}^{-1}(\tilde{A}) \subseteq (\tilde{\Psi})$.

Sufficiency: Assume $(\tilde{\Psi})$ is a $Q\text{-}N_s os$ of (Z_1, Γ_Q). So $\mathrm{K}^{-1}((\mathrm{K}(\tilde{\Psi}))^c) \subseteq (\tilde{\Psi})^c$ and $(\tilde{\Psi})^c$ is $Q\text{-}N_s cs$ in (Z_1, Γ_Q). By presumption, there is a $Q\text{-}N_s\beta os$ $\tilde{A}$ of (Z_2, σ_Q) such that $(\mathrm{K}(\tilde{\Psi}))^c \subseteq (\tilde{A})$ and $\mathrm{K}^{-1}(\tilde{A}) \subseteq (\tilde{\Psi})^c$. Therefore $(\tilde{\Psi}) \subseteq (\mathrm{K}^{-1}(\tilde{A}))^c$ .Hence $(\tilde{A})^c \subseteq \mathrm{K}(\tilde{\Psi}) \subseteq \mathrm{K}((\mathrm{K}^{-1}(\tilde{A}))^c) \subseteq (\tilde{A})^c$ which implies $\mathrm{K}(\tilde{\Psi}) = (\tilde{A})^c$.As $(\tilde{A})^c$ is $Q\text{-}N_s\beta cs$ of (Z_2, σ_Q), $\mathrm{K}(\tilde{\Psi})$ is $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q) and hence K is $Q\text{-}N_s C\beta O$ mapping.

Theorem 5.9. A mapping $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is $Q\text{-}N_s C\beta O$ iff $\mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi})) \supseteq Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi}))$ for every $Q\text{-}N_s s$ $\tilde{\Psi}$ of (Z_2, σ_Q).

Proof. Necessity: Let K be a $Q\text{-}N_s C\beta O$ mapping. For any $Q\text{-}N_s s$ $\tilde{\Psi}$ of (Z_2, σ_Q), $\mathrm{K}^{-1}(\tilde{\Psi}) \subseteq Q\text{-}N_s cl(\mathrm{K}^{-1}(\tilde{\Psi}))$. Therefore by Theorem 5.8, there exists a $Q\text{-}N_s\beta os$ $(\tilde{\Psi})$ in $(Z_2, \sigma_Q) \ni (\tilde{\Psi}) \supseteq (\tilde{\Psi})$ & $\mathrm{K}^{-1}(\tilde{\Psi}) \supseteq Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi}))$. Hence $\mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi})) \supseteq \mathrm{K}^{-1}(\tilde{\Psi}) \supseteq Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi}))$.

Sufficiency: Let $\tilde{\Psi}$ be a $Q\text{-}N_s s$ in (Z_2, σ_Q) and $(\tilde{\Psi})$ be a $Q\text{-}N_s cs$ of (Z_1, Γ_Q) containing $\mathrm{K}^{-1}(\tilde{\Psi})$. Put $(\tilde{A}) = Q\text{-}N_s cl(\tilde{\Psi})$, then $(\tilde{\Psi}) \subseteq (\tilde{A})$ and $\tilde{A}$ is $Q\text{-}N_s\beta c$ and $\mathrm{K}^{-1}(\tilde{A}) \subseteq Q\text{-}N_s int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq (\tilde{\Psi})$. Thus by Theorem 5.8, K is $Q\text{-}Q\text{-}N_s$ɕβO mapping.

Theorem 5.10. If $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ and $\mathrm{G} : (Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ be two quadripartitioned neutrosophic mappings and $\mathrm{G} \circ \mathrm{K} : (Z_1, \Gamma_Q) \to (Z_3, \rho_Q)$ is $Q\text{-}N_s$ɕβO. If $\mathrm{G} : (Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ is $Q\text{-}N_s$ɕβIrr, then $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is $Q\text{-}N_s$ɕβO mapping.

