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Distance-Based Robust Fuzzy Topological Mathematical Modelling for Spider Networks in Edge Detection Techniques

[1]E. Jayanthi, [2]S. Shunmugapriya

[1]PG and Research Department of Mathematics,A.V.V.M Sri Pushpam College (Autonomous), (Affiliated to Bharathidasan University),Poondi, Thanjavur – 613503, Tamilnadu, India

[2] PG and Research Department of Mathematics,Rajah Serfoji Government College (Autonomous), (Affiliated to Bharathidasan University),Thanjavur- 613005, Tamilnadu, India

[1]jayanthimax@gmail.com , [2] sspriya1969@gmail.com

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Abstract:

Introduction: Topological indices are numerical numbers used to represent a graph and describe its characteristics. In this method to find the distance based fuzzy topological spider network and it is implemented in robust regression. It will give the new proposed robust fuzzy topological model (RFTM). This model connected with the concept of image processing specially in edge detection techniques and verify our proposed model performance.

Objectives: Distance-based fuzzy topological indices are widely used in chemistry for quantitatively describing molecular structure and correlating it with various physicochemical properties. These indices are derived from the molecular graph, where atoms are represented as nodes and bonds as edges. There are several distances based topological indices, as well as a diverse set of distance-based molecular structure descriptors. Structure-based topological descriptors of chemical networks allow us to predict physicochemical attributes and bioactivities of compounds using QSAR/QSPR approaches. Topological indices are numerical numbers used to represent a graph and describe its characteristics. A robust fuzzy graph is a concept that integrates the robustness of graph structures with the flexibility and uncertainty modelling provided by fuzzy graph theory. It is used in applications where the system's topology or relationships among elements (nodes) are prone to noise, uncertainty, or partial truth, and there is a need for a reliable representation and analysis despite these uncertainties.

Methods: To find the distance based fuzzy topological spider network and it is implemented in robust regression. It will give the new proposed robust fuzzy topological model (RFTM). This model connected with the concept of image processing specially in edge detection techniques and verify our proposed model performance.

Results: In this paper, we provide analytical formulas for the Szeged index, edge Szeged index, edge vertex Szeged index, and PI index of spider networks and Augmented spider networks. Our proposed result also used in the concept of edge detection techniques.

Conclusions: The discrete characteristics of spider networks and augmented spider networks are determined using distance-based topological indices. We determined the analytical formulations for Wiener, Szeged, and their numerous versions, as well as PI of spider networks and augmented spider networks. The approach we utilized to calculate the topological networks is useful for evaluating the topological indices of various interconnection networks.The proposed model RFTM perform well compared to existing model. Also, it is the maximum no of true edges detected and reduced the number of false edges. The TLS MATLAB toolbox was used to simulate the previously provided data. Depending on the sample size, the data is generated randomly. Furthermore, the threshold settings in the toolbox in the image are set to two.

Keywords: Distance, topological indices, Szeged index, edge Szeged index, edge vertex Szeged index, spider, edge detection and PI index.

1. Introduction

A topological index is a molecular graph invariant that aims to give an easily calculable method of approximating a physicochemical feature that is connected to the molecular graph's structure. The Wiener index, the total of distances between all pairs of vertices in a graph, has been demonstrated by Harry Wiener to correlate well with different characteristics of alkanes in a series of studies in 1947 and 1948 [1–4]. Key indices such as the Wiener index, the Harary index, and the Randić index help researchers understand properties like boiling points, stability, and reactivity, by providing numerical values based on distances between atoms.

The study of topological indices is a central part of cheminformatics and mathematical chemistry, where these indices help to describe molecular structures quantitatively and predict various properties of chemical compounds. Topological indices are scalar values derived from the graph representation of molecules, where atoms are treated as nodes and bonds as edges.

While topological indices are powerful tools, their application in molecular property prediction is often limited by the complexity of large biological molecules and networks. Current research is expanding to include weighted indices, incorporating 3D spatial information, and developing indices that account for electronic effects in atoms.

Additionally, machine learning techniques are increasingly combined with topological indices, enhancing the accuracy of predictive models. This integration is pushing the field toward more reliable, rapid predictions in materials science, drug discovery, and environmental chemistry.

The next section describes the preliminaries and followed by in section 3 describe the spider networks. In section 4 indicate that distance-based spider networks. Robust fuzzy topological model (RFTM) describes in the section 5. In section 6 reveals that experimental results demonstrated in edge detection techniques and followed by final section is conclusion.

2. Objectives

In this study, all graphs G are assumed to be loopless, finite, and undirected. Graph G refers to the collection of vertices (also known as nodes) linked by edges (also referred to as links). It is divided into two sets: V and E(V is the set of vertices, while E is the set of edges). Over the last several decades, a wide range of numerical values known as structural invariants, topological indices, or topological descriptors have been developed and investigated in order to get a better understanding of the properties and information contained in graph connection patterns. The chemical networks are typically characterized using numerical and topological indices. They establish the link between a molecule's structure and characteristics. Topological indices are extensively used in QSAR and QSPR research studies [1].

If G and H are isomorphic graphs, then Top(G) = Top(H) [2]. Hundreds of different topological indices have emerged as a result of the widespread usage of fundamental topological indices such as the Wiener Index [3], Zagreb Index [4], and Szeged Index [5]. One of the earliest indices was the Wiener index, created by Harold Wiener in 1947 [6] while researching the boiling point of paraffins. The symbols V (G) and E(G) denote a graph's vertex and edge sets, respectively. We show that m = |E(G)| and n = |V(G)|. The distance between u and v in G is represented by dG(u,v) for vertices u,v∈ V (G), and the transmission of x ∈ V(G), also known as the status or simply the distance of a vertex, is defined as follows,

$$w(x) = \sum_{y \in V(G)} d_G(x, y)$$

To distinguish between isomers, the Balaban index provides information on the molecule's branching and compactness. The formula's denominator, m - n + 2, improves the comparability of cyclic and acyclic networks with equal number of vertices.

Harris Operator

To address the issue of noisy response induced by a binary window function, Harris proposed utilizing a Gaussian noisy filter in his Harris detector. The Harris operator detector is based on the local autocorrelation function of a signal, which looks at local changes in the signal with patches slightly moved in various directions [5, 6]. Given a point and a shift (x, y), the autocorrelation functionis,

C(x,y) = Σw[I(xi,yi) - I(xi+Δx, yi+ Δy)]2

A Taylor expansion with just first-order terms can approach the image function I(.). The autocorrelation matrix C(x, y) represents the intensity structure of the immediate neighborhood. Finally, define the operator points as local maxima of the operator response using C(x, y) Eigenvalues.

FAST Operator

Many of the various feature detectors are really good. However, they are not fast enough for real-time applications. For example, mobile robots with low computing power that employ SLAM (Simultaneous Localization and Mapping). To solve this, Edward Rosten and Tom Drummond [11-13] developed the FAST (Features from Accelerated Segment Test) technique, which may be summarized as follows. Find an area of interest in a photograph, then consider around it with sixteen pixels. Determine the number of pixels next to the interest point using the threshold value T.

When there are less than twelve consecutive pixels, this variation performs badly. The author proposed using machine learning to improve speed and efficiency. After identifying the interest sites, non-maximal suppression can be utilized to address the identification of several interest points adjacent to one another.

Canny Operator

The Canny approach [17] is an important strategy for recognizing edges since it removes noise from the image before identifying the edges while maintaining the edges' properties. The inclination to find edges is then applied to establish the critical value of the threshold. The intelligent edge detection approach uses the following algorithmic stages: 1. Convolution using a Gaussian function is used to generate a smooth image of f (r, c),

f(r,c).f(r,c)=f(r,c)*G(r,c,6)

3. Methods

Spider network

Definition 3.1

A Spider graph joined another spider graph by based on edge between them is called a n^{th} joint spider graph and it is denoted by $SP(n^{2t}, 2^t)$, n=1,2,….. and t=1,2,…n.

Example 3.2

The following is an example of the labeling process used above to confirm that the spider graph $SP(1^4,2^2)$ is an integer.

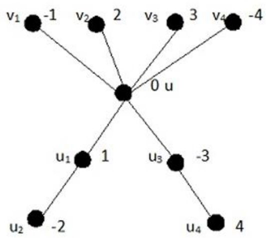

Fig. 1: Spider graph $SP(1^4,2^2)$ integer I cordial labeling of graph

Spider networks are connecting networks used in communication networks. They are referred to as the underlying graphs of networks that efficiently conduct the Fast Fourier Transform.