Proof. Let $(\tilde{\Psi})$ be a Q-$N_s os$ in (Z_1, Γ_Q). Then $G \circ K(\tilde{\Psi})$ is Q-$N_s\beta cs$ of (Z_3, ρ_Q) because $G \circ K$ is Q-N_sɕβO mapping. As G is Q-$N_s C\beta \boldsymbol{Irr}$ and G K$(\tilde{\Psi})$ is Q-$N_s\beta cs$ of (Z_3, ρ_Q), $G^{-1}(G \circ K(\tilde{\Psi})) = K(\tilde{\Psi})$ is Q-$N_s\beta os$ in (Z_2, σ_Q). Hence K is Q-N_sɕβO mapping.

Theorem 5.11. If $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is Q-$N_s O$ & $G : (Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ is Q-N_sɕβO mappings, then $G \circ K : (Z_1, \Gamma_Q) \to (Z_3, \rho_Q)$ is Q-N_sɕβO.

Proof. Let $(\tilde{\Psi})$ be a Q-$N_s os$ in (Z_1, Γ_Q). Then $K(\tilde{\Psi})$ is a Q-$N_s os$ of (Z_2, σ_Q) because K is a Q-$N_s O$ map- ping. As G is Q-N_sɕβO, $G(K(\tilde{\Psi})) = (G \circ K)(\tilde{\Psi})$ is a Q-$N_s\beta cs$ of (Z_3, ρ_Q). Thus G K is Q-N_sɕβO mapping.

6 Quadripartitioned Neutrosophic contra β-closed mapping

In this section, quadripartitioned neutrosophic contra β-closed maps are introduced and some of its properties are discussed.

Definition 6.1. A mapping $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is quadripartitioned neutrosophic contra (resp. semi, pre, b & β) closed (in short, Q-$N_s CC$ (resp. Q-$N_s CSC$, Q-$N_s CPC$, Q-$N_s CbC$ & Q-$N_s C\beta C$)) if the image of each Q-$N_s c$ set of (Z_1, Γ_Q) is Q-$N_s o$ (resp. Q-$N_s So$, Q-$N_s Po$, Q-$N_s bo$ & Q-$N_s\beta o$) set in (Z_2, σ_Q).

Proposition 6.2. A map $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$, then the statements are hold but the converse does not true. Every

(i) Q-N_sɕC is a Q-N_sɕSC.

(ii) Q-N_sɕC is a Q-N_sɕPC.

(iii) Q-N_sɕSC is a Q-N_sɕbC.

(iv) Q-N_sɕPC is a Q-N_sɕbC.

(v) Q-N_sɕbC is a Q-N_sɕβC.

Proof. (i) Let η be a Q-$N_s c$ set in Z_1. Since K is Q-N_sɕC, $K(\eta)$ is a Q-$N_s o$ set in Z_2. Since every Q-$N_s o$set is a Q-$N_s So$ set, $K(\eta)$ is a Q-$N_s So$ set in Z_2. Hence K is a Q-N_sɕSC.

(ii) Let η be a Q-$N_s c$ set in Z_1. Since K is Q-N_sɕC, $K(\eta)$ is a Q-$N_s o$ set in Z_2. Since every Q-N_s ρo set is a Q-$N_s Po$ set, $K(\eta)$ is a Q-$N_s Po$ set in Z_2. Hence K is a Q-N_sɕPC.

(iii) Let η be a Q-$N_s c$ set in Z_1. Since K is Q-N_sɕC, $K(\eta)$ is a Q-$N_{sS}o$ set in Z_2. Since every Q-N_s So set is a Q-$N_s bo$ set, $K(\eta)$ is a Q-$N_s bo$ set in Z_2. Hence K is a Q-N_sɕbC.

(iv) Let η be a Q-$N_s c$ set in Z_1. Since K is Q-N_sɕC, $K(\eta)$ is a Q-$N_s b\,o$ set in Z_2. Since every Q-$Ns\rho o$ set is a Q-$N_s bo$ set, $K(\eta)$ is a Q-$N_s bo$ set in Z_2. Hence K is a Q-$N_s CbC$.

(v) Let η be a Q-$N_s c$ set in Z_1. Since K is Q-N_sɕbC, $K(\eta)$ is a Q-$N_s bo$ set in Z_2. Since every Q-$N_s bo$ setis a Q-$N_s\beta o$ set, $K(\eta)$ is a Q-$N_s\beta o$ set in Z_2. Hence K is a Q-N_sɕβC.

Example 6.3. In Example 5.3, K is Q-N_sɕbC but not Q-N_sɕPC.