Definition 3.3

The spider network uses a sequence of switches and connection patterns to connect n inputs and n outputs. An n-dimensional spider is made up of vertex sets,

$$V = \{(u,i): u \in V(Q^*{}_n), 0 \leq i \leq n\}$$

such that if and only if l = i + 1 and either u=v or u varies from v in the lth bit, then an edge connects two vertices (u,i) and (v,l). BF(n) contains $n2^{n+1}$ edges with n levels and $(n + 1)2^n$ vertices. It is not Eulerian because no odd-degree vertices exist.

Remarks: 3.4

(i). The spider network with one edge along the antipodal vertices in each cycle of BF(n) is called the augmented spider network of dimension n, or ABF(n). The bulk of BF(n)'s properties remain in the upgraded spider network. By finding their topological indices, we may investigate some of the properties of augmented spiders.

(ii). This post discusses a simple yet useful graph. The shortest path length between two verticesμ and ν is μ*,v* ∈ V*(G), is indicated as d(v*,μ*), which may be calculated using by,

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$$N_{v*}(e \setminus G) = \{v * \in V * (G): d(v *, u *) < d(\mu, v *)\}$$
$$M_{v*}(e \setminus G) = \{f \in E(G): d(v *, f) < d(\mu, f)\}$$

Where $e = \mu(v *)$ and the symbol for their cardinality is$n_{v*}(e)$ and $m_{v*}(e)$.
The concept of weighted graph,$\mathcal{G}_{sm} = (\mathcal{G}, (m_{v*}, s_{v*}), s_e)$
Where m_{v*} vertex weight, s_{v*} is strength weight and s_e is edge strength.

Definition 3.5

Let, $v^*\mu \in E_{\mathcal{G}_{sm}}$ is defined by,

$$n_{v*}(e \setminus \mathcal{G}_{sm}) = \sum_{p \in n_{v*}(e \setminus \mathcal{G}_{sm})} m_{v^*}(p)$$
$$m_{v*}(e \setminus \mathcal{G}_{sm}) = \sum_{p \in n_{v*}(e \setminus \mathcal{G}_{sm})} m_{v^*}(p) + \sum_{f \in m_{v^*}(e \setminus \mathcal{G}_{sm})} s_e(f)$$

Let consider X as the topological Balaban index, with $s_{v^*} = 0$ and $w_{v^*} = s_e = 1$. Strength-weighted graphs form the foundation for distance-based topological indices. The cut approach is a highly useful tool for comparing various distance-based topological Balaban indices. The notion of the cut method is given and discussed.If X_i is the union of one or more sets in Y, then the partition $X=\{X_1,X_2,...,X_r\}$ of E(G) is coarser than $Y=\{Y_1,Y_2,...,Y_s\}$. The quotient graphs of G are accessible for the Wiener, Szeged, Edge, Edge-vertex, and Padmakar Ivan indexes.

4.Results

Distance based Topological indices of certain spider network

Definition 4.1

Assume two graphs, G_1 and G_2, each with two vertices ($n_1, n_2 \geq 2$). The G* is the edge transformation of G, and J(G) < J(G*) if G is the graph created by adding an edge between a vertex u∗ of G_1 and a vertex v∗ of G_2. The G is the graph formed by connecting u∗ of G_1 and v∗ of G_2 and adding an edge to u∗(v∗).

Theorem 4.2

Let $P = v_1, v_2, ...v_r$ be a route of length r −1 >=2, and G be a graph with n ≥ 2 vertices. The G* is edge transformation of G, and we have J(G) < J(G*) if G (resp. G*) is the graph created by finding a vertex v∗ of G and v^*_{k-1} in p.

Proof: Let,

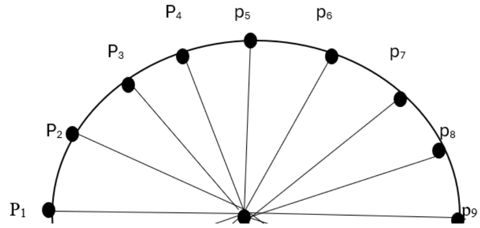


Fig.2: half of the spider graph (1^4, 2^3)

A Half of the spider network has two a + b vertices with degrees a and b, whereas the other half has degree 1 (assuming a ≥ b for symmetry). Deng established that J is a convex (continuous) function of x. To understand how this result might be expanded, we must first define a discrete convex function. A (discrete) function f is strictly convex if it holds for every $x_0 < x_1 < x_2$in its domain.

$$f(x_1) < \frac{x_2 - x_1}{x_2 - x_0} f(x_0) + \frac{x_1 - x_0}{x_2 - x_0} f(x_2)$$

The rule holds true even if the domain is a collection of numbers greater than or equal to b.

Theorem 4.3

The spider graph $J(1^{2t},2^t)$ has an integer I-Cordial labeling graph at the n-part 1-join.

Proof:

We establish the theorem by calculating the number of pieces added to the basic Spider graph by mathematical induction. We know that when a basic spider graph is linked to another basic spider network using a 1-join, an integer I Cordial Labeling graph is created. Assume the theory holds for n-1 parts, which means that an integer I cordial labeling graph is the 1-join of one n-1 spider graph part to another n-1 spider graph part.

Let us now demonstrate that the theory works for n parts, i.e., a 1-join of n spider graph parts to another n spider graph parts, with integer I representing a pleasant labeling graph. To form a k-part spider graph, we connect one part to the n-1 spider graph, resulting in n parts, and then merge with another n-1 spider part, which is also connected to one part. Because the n-1 and 1 portions are both integer I cordial labelings, it follows that merging two n-part spider graphs is integer I cordial labeling.

We may now construct a 2-join n-part spider network in a similar fashion, resulting in an odd number of vertices. In this scenario, M represents the number of joins that are connected to the Spider graph's n-part. Because the number of vertices that must be labeled is odd if M is even, we use the technique provided. If M is odd, the number of vertices to label is even, therefore we repeat the technique used to name the 1-join of the n-part of the spider network.

Example4.4

The treatment can be continued in this manner. The table below includes instances of the following. Figure 2's reference indicates that,

Table 1. Spider Graph SP(n^{2t}, 2^t) with number of Vertices and Edges

Type of 1- Join of Spider graph SP(1m,2t)	Number of Vertices	Number of Edges
$G(1^2,2^1)$	5	6
$G(1^3,2^2)$	7	8
$G(1^4,2^3)$	9	9
$G(1^5,2^4)$	14	13
and so on..	….	….

Corollary4.5

(i). Edge vertex Balaban index of BF(n)

$$Sz(G) = \sum_{e=uv \in E(G)} s_e n_u(e) n_v(e)$$

(ii). Padmakar Ivan index of BF(n) is calculated as follows

$$PI(BF(n)) = \sum_{i=1}^{n} 2 \times 2^{2n}(n+1)$$

Robust Fuzzy Topological Model (RFTM)

In this section to treat the edges as relationships (dependencies) between variables, and the nodes represent the variables themselves.Determine the variables (nodes in the fuzzy graph) and their relationships (edges). Use fuzzy membership functions to quantify uncertainties in these relationships. These might represent expert opinions, probabilities, or vague correlations.Assign fuzzy weights to edges based on the strength of relationships. The robust fuzzy variables in the regression equationis,

$$\tilde{y} = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_1 + \cdots + \tilde{\beta}_n \tilde{x}_n + \tilde{\epsilon}$$

Where $\widetilde{\beta_\iota}$ and $\tilde{\varepsilon}$ are the fuzzy coefficients and residuals.

Use a fuzzy linear regression Fuzzy Least Squares approach to estimate coefficients. That is Modify the least squares method to accommodate fuzziness in the dependent or independent variables.

Algorithm

(i). Extract regression features based on the fuzzy graph structure.

(ii). Use the fuzzy adjacency matrix to weight relationships in the regression model.

(iii). Incorporate fuzzy graph centrality measures or influence scores as predictors.

(iv). Build regression equations that reflect these weighted fuzzy relationships.

(v). Convert the fuzzy regression output back to crisp values using defuzzification methods (e.g., centroid, maximum membership).

(vi). Validate the model using error metrics adapted for fuzzy systems (e.g., fuzzy mean squared error).