Example 6.4. In Example 5.4, K is Q-N_sɕbC but not Q-N_sɕSC.

Example 6.5. In Example 5.5, K is Q-$N_s\mathfrak{c}\beta C$ but not Q-$N_s\mathfrak{c}bC$.

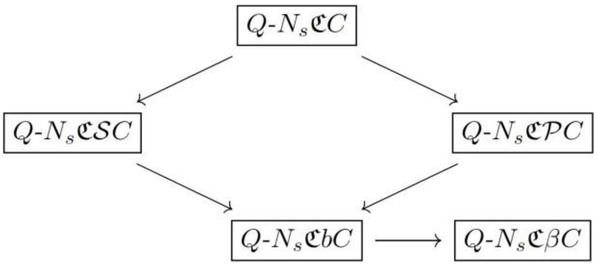

Figure 3: Q-$N_s\mathfrak{c}\beta C$ maps in Q-$N_s ts$

Theorem 6.6. A mapping K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is Q-$N_S\mathfrak{c}\beta C$ iff for each Q-$N_S s$ $\tilde{\Psi}$ of (Z_2, σ_Q) and for each Q-$N_S cs$ $(\tilde{\Psi})$ of (Z_1, Γ_Q) containing $K^{-1}(\tilde{\Psi})$, there is a Q-$N_s\beta cs$ $\tilde{A}$ of (Z_2, σ_Q) such that $(\tilde{\Psi}) \subseteq (\tilde{A})$ and $K^{-1}(\tilde{A}) \subseteq (\tilde{\Psi})$.

Proof. Necessity: Let K be a Q-$N_S C\beta C$ mapping. Consider a Q-$N_S os$ $\tilde{\Psi}$ in (Z_2, σ_Q) and a Q-$N_S cs$ in (Z_1, Γ_Q) such that $K^{-1}(\tilde{\Psi}) \subseteq (\tilde{\Psi})$ .Then $(\tilde{\Psi}) = 1_{QN}$– $K^{-1}((\tilde{\Psi})^c)$ is Q-$N_s\beta cs$ of (Z_2, σ_Q) such that

Sufficiency: Assume $(\tilde{\Psi})$ is a Q-$N_S cs$ of (Z_1, Γ_Q). Then $(K(\tilde{\Psi}))^c$ is a Q-$N_S s$ of (Z_2, σ_Q) and $(\tilde{\Psi})^c$ is$Q$-$N_S os$ in (Z_1, Γ_Q) such that $K^{-1}((K(\tilde{\Psi}))^c) \subseteq (\tilde{\Psi})^c$. By presumption, there is a Q-$N_s\beta cs$ $\tilde{A}$ of (Z_2, σ_Q) such that $(K(\tilde{\Psi}))^c \subseteq (\tilde{\Psi})$ and $K^{-1}(\tilde{\Psi}) \subseteq (\tilde{\Psi})^c$. Therefore $(\tilde{\Psi}) \subseteq (K^{-1}(\tilde{\Psi}))^c$. Hence $(\tilde{\Psi})^c \subseteq K(\tilde{\Psi})$ $K \subseteq ((K^{-1}(\tilde{\Psi}))^c) \subseteq (\tilde{\Psi})^c$ which implies $K(\tilde{\Psi}) = (\tilde{\Psi})^c$. As $(\tilde{\Psi})^c$ is Q-$N_s\beta os$ of (Z_2, σ_Q), $K(\tilde{\Psi})$ is Q-$N_s\beta o$ in (Z_2, σ_Q) and hence K is Q-$N_s\mathfrak{c}\beta C$ mapping.

Theorem 6.7. If K : $(Z_1, \Gamma_Q) \longrightarrow (Z_2, \sigma_Q)$ is Q-$N_s C$ and G : $(Z_2, \sigma_Q) \longrightarrow (Z_3, \rho_Q)$ is Q-$N_s\mathfrak{c}\beta C$. Then $G \circ K : (Z_1, \Gamma_Q) \longrightarrow (Z_3, \rho_Q)$ is Q-$N_s\mathfrak{c}\beta C$.

Proof. Let $(\tilde{\Psi})$ be a Q-$N_S cs$ in (Z_1, Γ_Q). As K is Q-$N_S C$ mapping, $K(\tilde{\Psi})$ is Q-$N_S cs$ in (Z_2, σ_Q). As G is Q- $N_S C\beta C$ mapping, $(G \circ K)(\tilde{\Psi}) = G(K(\tilde{\Psi}))$ is Q-$N_s\beta os$ in (Z_3, ρ_Q). Hence $G \circ K$ is Q-$N_S\mathfrak{c}\beta C$ mapping.

Theorem 6.8. If K : $(Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is Q-$N_S C\beta C$ map, then Q-$N_s\beta int(K(\tilde{\Psi})) \supseteq K(Q$-$N_S int(\tilde{\Psi}))$.

Proof. The proof is obvious from Definition 2.7 and Definition 6.1.

Theorem 6.9. Let K : $(Z_1, \Gamma_Q) \longrightarrow (Z_2, \sigma_Q)$ and G : $(Z_2, \sigma_Q) \longrightarrow (Z_3, \rho_Q)$ be Q-$N_s\mathfrak{c}\beta C$ mappings. If every Q-$N_s\beta os$ of (Z_2, σ_Q) is Q-$N_s os$, then $G \circ K : (Z_1, \Gamma_Q) \longrightarrow (Z_3, \rho_Q)$ is Q-$N_s\beta C$.

Proof. Let $(\tilde{\Psi})$ be a Q-$N_S cs$ in (Z_1, Γ_Q). As K is Q-$N_s\mathfrak{c}\beta C$ mapping, $K(\tilde{\Psi})$ is Q-$N_s\beta os$ in (Z_2, σ_Q). By presumption, $K(\tilde{\Psi})$ is Q-$N_s os$ of (Z_2, σ_Q). As G is Q-$N_s\mathfrak{c}\beta C$ mapping, $G(K(\tilde{\Psi})) = (G \circ K)(\tilde{\Psi})$ is Q-$N_s\beta cs$ in (Z_3, ρ_Q). Hence $G \circ K$ is Q-$N_s\beta C$ mapping.

Theorem 6.10. Consider a bijective mapping K : $(Z_1, \Gamma_Q) \longrightarrow (Z_2, \sigma_Q)$. Then the following statements areequivalent:

(i) K is a $Q\text{-}N_s\mathfrak{c}\beta O$ mapping.

(ii) K is a $Q\text{-}N_s\mathfrak{c}\beta C$ mapping.

(iii) K^{-1} is $Q\text{-}N_s\beta Cts$ mapping.

Proof. (i) ⇒ (ii): Assume K is a $Q\text{-}N_s\mathfrak{c}\beta O$ mapping. If $Q\text{-}N_sos$ $(\tilde{\Psi})$ in (Z_1, Γ_Q), by presumption $K(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q). But now, $(\tilde{\Psi})$ is $Q\text{-}N_scs$ in (Z_1, Γ_Q). So, $1_{QNS} - (\tilde{\Psi})$ is a $Q\text{-}N_sos$ in (Z_1, Γ_Q). By assumption, $K(1_{QNS} - (\tilde{\Psi}))$ is a $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q). Hence, $1_{QNS} - K(1_{QNS} - (\tilde{\Psi}))$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). Thus, K is a $Q\text{-}N_s\mathfrak{c}\beta C$ mapping.

(ii) ⇒ (iii): Consider a $Q\text{-}N_scs$ $(\tilde{\Psi})$ in (Z_1, Γ_Q). By assumption, $K(\tilde{\Psi})$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). Hence, $K(\tilde{\Psi}) = (K^{-1})^{-1}(\tilde{\Psi})$. So K^{-1} is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). Thus, K^{-1} is $Q\text{-}N_s\beta Cts$.

(iii) ⇒ (i): Consider a $Q\text{-}N_sos$ $(\tilde{\Psi})$ in (Z_1, Γ_Q). By assumption, $(K^{-1})^{-1}(\tilde{\Psi}) = K(\tilde{\Psi})$ is a $Q\text{-}N_sC\beta O$ mapping.

7 Quadripartitioned Neutrosophic contra *β*-homeomorphism

In this section, the concept of quadripartitioned neutrosophic contra *β*-homeomorphism is introduced and itsproperties are discussed.