5.Discussion

Experimental Study

All algorithms were implemented using MATLAB software. Images are used to assess the efficiency of various edge detection systems. Edge points are retrieved using several edge detectors, and edge features are identified by filling in one-pixel gaps. Table 2 summarizes the processing time required to detect edges in photos. Figure 3 depicts the edge shown in the images.

Algorithm: 5.1

- Input image.
- Guassian Smoothing.
- Grandient in x & y.
- Non-maximum suppression.
- Apply proposed RFTM.
- Hysteresis threshold(T) value.
- Fill small gaps in edge contours.
- Compare edges found using the procedure.

Table 2: Time taken for various edge detection operators

Image	**Processing Time (in Seconds)**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Wall image	0.514	0.494	0.475	0.383	0.366	0.355	0.331	0.279	0.265	0.254	0.241
Bacteria image	0.888	0.776	0.610	0.559	0.531	0.500	0.462	0.433	0.411	0.387	0.311

(1) Wang Brady **(2) Trajkovic Hedley** **(3) Robert** **(4) Prewitt**

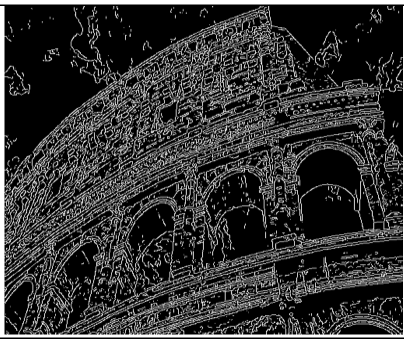
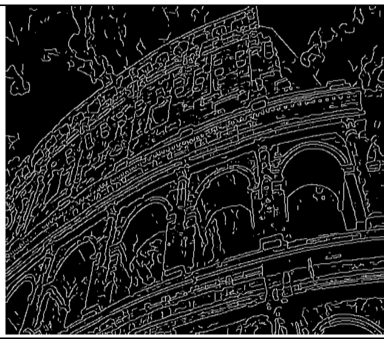
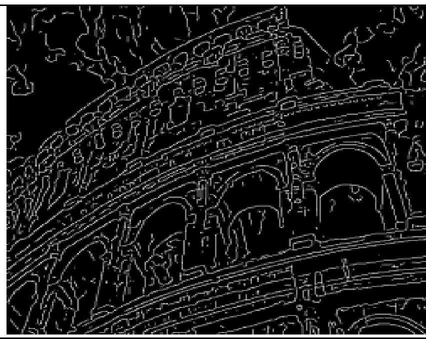
(5) Harris **(6) Susan** **(7) LOG** **(8) Zero Crossing**

(9) Canny **(10) Sobel** **(11) RFTM**

Table 2 demonstrates that the recommended RFTM outperforms the other edge detection approaches. Given that the recommended technique employs 36 pixels, the RFTM procedure's edge identification and curve extraction durations are significantly reduced. The data described here was simulated using the TLS MATLAB toll box. Depending on the sample size, the data is generated randomly. In addition, 2% of salt and pepper Gaussian noise is applied to the image, and the toolbox's threshold value is set to two.