Definition 7.1. A bijection $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is called a

(i) quadripartitioned neutrosophic contra homeomorphism (briefly $Q\text{-}N_sCHom$) if K and K^{-1} are $Q\text{-}N_sCCts$ mapping.

(ii) quadripartitioned neutrosophic contra *β*-homeomorphism (briefly $Q\text{-}N_sC\beta Hom$) if K and K^{-1} are $Q\text{-}N_sC\beta Cts$ mapping.

Theorem 7.2. Each $Q\text{-}N_s\mathfrak{c}Hom$ is a $Q\text{-}N_s\mathfrak{c}\beta Hom$. But the converse not true.

Proof. Assume K is $Q\text{-}N_s\mathfrak{c}Hom$. Then K and K^{-1} are $Q\text{-}N_s\mathfrak{c}Cts$. We know that each $Q\text{-}N_s\mathfrak{c}Cts$ functionis $Q\text{-}N_s\mathfrak{c}\beta Cts$. So, K and K^{-1} are $Q\text{-}N_s\mathfrak{c}\beta Cts$. Thus, K is a $Q\text{-}N_s\mathfrak{c}\beta Hom$.

Example 7.3. Let $V = \{a, b, c\} = W$ and define $Q\text{-}N_ss$'s V_1, V_2 & V_3 in V and W_1 in W are

$V_1 = \{(a, 0.2, 0.5, 0.5, 0.8), (b, 0.3, 0.5, 0.5, 0.7), (c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_2 = \{(a, 0.1, 0.5, 0.5, 0.9), (b, 0.1, 0.5, 0.5, 0.9), (c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_3 = \{(a, 0.2, 0.5, 0.5, 0.8), (b, 0.4, 0.5, 0.5, 0.6), (c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(a, 0.2, 0.5, 0.5, 0.8), (b, 0.4, 0.5, 0.5, 0.6), (c, 0.4, 0.5, 0.5, 0.6)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, V_2, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$. Let $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ bean identity mapping, then K is $Q\text{-}N_sHom$ but not $Q\text{-}N_s\mathfrak{c}Hom$.

Theorem 7.4. Consider a bijective mapping $K : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$. The followings statements are equivalent if K is $Q\text{-}N_s\mathfrak{c}\beta Cts$.

(i) K is a $Q\text{-}N_s\mathfrak{c}\beta C$ mapping.

(ii) K is a Q-N_sɕβO mapping.

(iii) K is a Q-N_sɕβHom.

Proof. (i)⇒(ii) : Let K be a bijective mapping and a Q-N_sɕβC mapping. Therefore, K^{-1} is a Q-N_sɕβCts mapping. As each Q-$N_s os$ in (Z_1, Γ_Q) is a Q-$N_s\beta cs$ in (Z_2, σ_Q), K is a Q-N_sɕβO mapping.

(ii) ⇒ (iii) : Assume K is a bijective and Q-N_sɕβO mapping. Also, K^{-1} is a Q-N_sCβCts mapping.Therefore, K and K^{-1} are Q-N_sɕβCts. Thus, K is a Q-N_sɕβHom.

(iii) ⇒ (i): Assume K is a Q-N_sɕβHom. So, K and K^{-1} are Q-N_sɕβCts. As every Q-$N_s cs$ in (Z_1, Γ_Q) is a Q-$N_s\beta os$ in (Z_2, σ_Q), K is a Q-N_sɕβC mapping.

Theorem 7.5. Let K : (Z_1, Γ_Q) (Z_2, σ_Q) be a Q-N_sɕβHom. If (Z_1, Γ_Q) and (Z_2, σ_Q) are Q-$N_s\beta T1$ - spaces, then K is a Q-N_sɕHom.

Proof. Consider a Q-$N_s cs$ $\tilde{\Psi}$ in (Z_2, σ_Q). So, $K^{-1}(\tilde{\Psi})$ is a Q-$N_s\beta os$ in (Z_1, Γ_Q). As (Z_1, Γ_Q) is a Q-$N_s\beta T_{1/2}$-space, $K^{-1}(\tilde{\Psi})$ is a Q-$N_s os$ in (Z_1, Γ_Q). Therefore, K is Q-N_sɕCts. By hypothesis, K^{-1} is Q-N_sɕβCts. Let $(\tilde{\Psi})$ be a Q-$N_s cs$ in (Z_1, Γ_Q). Then, $K(\tilde{\Psi})$ is a Q-$N_s\beta os$ in (Z_2, σ_Q), by presumption. Since (Z_2, σ_Q) is a Q-$N_s\beta T_{1/2}$-space, $K(\tilde{\Psi})$ is a Q-$N_s os$ in (Z_2, σ_Q). Therefore, K^{-1} is Q-N_sɕCts. Thus, K is a Q-N_sɕHom.

Theorem 7.6. Let K : $(Z_1, \Gamma_Q) \longrightarrow (Z_2, \sigma_Q)$ be a Q-$N_s ts$. If (Z_2, σ_Q) is a Q-$N_s\beta T_{1/2}$-space, then the following are equivalent:

(i) K is Q-N_sɕβC mapping.

(ii) If $(\tilde{\Psi})$ is a Q-$N_s os$ in (Z_1, Γ_Q), then $K(\tilde{\Psi})$ is Q-$N_s\beta cs$ in (Z_2, σ_Q).

(iii) $K(Q\text{-}N_s int(\tilde{\Psi})) \subseteq Q\text{-}N_s cl\ (Q\text{-}N_s int(K(\tilde{\Psi})))$ for every Q-$N_s s$ $(\tilde{\Psi})$ in (Z_1, Γ_Q).

Proof. (i) ⇒ (ii): Obvious.

(ii)⇒ (iii): Consider a Q-$N_s s$ (Ψ) in (Z_1, Γ_Q). We know that, Q-$N_s int(\Psi)$ is a Q-$N_s os$ in (Z_1, Γ_Q). Then, $K(Q\text{-}N_s int(\tilde{\Psi}))$ is a Q-$N_s\beta cs$ in (Z_2, σ_Q). Since (Z_2, σ_Q) is a Q-$N_s\beta T_{1/2}$-space, $K(Q\text{-}N_s int(\tilde{\Psi}))$ is a Q-$N_s cs$ in (Z_2, σ_Q). Therefore, $K(Q\text{-}N_s int(\tilde{\Psi})) = Q\text{-}N_s cl(K(Q\text{-}N_s int(\tilde{\Psi}))) \subseteq Q\text{-}N_s cl(Q\text{-}N_s int(K(\tilde{\Psi})))$.

(iii)⇒ (i): Let $(\tilde{\Psi})$ be a Q-$N_s cs$ in (Z_1, Γ_Q). Then, $(\tilde{\Psi})^c$ is a Q-$N_s os$ in (Z_1, Γ_Q). As $K(Q\text{-}N_s int(\tilde{\Psi})^c) \subseteq Q\text{-}N_s cl(Q\text{-}N_s int(K(\tilde{\Psi})^c))$, we get $K((\tilde{\Psi})^c) \subseteq Q\text{-}N_s cl(Q\text{-}N_s int(K(\tilde{\Psi})^c))$. Therefore, $K((\tilde{\Psi})^c)$ is Q-$N_s\beta cs$ in (Z_2, σ_Q). Thus, $K(\tilde{\Psi})$ is a Q-$N_s\beta os$ in (Z_1, Γ_Q). Hence, K is a Q-N_sCβC mapping.

Theorem 7.7. Let K : $(Z_1, \Gamma_Q) \rightarrow (Z_2, \sigma_Q)$ and G : $(Z_2, \sigma_Q) \rightarrow (Z_3, \rho_Q)$ be Q-N_sɕβC, where $(Z_1, \Gamma_Q)$and (Z_3, ρ_Q) are two Q-$N_s ts$'s and (Z_2, σ_Q) a Q-$N_s\beta T_{1/2}$-space, then the composition $G \circ K$ is Q-$N_s\beta C$.

Proof. Consider a $Q\text{-}N_{S}cs$ $(\tilde{\Psi})$ in (Z_1, Γ_Q). As K is $Q\text{-}N_s\mathfrak{c}\beta C$ and $\mathrm{K}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q), by assumption, $\mathrm{K}(\tilde{\Psi})$ is a $Q\text{-}N_{S}os$ in (Z_2, σ_Q). Since G is $Q\text{-}N_s\mathfrak{c}\beta C$, then $\mathrm{G}(\mathrm{K}(\tilde{\Psi}))$ is $Q\text{-}N_s\beta cs$ in (Z_3, ρ_Q) and $\mathrm{G}(\mathrm{K}(\tilde{\Psi})) = (\mathrm{G} \circ \mathrm{K})(\tilde{\Psi})$. Thus, $\mathrm{G} \circ \mathrm{K}$ is $Q\text{-}N_s\beta C$.