(0) *(1)* *(2)*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	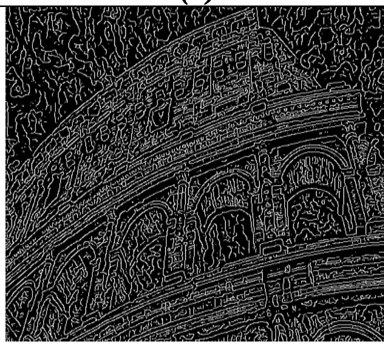	
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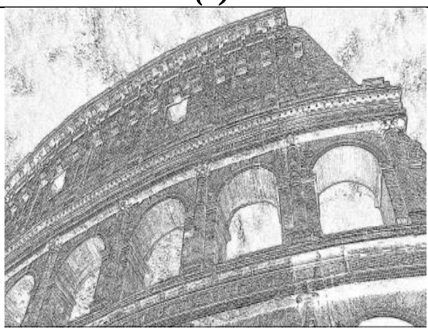
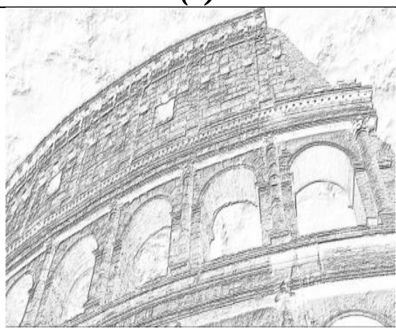
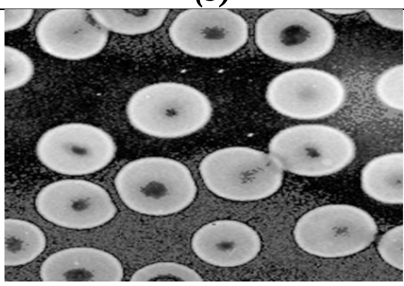
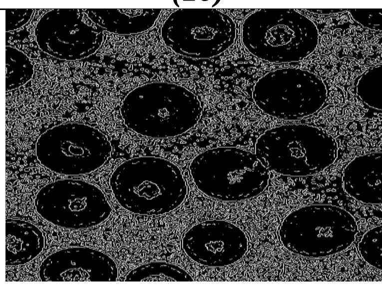
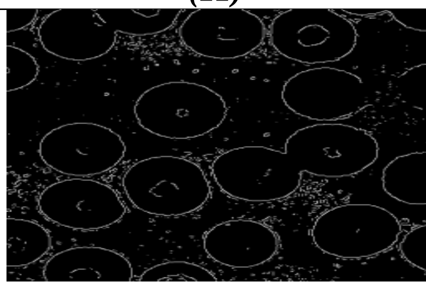
		
(9)	***(10)***	***(11)***
		
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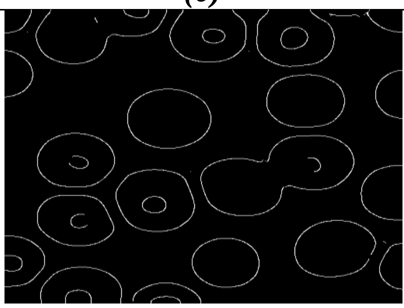
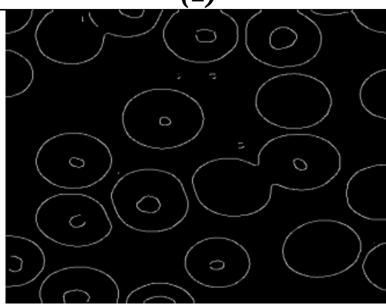
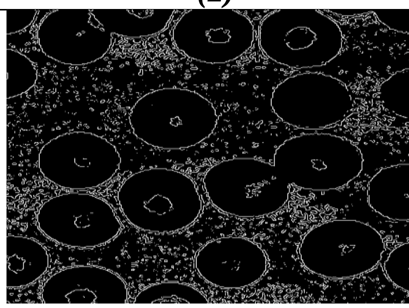
		

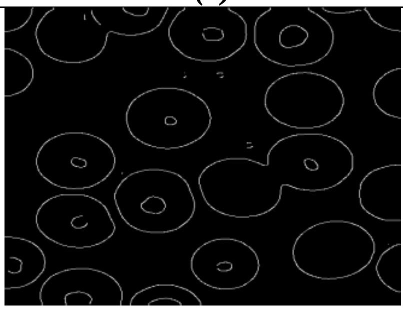
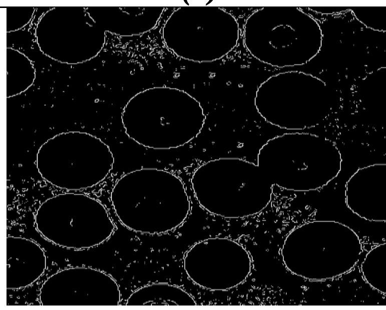
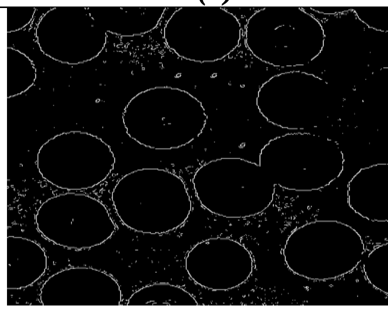
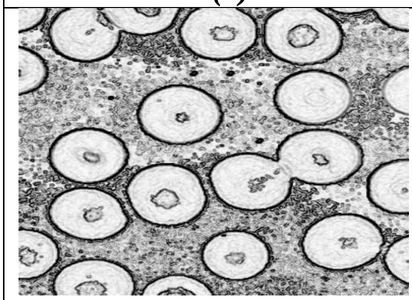
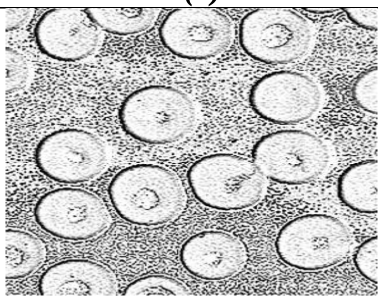
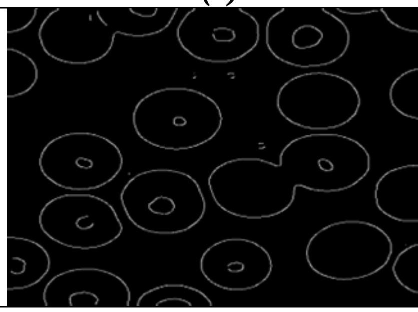
(3)	***(4)***	***(5)***
		