Theorem 7.8. Let $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ and $\mathrm{G} : (Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ be two $Q\text{-}N_sts$'s, then the following hold:

(i) If $\mathrm{G} \circ \mathrm{K}$ is $Q\text{-}N_s\mathfrak{c}\beta O$ and K is $Q\text{-}N_sCts$, then G is $Q\text{-}N_s\mathfrak{c}\beta O$.

(ii) If $\mathrm{G} \circ \mathrm{K}$ is $Q\text{-}N_sO$ and G is $Q\text{-}N_s\mathfrak{c}\beta Cts$, then K is $Q\text{-}N_s\mathfrak{c}\beta O$.

Proof. The proof is obvious from Definition 3.1 and Definition 5.1.

8 Quadripartitioned neutrosophic contra β-C homeomorphism

The quadripartitioned neutrosophic contra β-C homeomorphism is introduced in this section and some of its properties are analyzed.

Definition 8.1. A bijection $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is called a quadripartitioned neutrosophic contra β-Completely homeomorphism (briefly, $Q\text{-}N_s\mathfrak{c}\beta CHom$) if K and K^{-1} are $Q\text{-}N_s\mathfrak{c}\beta Irr$ mappings.

Theorem 8.2. Each $Q\text{-}N_s\mathfrak{c}\beta CHom$ is a $Q\text{-}N_s\mathfrak{c}\beta Hom$. But not conversely.

Proof. Consider a $Q\text{-}N_{S}os$ $\tilde{\Psi}$ in (Z_2, σ_Q). Then $\tilde{\Psi}$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). By presumption, $\mathrm{K}^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in (Z_1, Γ_Q). Therefore, K is a $Q\text{-}N_s\mathfrak{c}\beta Cts$ mapping. So, K and K^{-1} are $Q\text{-}N_s\mathfrak{c}\beta Cts$ mappings. Thus, K is a $Q\text{-}N_s\mathfrak{c}\beta Hom$.

Example 8.3. Let $V = \{a, b, c\} = W$ and define $Q\text{-}N_sS$'s V_1 & V_2 in V and W_1 in W are

$V_1 = \{(a, 0.2, 0.5, 0.5, 0.8), (b, 0.3, 0.5, 0.5, 0.7), (c, 0.4, 0.5, 0.5, 0.6)\}$,

$V_2 = \{(a, 0.1, 0.5, 0.5, 0.9), (b, 0.1, 0.5, 0.5, 0.9), (c, 0.4, 0.5, 0.5, 0.6)\}$,

$W_1 = \{(a, 0.4, 0.5, 0.5, 0.6), (b, 0.3, 0.5, 0.5, 0.7), (c, 0.2, 0.5, 0.5, 0.8)\}$.

Then we have $\Gamma_Q = \{0_{QNS}, V_1, V_2, 1_{QNS}\}$ and $\sigma_Q = \{0_{QNS}, W_1, 1_{QNS}\}$. Let $\mathrm{K} : (Z_1, \Gamma_Q)$ (Z_2, σ_Q) be a mapping, defined as $\mathrm{K}(a) = c$, $\mathrm{K}(b) = b$ & $\mathrm{K}(c) = a$, then K is $Q\text{-}N_s\mathfrak{c}\beta Hom$ but not $Q\text{-}N_s\mathfrak{c}\beta CHom$.

Theorem 8.4. If $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ is a $Q\text{-}N_s\mathfrak{c}\beta CHom$, then $Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))$ for every $Q\text{-}N_{S}s$ $\tilde{\Psi}$ in (Z_2, σ_Q).