(6)	***(7)***	***(8)***
		
(9)	***(10)***	***(11)***

Figure 1.1: Edge detectedfor (0) original Image, (1) Wang Brady, (2) Trajkovic Hedley, (3) Robert,
(4) Prewitt, (5) Harris, (6) Susan, (7) LOG, (8) Zero Crossing, (9) Canny, (10) Sobel, (11) RFTM

Refrences

[1] M Arockiaraj, J Clement, and K Balasubramanian. Topological indices and their applications to circumcised donut benzenoid systems, keku lenes and drugs. Polycyclic Aromatic Compounds, 40(2):280–303, 2020.
[2] M Arockiaraj, J Clement, and N Tratnik. Mostar indices of carbon nanostructures and circumscribed donut benzenoid systems. International Journal of Quantum Chemistry, 119(24):e26043, 2019.
[3] J A Bondy, U S R Murty, et al. Graph theory with applications, volume 290. Macmillan London, 1976.
[4] Estrada, E., & Rodriguez-Velazquez, J. A. (2005). "Subgraph centrality and clustering in complex hyper-networks." Physical Review E, 71(5), 056103.
[5] Gutman, I., & Polansky, O. E. (1986). Mathematical Concepts in Organic Chemistry. Springer.
[6] Trinajstić, N. (1992). Chemical Graph Theory. CRC Press.
[7] Balaban, A. T. (1983). "Applications of graph theory in chemistry." Journal of Chemical Information and Computer Sciences, 23(3), 165-175.
[8] Randić, M. (1975). "On characterization of molecular branching." Journal of the American Chemical Society, 97(23), 6609-6615.
[9] Muthukrishnan,R and Ravi,J (2015), Image type-based Assessment of SIFT and FAST Algorithms, International Journal of Signal Processing, Image Processing and Pattern Recognition, 8(3), 211-216.
[10] Dobrynin, A. A., Entringer, R., & Gutman, I. (2001). "Wiener index of trees: Theory and applications." Acta ApplicandaeMathematicae, 66(3), 211-249.
[11] Diudea, M. V. (2001). QSPR/QSAR Studies by Molecular Descriptors. Nova Science Publishers.
[12] Estrada, E., & Gutman, I. (1996). "Topological indices based on the line graph of the molecular graph." Journal of Chemical Information and Computer Sciences, 36(4), 598-603.
[13] Hansen, P., &Mélot, H. (1999). "Variable neighborhood search for extremal graphs. 6. Analyzing bounds for the connectivity index." Journal of Chemical Information and Computer Sciences, 39(6), 1163-1169.
[14] S.Mohan Kumar, Venkatesh A(2017) ."Step –Stress and Truncated Acceptance Sampling Plan Model for the analysis of Vasopressin", International Journal of Pure and applied Mathematics, 117(6), 107 -114.
[15] R.Karthikeyan , S.Mohan kumar et.al.,(2025), "Viscous Dissipation effects on MHD free convective effects of unsteady flow over a Vertical Porous Plate with effects of Radiation" .