Proof. Consider a $Q\text{-}N_{S}s$ $\tilde{\Psi}$ in (Z_2, σ_Q). Since, $Q\text{-}N_{S}cl(\tilde{\Psi})$ is a $Q\text{-}N_{S}cs$ in (Z_2, σ_Q) and every $Q\text{-}N_{S}cs$ is a $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q). As K is $Q\text{-}N_s\mathfrak{c}\beta Irr$, $\mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))$ is a $Q\text{-}N_s\beta os$ in (Z_1, Γ_Q). Then, $Q\text{-}N_sint(\mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))) = \mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))$. Here, $Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq Q\text{-}N_s\beta int(\mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))) = \mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))$. Therefore, $Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(Q\text{-}N_{S}cl(\tilde{\Psi}))$ for every $Q\text{-}N_{S}s$ $\tilde{\Psi}$ in (Z_2, σ_Q).

Theorem 8.5. Let $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ be a $Q\text{-}N_s\beta CHom$. Then $Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi}))$ for every $Q\text{-}N_{S}s$ $\tilde{\Psi}$ in (Z_2, σ_Q).

Proof. As K is a $Q\text{-}N_s C\beta CHom$, K is a $Q\text{-}N_s C\beta Irr$ mapping. Consider a $Q\text{-}N_s s$ $\tilde{\Psi}$ in (Z_2, σ_Q). It is obvious that, $Q\text{-}N_s\beta cl(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in (Z_2, σ_Q). As $\mathrm{K}^{-1}(\tilde{\Psi}) \subseteq \mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi}))$, we have

$$Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq Q\text{-}N_s\beta int(\mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi})))$$

$$\subseteq \mathrm{K}^{-1}(Q\text{-}N_s\beta cl(\tilde{\Psi})).$$

$$\Rightarrow Q\text{-}N_s\beta int(\mathrm{K}^{-1}(\tilde{\Psi})) \subseteq \mathrm{K}^{-1}(N_s\beta cl(\tilde{\Psi})).$$

Theorem 8.6. If $\mathrm{K} : (Z_1, \Gamma_Q) \to (Z_2, \sigma_Q)$ and $\mathrm{G} : (Z_2, \sigma_Q) \to (Z_3, \rho_Q)$ are $Q\text{-}N_s\mathfrak{c}\beta CHom$'s, then $\mathrm{G}\circ\mathrm{K}$ is a $Q\text{-}N_s\beta CHom$.

Proof. Assume that K and G are two $Q\text{-}N_s\mathfrak{c}\beta CHom$'s. Let $\tilde{\Psi}$ be a $Q\text{-}N_s\beta cs$ in (Z_3, ρ_Q). Then, $\mathrm{G}^{-1}(\tilde{\Psi})$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). By presumption, $\mathrm{K}^{-1}(\mathrm{G}^{-1}(\tilde{\Psi}))$ is a $Q\text{-}N_s\beta cs$ in (Z_1, Γ_Q). Therefore, $(\mathrm{G}\circ\mathrm{K})^{-1}$ is a $Q\text{-}N_s\beta Irr$ mapping. Assume $(\tilde{\Psi})$ is a $Q\text{-}N_s\beta cs$ in (Z_1, Γ_Q). Then, by hypothesis, $\mathrm{K}(\mathrm{G})$ is a $Q\text{-}N_s\beta os$ in (Z_2, σ_Q). Hence, $\mathrm{G}(\mathrm{K}(\tilde{\Psi}))$ is a $Q\text{-}N_s\beta cs$ in (Z_3, ρ_Q). This implies that $\mathrm{G}\circ\mathrm{K}$ is a $Q\text{-}N_s\beta Irr$ mapping. Thus, $\mathrm{G}\circ\mathrm{K}$ is a $Q\text{-}N_s\beta CHom$.

9 Conclusions

In this paper, the new concept of a quadripartitioned neutrosophic contra β-continuous mappings, quadri- partitioned neutrosophic contra β-open mappings, a quadripartitioned neutrosophic contra β-closed map- pings, a quadripartitioned neutrosophic contra β-homeomorphism, a quadripartitioned neutrosophic contra β-completely homeomorphism in $Q\text{-}N_sts$ are discussed and also derived some of their related attributes. In future, we can carry out the further research on a quadripartitioned neutrosophic β-compactness, a quadripar- titioned neutrosophic β-connectedness and a quadripartitioned neutrosophic β-regular and normal spaces in $Q\text{-}N_sts$.

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